

Essays on Innovation and Contest Theory

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The Faculty of Business, Economics and Informatics of the University of Zurich hereby authorizes the printing of this dissertation, without indicating an opinion of the views expressed in the work.

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Part I

Dissertation Overview

Dissertation Overview

This dissertation consists of four separate chapters which are centered around two themes — innovation and contests. At first blush it may seem that the two themes are not related. However, a closer examination reveals that contests, as mechanisms, are often used to induce individuals and firms to innovate. The idea that we can design institutions in a way that induces some desirable behavior underlies all four chapters of this dissertation.

Chapters 1-3 examine questions related to how institutions should be designed in order to induce discovery of innovations (see Figure 1). It is difficult to overstate the importance of innovations on the evolution of the human standard of living, as essentially every aspect of human life has been made better by some innovation. But innovations are rarely products of mere serendipity. Rather, they are a response to the environment of the innovator, who seeks a way to make his labor easier or profits larger by innovating. As innovations become more complex and difficult, having the proper institutional environment that encourages innovative activities becomes more and more important.

The influence of institutions on innovative activity has been extensively studied by economists. However, for the most part the focus of the literature has been on inducing innovators to work harder, or inducing firms to invest more resources in innovation. This dissertation begins with an observation that discoveries are not merely the outcome of trying harder — often they are the result of attempting the right, and sometimes surprising, approach to solving the problem at hand. One implication of this is that we need to design our institutions not only with the goal of increasing the amount of resources invested in innovations, but also to make sure that the resources are invested in the right combination of approaches.

Chapter 1 considers the question of the R&D portfolio from the perspective of firms competing on a product market and seeking to innovate as a way of increasing their profits. It develops a model in which firms can invest in multiple research approaches and examines the duplication and variety of research in equilibrium. I show that an equilibrium always exists and characterize both the (unique) equilibrium R&D portfolio and the socially optimal portfolio. These results are then used for comparative statics and to derive policy implications.

The results in this chapter offer insights relevant to competition and innovation policy. This chapter shows that a merger will likely decrease the variety of research approaches, which is a question that has been asked in several merger cases (for example, in the blocked merger between Lockheed Martin and Northrop Grumman). Hence, a competition authority reviewing a merger proposal in an industry in which innovation is particularly important should take this negative effect into account. Comparing the market equilibrium with the socially optimal outcome, I show that firms competing on the product market will tend to overinvest in duplication of research approaches and underinvest in the variety of research approaches. This implies that innovation policy should be aimed at increasing the variety of research approaches that the firms

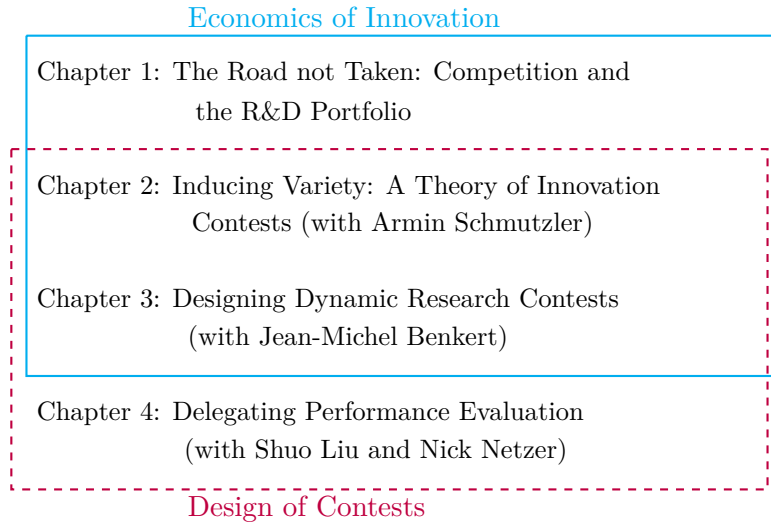


Figure 1: Overview of the main themes.

pursue. The chapter concludes by relaxing several important assumptions and shows that the main results are in general robust.

Chapter 2 continues with the examination of how institutions affect the choice of research approaches. We assume that there is a sponsor who is interested in obtaining an innovation. To stimulate research, the sponsor commits to an innovation contest. Examples of such innovation contests abound — from the 1714 Longitude Prize for accurate determination of the latitude of a ship at sea to the 2015 EU Horizon prize for a test to determine whether antibiotics should be prescribed to a patient with flu. In this chapter, Armin Schmutzler and I study the optimal contest design when the research approach of the participants matters. We are in particular interested in how contest design can be used to influence the variety of research approaches taken by the contest participants. The main result we obtain is that a bonus tournament — a tournament offering two prizes, a high ("bonus") prize when one participant significantly outperforms others, and a low prize when the best and the second-best solutions are of similar quality — is optimal. The intuition behind this result is simple. Since the high prize is only paid out in the case of significant difference in performance, the participants will seek to pursue those research approaches which are likely to succeed when the ones pursued by their competitors fail. This leads to diversification of research outcomes. By changing the difference between the low and the high prize, the contest designer can affect the level of diversification of research outcomes. In particular, the socially optimal level of diversification can be implemented, but it will not always be in the best interest of the contest sponsor to do so. We also examine how some commonly studied and used contests perform in our setting. A fixed prize tournament, where a fixed prize is paid to the winner of the tournament is not good for inducing diversity. In particular, when there are only two contestants there will be no diversity (and full duplication) in equilibrium. An auction, which is optimal in the setting of Che and Gale (2003), induces socially optimal diversity but at the cost of high payments to contestants. Since the payments are lower, a bonus tournament outperforms an auction. We show that our analysis can be generalized beyond the baseline model and discuss implications that go beyond innovation contests.

In Chapter 3, Jean-Michel Benkert and I continue the study of optimal design of innovation contests. The focus in this chapter is on the dynamic incentives to invest in costly effort. Once dynamics are taken into account a novel problem arises. Namely, when should the contest end? Taylor (1995) — which is the seminal paper on dynamic innovation contests — assumes that the contest ends after an exogenously fixed number of periods. However, the ending time could also be a design parameter. The longer the contest lasts, the more likely that a valuable innovation will be discovered. However, a long contest is also likely to lead to inefficient duplication of research costs. Of course, a contest designer could set the contest length that is optimal in expectation. In this chapter, we show that she can do better. Namely, we show that it is possible to implement a quality-contingent stopping rule, where the contest ends only once an innovation of sufficient quality has been discovered. We further show that the implementation result is robust, and provide sufficient conditions under which the optimal outcome is achieved. The mechanism that achieves this quality-contingent stopping rule is a dynamic prize profile. That is, the principal commits to pay out a prize to the winner which depends only on the period in which the prize is awarded. If the overall level of prizes is high enough, then the contestants will have an incentive to invest in innovation and to reveal the winning innovation as soon as they have it. The change in the prizes paid out from one period to the next provides the incentives to the principal to stop the contest as soon as the innovation of sufficient quality is discovered.

While Chapters 2 and 3 have examined the way contests should be designed, they do not tackle the question of whether some other feasible mechanism could do better than any contest. Surprisingly, there are very few results of this kind in the literature. Among other results, Chapter 4 provides a setting in which contests are actually optimal in the class of all feasible mechanisms. In this chapter, joint with Shuo Liu and Nick Netzer, we consider a problem of a principal who wants to incentivize agents to work. However, the principal does not observe the effort exerted by the agents but there is a reviewer who does. The principal then designs a remuneration scheme and delegates its implementation to the reviewer. Furthermore, we assume that the reviewer takes the payoff of agents into account when implementing the remuneration scheme and that the intensity of this bias is private information of the reviewer. While none of our results depend on the direction of reviewer's bias (that is, the reviewer can receive both positive and negative utility from agents' payoffs), our favored interpretation is that the reviewer's bias is positive. This is referred to as the leniency bias, since it makes the reviewer reluctant to punish the agents.

In this setting, a contest is always in the class of optimal mechanisms. The reason for this is as follows. The leniency bias makes the reviewer reluctant to punish shirking agents, which is necessary to sustain the incentives of the agents to work. The contest acts as the commitment to punishment — one of the agents has to receive the lowest prize, which acts as a punishment for shirkers. We further characterize the optimal contest and show that it features $n - 1$ equal prizes and one zero prize. The zero prize is used as the commitment to punishment. Next we show that the optimum can be implemented by familiar allocation rules: (modified) all-pay auctions, Tullock contests, and rank-order tournaments with both additive and multiplicative noise can all be used to implement the optimum. In particular, an all-pay auction with censoring implements

the optimum as the unique equilibrium. Our results offer one explanation for the widespread use of contests and in addition identify a set of conditions under which contests could perform better than institutions currently in use.

The common thread permeating this dissertation is the question of institutional design, with a particular focus on rules and institutions aimed at providing incentives for firms and individuals to engage in innovative activity. Chapter 1 focuses on competition policy as one such institutions. Chapters 2 and 3 examine the design of innovation contests. Chapter 4 examines the delegation of performance evaluation more broadly. While the models examined in this dissertation offer clear-cut answers, they should not be interpreted literally. As any theoretical exercise, this dissertation captures some factors while ignoring others. This should be kept in mind when interpreting these results and in particular in any attempt to apply them.

Part II

Research Papers

1 The Road not Taken: Competition and the R&D Portfolio¹

1.1 Introduction

In 1998, the U.S. Department of Justice blocked the proposed merger of Lockheed Martin and Northrop Grumman, the largest blocked merger in the U.S. history at the time. The merger would have reduced the number of firms supplying aircraft and electronic systems to the Department of Defense from three (including Boeing) to only two. According to Robinson (1999) and Rubinfeld and Hoven (2001), one of the main reasons why the Department of Justice, supported by the Department of Defense, opposed the merger was the concern that the merger would have had negative effects on innovation. However, the issue was not so much with the amount of funds invested in innovation, the bulk of which comes from the Department of Defense anyway (Rubinfeld and Hoven, 2001). Rather, the principal concern was that reducing the number of firms in the industry would reduce the diversity of approaches to innovation.

This article develops a model where the effects of such a merger on the variety of approaches to innovation and the amount of duplicative research can be studied explicitly. From society's point of view, higher variety of research projects being developed is desirable because it increases the probability that the innovation will be discovered. On the other hand, more duplication of research projects is also desirable because it implies stronger product market competition ex post and lower deadweight loss. The market R&D portfolio is a function which captures how many firms are investing in each of the possible projects, and the variety of approaches is the fraction of projects which are developed by at least one firm. Of course, both more variety and more duplication are costly. The main object of analysis will be the market R&D portfolio. The model will allow us to study how a change in the market structure will change the equilibrium R&D portfolio.

The main model assumes that there are N symmetric firms competing in a market. In the first stage, the firms can invest in innovation. There is a set of heterogeneous research projects and firms simultaneously choose the subset they wish to develop. The innovation is assumed to be drastic² and the discovery procedure is stochastic. All approaches are ex ante equally likely to be successful, but ex post only one approach will be successful. The approaches differ only in the cost needed to pursue them. There are no spillovers or patents. Each firm which invested in the successful approach receives the innovation whereas each firm that did not invest in the

¹A version of this paper is published as Letina, I. (2016), "The Road not Taken: Competition and the R&D Portfolio," *RAND Journal of Economics*, 47, 433–460.

²Innovation is drastic if whenever at least one firm innovates, firms without the innovation cannot compete. This assumption was introduced by Arrow (1962); see also Gilbert (2006). This assumption is not needed for the characterization of the equilibrium and is relaxed in Section 1.7.

successful project receives nothing from its research. In the second stage, the firms compete on the product market either with or without the innovation.

As all approaches are *ex ante* equally likely to be successful, the firms have an incentive to develop only the cheapest projects. However, the number of firms developing any given project also determines the number of firms which will compete on the product market with the new technology. Thus, when choosing which projects to develop the firms face a trade off — cheaper approaches cost less to develop but will in equilibrium attract more competitors. I show that an equilibrium of the investment game always exists and that the equilibrium market R&D portfolio is uniquely determined. I provide a simple characterization of the equilibrium R&D portfolio and show that it follows a step function — with more expensive approaches being developed by fewer firms.

The characterization of the R&D portfolio is then used to derive comparative statics. I show that a decrease in the number of firms weakly decreases the variety of approaches to innovation and also weakly decreases the amount of duplication. Hence, a merger leads to a weakly decreasing variety of approaches to innovation. A policy implication drawn from this analysis is that the competition authorities should take into account the negative effects of a merger on the variety of approaches to innovation, in part giving theoretical foundation to the concern expressed in the Lockheed-Northrop case. However, if a merger leads to efficiency gains, this result need not hold.

Next, I consider the effects of a change in the intensity of competition between firms. I define an increase in the intensity of competition as any exogenous change which decreases firm profits.³ An increase in the intensity of competition is shown to increase the variety of approaches to innovation and to decrease the amount of duplication in equilibrium. Thus, an increase in the intensity of product market competition leads to more specialized R&D portfolios. This illustrates why an increase in the intensity of competition can both increase and decrease the amount of resources invested in R&D — if the reduction in duplication of research efforts is greater than the increase in variety of research efforts, the total amount invested in R&D will decrease. If the opposite is true, the total amount invested in R&D will increase.

I provide a characterization of the socially optimal R&D portfolio and compare it with the market R&D portfolio. I derive the condition under which the market investment in the variety of research approaches is optimal, too low or too high. Similarly, I derive the condition under which the market duplication of research approaches is optimal, too low or too high. I show that in a large class of homogeneous goods models, the market will always underinvest in the variety of approaches to process innovation. This result implies that there is a role for government subsidies of R&D. Furthermore, it implies that the subsidies should be targeted at research projects with high development costs and high potential payoffs.

The main body of the article assumes that the innovation is drastic, firms are symmetric, have unlimited budgets and use only pure strategies. I consider the effects of relaxing these assumptions in turn and show that the equilibrium structure is in general robust. In particular, the assumption that the innovation is drastic is not necessary. In Section 1.7, I provide a characterization of the equilibrium portfolio without assuming drastic innovation and show that

³Similar approach is taken in Schmidt (1997) and Schmutzler (2013).

it is qualitatively similar to the equilibrium portfolio when the innovation is assumed to be drastic. However, comparative statics become significantly more complex. Section 1.7 shows that similar comparative static results can be obtained without the drastic innovation assumption if instead additional assumptions are imposed on the reduced form payoffs.

The outline of the article is as follows. In Section 1.2, a brief overview of the related literature is provided. Section 1.3 describes the model. The equilibrium is characterized in Section 1.4. Comparative statics are analyzed in Section 1.5. The socially optimal portfolio and its relation to the market portfolio are analyzed in Section 1.6. In Section 1.7, I relax a number of assumptions made in the main body of the article. Section 1.8 concludes. All proofs and additional extensions are in the appendix.

1.2 Related literature

This article contributes to the literature on the relationship between market structure and the incentives of the firms to invest in innovation. A large part of this literature studies the *amount of resources* that firms invest in R&D. Depending on the specifics of the model used, the literature finds that the competition in the marketplace can increase, decrease or have non-monotone effects on the amount invested in R&D. For surveys of this vast literature see Gilbert (2006), Sena (2004) and van Cayseele (1998). Vives (2008) and Schmutzler (2013) provide comprehensive studies for a range of market competition models and demand structures. Important contributions to this literature have been made from the endogenous growth literature, particularly from the models of *step-by-step innovations* (see for example Aghion et al. (2001) and Aghion et al. (2005)). This article contributes to this literature by providing a model which allows us to consider how the variety and duplication of approaches to research change as the market structure changes, and how it relates to the socially optimal amount of variety and duplication of research.

This article is more closely related to the part of the literature on competition and innovation that examines how competing firms choose some aspect of the research strategy. Bhattacharya and Mookherjee (1986) and Klette and de Meza (1986) consider a model where undertaking research is like drawing a random variable. The maximum realization of the random variables determines the winner of the race (winner takes all) but also both private and social payoff. Firms choose a parameter of the density function, which determines the variance and in some scenarios the correlation of the research output. This parameter is interpreted as a research strategy of the firm. Dasgupta and Maskin (1987) consider a similar model. Results obtained by these models depend on the assumptions made about the distribution of research outcomes, but in a large class of cases, firms undertake excessive risk (because firms care who wins the race, whereas society only cares about the best research output; however see also Cabral (1994) and Kwon (2010) who find that the market is biased against risky research). At the same time, if reducing correlation is costly, firms will choose research strategies that are too correlated, as firms will not internalize the benefit low correlation confers to its opponent when its own research output is low. In this setting, the firms can choose only one research project (i.e., each firm chooses one parameter of the density function), hence these models cannot examine the variety of research projects. At the same time, in these articles the choice of correlation of the outcomes is interpreted as a measure of duplication. In the present article, duplication of

research is literal — the firms can choose to pursue the same project.

Fershtman and Rubinstein (1997) study an interesting model where two players search for a treasure which is hidden in one box among a set of boxes. Each player has an endogenously chosen capacity determining the number of boxes he can examine in each period. The player that first discovers the treasure, keeps it. The research strategy is the choice of the boxes which will be examined in each period. In both this article and in Fershtman and Rubinstein (1997), the players can choose both the intensity and the location of search. However, the equilibrium predictions are different. The main result of Fershtman and Rubinstein (1997) is that the search will be completely random, whereas in this article pure strategy equilibria will always exist. The reason for this difference is that in Fershtman and Rubinstein (1997) the search continues until the treasure is found and in this article the search lasts only one period. When there are multiple search periods, there is an incentive for players to preempt their opponent by searching exactly those boxes that the opponents intends to search in the next period. This incentive destroys any equilibrium in which the search is not completely random. Clearly, this incentive does not exist when search lasts only one period, in which case the choice of boxes is driven by their equilibrium net expected value. In this sense, these articles are complementary. In addition, this article is concerned with how the research incentives are affected by the market structure, which is not studied in Fershtman and Rubinstein (1997).

Chatterjee and Evans (2004) present a dynamic model where two firms are searching for an innovation in a model of a hidden treasure. There are two possible research projects and only one can yield the innovation, with the winner take all feature. However, unlike Fershtman and Rubinstein (1997) and this article, developing the “right” project yields the innovation only with some exogenously given probability. They find that the amount of correlation between research of the two firms can be too high or too low depending on the nature of asymmetry between the two research paths. Akcigit and Liu (2016) also consider a setup with two firms and two possible avenues for research, one is more profitable (in expectation) but may result in a dead-end and another which always yields a less profitable innovation if it is researched long enough. As opposed to Chatterjee and Evans (2004) they assume that firms cannot observe which research path their competitor is pursuing and they find that firms duplicate dead-end research and at the same time leave the risky research path too early. In contrast to the present article, this strand of literature assumes that firms can research only one project at a time, so the question of the choice of variety of research projects and the amount of duplicative research does not arise. This is, however, the main focus of the present article.

Most closely related to the present article is the literature on multiproject innovation, which has been studied by Sah and Stiglitz (1987) and in the related work by Reynolds and Isaac (1992) and Farrell et al. (2003). Sah and Stiglitz (1987) assume that all projects are identical. The probability of success of any individual project depends only on the effort invested in this project and is independent of anything that might be happening with other projects. Using this setting and the Bertrand model of the product market, Sah and Stiglitz (1987) show that the number of projects is invariant to the number of firms in the market, a result they refer to as the “strong invariance result.” Reynolds and Isaac (1992) and Farrell et al. (2003) explore this setting further and show that the invariance result is sensitive to type of product market

competition. In particular, they show that the invariance result does not hold under Cournot competition.

The main difference between this article and the literature in tradition of Sah and Stiglitz (1987) is that here projects are assumed to be heterogeneous and that more than one firm can invest in the same project. Hence, firms need to decide which projects to develop and have to do so in a strategic manner, keeping in mind which projects their competitors are developing. In this article, R&D portfolio is the main object of interest, whereas in the Sah and Stiglitz (1987) tradition it does not appear at all. There, projects are identical and it is immaterial which projects firms or their opponents develop. Thus, the model of Sah and Stiglitz (1987) does not capture the effects of variety of projects or the duplication of projects which is the main focus of analysis here.

1.3 A model of stochastic multiproject innovation

There are N symmetric firms⁴ that compete in the pre-innovation market and that can invest in innovation. There is a continuum of research projects Ω , but only one project $\hat{j} \in \Omega$ leads to the innovation.⁵ We can normalize the set of possible projects to the unit interval, that is $\Omega = [0, 1)$. I assume that the successful project is drawn from the uniform distribution over the set $[0, 1)$. Furthermore, each project has a fixed cost of development. Investing less than this cost means that firm will fail to develop the project and investing more will not improve the probability of the project being successful. In essence, the innovation mechanism is a lottery — developing different projects is akin to buying lottery tickets, the more lottery tickets you have the higher the probability you will win, but offering to pay more for a ticket will not increase its chances of winning. This fixed cost, fixed probability mechanism is similar to the one developed in Quirnbach (1993), the difference being that here firms can invest in multiple projects.

The projects are assumed to differ in terms of the investment cost needed to develop them. Denote the cost of developing project $j \in [0, 1)$ as $C(j)$. We can view C as a function such that $C : [0, 1) \rightarrow \mathbb{R}^+$. I assume that C is continuous, differentiable and strictly increasing. The fact that the function C is increasing is simply a matter of ordering the projects j in the right way, strictness is assumed so that marginal reasoning will yield unique results. Continuity is assumed to make the problem more tractable. Furthermore, assume $\lim_{j \rightarrow 1} C(j) = \infty$. As rewards from innovation are finite, this assumption ensures that firms will not want to invest in all possible projects. No exogenous restrictions are placed on the research budgets of firms, except in the Appendix A.2.3, which studies the consequences of limited research budgets and costly financing of research.

There are two possible levels of technology — old and new. The new technology is available only to the firms which invested in the successful innovation project, whereas the old technology is available to all firms. Let $n \leq N$ be the number of firms which developed the new technology. Denote with $R(n, N)$ the payoff of a firm with the new technology, where n is the number of firms with the new technology and N is the total number of firms. Analogously, denote with

⁴Asymmetric firms are studied in Section 1.7.

⁵The stochastic mechanism used to model innovation is adapted from Acemoglu and Zilibotti (1997).

$r(n, N)$ the profits of a firm with the old technology. The difference between process and product innovations is not explicitly modeled. As long as the product market payoffs can be expressed in terms of the reward functions, the present model can be used to study both types of innovation.

Next, I list assumptions that will be used in the article. However, note that only Assumption 1.1 is used throughout.

Assumption 1.1 (Non-increasing reward to subsequent innovators).

For all N and $n \in \{1, \dots, N-1\}$ it holds:

$$R(n, N) - r(n-1, N) \geq R(n+1, N) - r(n, N).$$

This assumption implies that the gain from innovation does not increase as the number of innovators increases. It captures the intuition that a firm prefers that its competitors do not innovate. Thus innovations are strategic substitutes. Although intuitive, this assumption needs to be checked for each model of product market competition. The consequences of relaxing this assumption will be considered in Section 1.7.

I assume that the innovation is drastic, in the sense that if there is at least one firm which has successfully developed the innovation, all firms which do not have the innovation cannot compete. That is, the laggards receive a payoff of zero and do not exert competitive pressure on the firms which have successfully innovated. For process innovations, this implies that the price of a monopolist with the innovation is below the marginal cost of any firm without the innovation. For product innovation this implies that the old product is made obsolete and it cannot be sold on the market. This assumption will be relaxed in a Section 1.7, where the equilibrium will be characterized for non-drastic innovations and robustness of comparative static effects will be discussed. In the notation used here we have:

Assumption 1.2 (Drastic innovation).

For all n, N and N' such that $1 \leq n \leq N \leq N'$ it holds: (i) $r(n, N) = 0$ and (ii) $R(n, N) = R(n, N')$.

Expression (i) ensures that laggards have zero profits and (ii) ensures that laggards do not exert competitive pressure on the innovators. Under Assumption 1.2, $R(n, N)$ is constant for any N , so from now on just $R(n)$ will be used to indicate the payoff of an innovator when there are n innovators. Furthermore, if Assumption 1.2 holds then Assumption 1.1 simplifies to the following two conditions: $R(n) \geq R(n+1)$ for all $n \geq 1$ and $R(1) - r(0, N) \geq R(2)$. The first expression states that the payoff per innovator weakly decreases as the number of innovators increases. The second expression states that the incentives of a prospective monopolist are greater than those of a single innovator when two firms innovate.

Assumption 1.3. For every N it holds: $r(0, N) \geq r(0, N+1)$.

This assumption states that as the number of firms which are active in the pre-innovation market increases, the profits of each individual firm do not increase. The intuition is simple: the additional firm will either not be competitive and have no effect on the profits of other firms, or it will put competitive pressure on other firms and decrease their profits, but it cannot increase their profits.

The unmodeled product market game that determines payoffs to the firms, $R(n, N)$ and $r(n, N)$, also determines the consumer surplus. This consumer surplus is the result of the competition among N firms, n of which have the new technology, and who face some demand curve on the product market. Denote this consumer surplus in reduced form with $CS(n, N)$. Then, the social welfare, when there are n innovators, is the sum of consumer and producer surplus: $W(n) = CS(n, N) + nR(n, N) + (N - n)r(n, N)$.⁶ That is, $W(0)$ denotes the welfare without the innovation, $W(1)$ denotes the welfare when there is only one firm with the innovation, and so on.

Assumption 1.4 (Non-increasing welfare returns).

For every $n \in \{1, \dots, N - 1\}$ it holds $W(n) - W(n - 1) \geq W(n + 1) - W(n)$.

Each firm is assumed to be risk neutral and to maximize its expected profits. Profit maximization requires that firms either invest zero in a project or exactly the amount that is required to open the project. Thus, we can identify the strategy of a firm simply by the set of the projects in which it invests. Denote the strategy of a firm i with I_i and call it the investment plan of firm i . In principle, I_i could be any measurable subset of the unit interval or the empty set. To simplify exposition, assume that unless it is empty, the set I_i consists only of a finite number of intervals, each closed from below and open from above.⁷ Formally, the strategy space of firm i is the set \mathcal{I}_i where:

$$\mathcal{I}_i := \left\{ I_i \subset [0, 1) : I_i = \bigcup_{k=1}^{\bar{k}} [a_k, b_k) \right\} \cup \{\emptyset\},$$

for $\bar{k} \in \mathbb{N}$ and $0 \leq a_k < b_k < 1$ for all $k \in \{1, \dots, \bar{k}\}$.

In particular, note that this assumption ensures that the investment plan will not contain any isolated zero-mass points.

Let $I = [I_1, \dots, I_N]$ be a vector of investment plans of all N firms. Define the function indicating the number of firms investing in a project, given a vector of investment plans I , as $n(j, I) : [0, 1) \rightarrow \mathbb{N}^0$ as:

$$n(j, I) = \sum_{i=1}^N \mathbf{1}(j \in I_i),$$

where $\mathbf{1}(\cdot)$ is the indicator function.

Let $I_i^c := [0, 1) \setminus I_i$. The expected profit of a firm i is then

$$\pi_i(I) = - \int_{I_i} C(j) dj + \int_{I_i} R(n(j, I)) dj + \int_{I_i^c} r(n(j, I), N) dj. \quad (1.1)$$

The first part of the equation above represents the investment costs of firm i , the second part gives the expected profits from the new technology, whereas the third part gives the expected profits from the old technology. By Assumption 1.2, $r(n(j, I), N) = 0$ whenever $n(j, I) > 0$.

⁶We can suppress the dependence of W on N as N will be fixed whenever welfare is analyzed.

⁷Because adding or removing zero-mass points does not change the payoff of any of the firms, allowing I_i to be general would mean that all statements regarding the properties of the equilibrium would have to be qualified by "almost everywhere". This assumption does not affect the mechanics of the model. For any measurable investment plan I_i which does not satisfy the assumption above, there always exists plan I'_i which does satisfy the assumption and only differs from I_i by zero-mass points, hence delivers the same payoff to all firms.

However, it will be positive whenever $n(j, I) = 0$, which will occur in equilibrium with positive probability.⁸

When $N = 1$, that is, when there is a monopolist in the market, the above becomes a pure maximization problem. When there are more firms in the market we have to consider the effects of strategic interaction among firms. Specifically, $n(j, I)$ depends on the actions of other firms and thus the expected profit of one firm depends on the actions of other firms.

Finally, assume that investment in innovation is profitable. That is $R(1) - r(0, N) > C(0)$. This assumption guarantees positive investments in the equilibrium. If this assumption was not met, even the monopolist's return on the investment in the cheapest project would not justify its cost. As $C(j)$ is strictly increasing and rewards are non-increasing in n , then no project could be profitable. Thus, if this assumption failed there would be a simple equilibrium in which firms did not invest at all.

The model is developed and analyzed in general terms. In the appendix A.1.10, I provide an example with three firms, process innovation and Cournot competition, which shows how the general model can be applied to a more specific setting. In the example, I analyze the effects of a merger, a switch from Cournot to Bertrand competition as well as compare the market portfolio of R&D projects with the socially optimal portfolio.

1.4 Equilibrium

The vector of investment plans summarizes all decisions of all firms that are relevant for this problem. A vector of investment plans I^* is a pure strategy equilibrium (PSE) if no firm can increase its expected profit by unilaterally choosing an alternative investment plan I'_i . That is I^* is a pure strategy equilibrium if, for any firm i , there does not exist an investment plan I'_i such that $\pi_i(I'_i, I^*_{-i}) > \pi_i(I^*)$. In the main text, I will only consider pure strategy equilibria. Mixed strategy equilibria exist and are considered in a special case in the Appendix A.2.2. As it is shown there, the insights from pure strategy equilibria, both in terms of the structure of the equilibrium and the comparative static effect, are robust.

Proposition 1.1 (Existence, non-uniqueness and equivalence of equilibria).

Suppose that Assumption 1.1 holds. Then:

1. *A pure strategy equilibrium always exists.*
2. *If I^* is a PSE and $0 < n(j, I^*) < N$ for some $j \in [0, 1)$, then infinitely many PSE exist.*
3. *If there are multiple PSE they all result in the same market portfolio of research projects. That is, if I_1^* and I_2^* are PSE investment plans, then $n(j, I_1^*) = n(j, I_2^*)$ for all $j \in [0, 1)$. Furthermore, if I_1^* is a PSE then any investment plan I_3^* such that $n(j, I_1^*) = n(j, I_3^*)$ for all $j \in [0, 1)$ is also a PSE.*

An equilibrium in pure strategies will always exist. However, typically there will also exist infinitely many equilibria. The proof of statement 2 in Proposition 1.1 (see the Appendix A.1.1)

⁸Furthermore, the magnitude of $r(0, N)$ will determine the strength of the Arrow replacement effect, which is crucial for the equilibrium variety of research projects.

reveals the nature of the multiplicity. In equilibrium, identities of firms investing in any given project are in general not determined, only the number of firms investing is determined. Only when either all firms invest in a project or no firm invests in a project, we can infer the behavior of individual firms. Thus, when $0 < n(j, I^*) < N$ for some $j \in [0, 1]$, there are projects for which the identities of firms investing are not determined and as there is an infinite number of ways to assign investments to firms, there must be infinitely many equilibria.

Statement 3 of Proposition 1.1 clarifies this point further. It states that every equilibrium induces the same market portfolio of research projects — that is in every equilibrium the set of developed projects will be the same and the number of firms investing in each project will be the same. Thus, although there is a multiplicity of equilibria, the equilibrium market portfolio is unique. As firms are identical, welfare does not depend on the identity of firms doing research. From the social welfare perspective, any two equilibria are equivalent.

Furthermore, not only do all equilibria induce the same market portfolio of research projects, but any investment that induces the equilibrium portfolio is itself an equilibrium. The intuition for this result is straightforward — the profitability of any research project depends only on the cost of the project and the number of competitors who are investing in the same project. In particular, it does not depend on any other investment that the firm or its competitors may be making. Hence, if in an equilibrium all profitable investments are exhausted and no unprofitable investments are made, then any other investment plan that prescribes the same investment portfolio in the same manner exhausts all profitable investments and has no superfluous investments.

Statement 3 of Proposition 1.1 implies that if I^* is an equilibrium then the function $n(j, I^*)$ fully characterizes the equilibrium portfolio of research projects. As $n(j, \cdot)$ is the same for any equilibrium, we can denote the function characterizing the equilibrium portfolio of research projects as $n^*(j)$. Using the equilibrium constructed in the proof of statement one of Proposition 1.1 and applying Assumption 1.2 yields the following result.

Proposition 1.2 (Characterization of equilibrium portfolio).

Suppose that Assumptions 1.1 and 1.2 hold. Denote with m the maximum number of firms investing in any project:

$$m = \max_{\{1, \dots, N\}} n$$

$$s.t. \ R(n) - r(n-1, N) - C(0) > 0$$

and with α_k for $k \in \{1, 2, \dots, m\}$ the most expensive project in which k firms can profitably invest. That is:

$$\begin{aligned} R(1) - r(0, N) - C(\alpha_1) &= 0 \\ R(2) - C(\alpha_2) &= 0 \\ &\vdots \\ R(m) - C(\alpha_m) &= 0. \end{aligned}$$

Let $\alpha_{m+1} = 0$ and $\alpha_0 = 1$. Then the PSE portfolio $n^(j)$ is given by*

$$n^*(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}, \alpha_k).$$

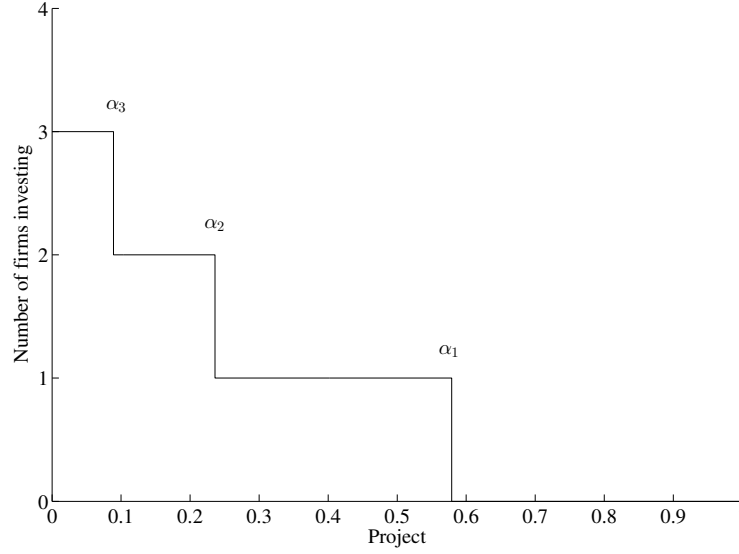


Figure 1.1: Equilibrium market portfolio.

An illustration of the equilibrium market portfolio for $N = 3$ and a process innovation in a Cournot market (which is the example from the appendix A.1.10) is provided in Figure 1.1. Here, $m = 3$ represents the maximum number of firms that can profitably invest in any project. Because project 0 is by assumption the cheapest to develop, then m firms will invest in this project. Each point α_k is constructed so that, at the margin, if k firms invested the profit from investment would be zero. As $C(j)$ is assumed to be strictly increasing, then at any point $j > \alpha_k$ strictly fewer than k firms can profitably invest. As rewards are finite and costs to innovation approach infinity as $j \rightarrow 1$, values $\alpha_1, \alpha_2, \dots, \alpha_m$ always exist. Furthermore, as $C(j)$ is increasing and by Assumption 1.1 the rewards to innovation are non-increasing it is easy to see that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_m$.⁹ From this observation it follows directly that the function $n^*(j)$ is weakly decreasing.

It is interesting to note that the present model would be equivalent to the level-of-investment models if (i) the successful project was drawn for each firm separately and (ii) the draws were independent. In such a setting, a firm would have no incentive to choose more expensive R&D projects. Thus, in equilibrium, all firms would invest in some interval of R&D projects $[0, j)$, where investing in a marginal project increases the probability of discovering the innovation at the marginal cost $C(j)$. In such a setting duplication of research and variety of research would be meaningless.

The set of all projects the market invests in is $[0, \alpha_1)$. Thus, I will refer to α_1 as the *variety of research projects* undertaken. I will say that the variety of research projects increases if α_1 increases. The probability that the market develops an innovation is equal to α_1 . Hence an increase in the variety of research projects implies an increase in the probability that the market will develop an innovation. The function $n^*(j)$ captures the number of firms investing in any given project j in equilibrium. Hence, I will refer to the number $n^*(j)$ as the market *amount of*

⁹Note that the inequality is weak (because the inequality in Assumption 1.1 is weak), so that it might happen for some $k \leq m$ that $\alpha_k = \alpha_{k+1}$. In this case, define $[\alpha_{k+1}, \alpha_k) = \emptyset$. Thus there will be no project that exactly k firms will develop.

duplication of project j .

1.5 Comparative statics

In this section, I will study how the market portfolio of research projects changes as the market structure changes. In particular, I will look at how a change in the number of active firms in the market and the intensity of competition among them affects the market portfolio of research projects. As can be seen from Proposition 1.2, the equilibrium portfolio is characterized by the maximum number of firms m investing in any project and the k -firm frontiers α_k , for $k \in \{1, \dots, m\}$. I will analyze how a change in N and a change in the intensity of competition affect these variables.

Change in the number of firms

Consider first the case where the number of active firms in the market changes, but all other characteristics of the market remain the same.

Proposition 1.3 (Increase in the number of firms).

Suppose that Assumptions 1.1, 1.2, and 1.3 hold. Let the number of firms in the pre-innovation market increase from N to N' so that the PSE investment plan changes from I to I' .

1. *In PSE, the variety of projects developed and the probability of developing an innovation weakly increases, that is $\alpha_1 \leq \alpha'_1$.*
2. *The maximum number of firms investing also increases, that is $m \leq m'$.*
3. *Apart from the increase in variety of projects developed and in the maximum number of firms investing, the PSE portfolio remains the same. That is, $n(j, I) = n(j, I')$ for all $j \in [0, 1) \setminus \{[0, \alpha'_{m'}] \cup [\alpha_1, \alpha'_1]\}$.*

If a firm innovates, it replaces its profits without the innovation with the profits with the innovation. Thus, holding everything else equal, the larger the pre-innovation profits of firms, the weaker its incentive to innovate. The increase in the variety of developed projects is driven solely by the Arrow replacement effect. In this setting, the firm investing near α_1 replaces $r(0, N)$ with $R(1)$. As $r(0, N) \geq r(0, N + 1)$, the Arrow replacement effect is weaker when there are $N + 1$ firms in the market. Consequently, firms attempt to escape the competition by investing in more expensive research projects than before and the variety of developed projects increases. This is equivalent to saying that the probability of discovering an innovation increases.

An increase in the number of active firms weakens the Arrow replacement effect both in this model and in the usual level-of-investment models. A difference, however, arises in the effect on the ex post profits of firms. Here the firms are free to choose in which projects to invest. The number of firms investing in any given project, and hence the ex post number of competitors, is endogenously determined. Following Vives (2008), call the reduction in innovation incentives due to competition ex post as the Schumpeter effect. Then, the Schumpeter effect in this model does not change as the number of ex ante active firms changes (except in those cases where the number of firms ex post was limited by the number of active firms). This leads to the clear effect

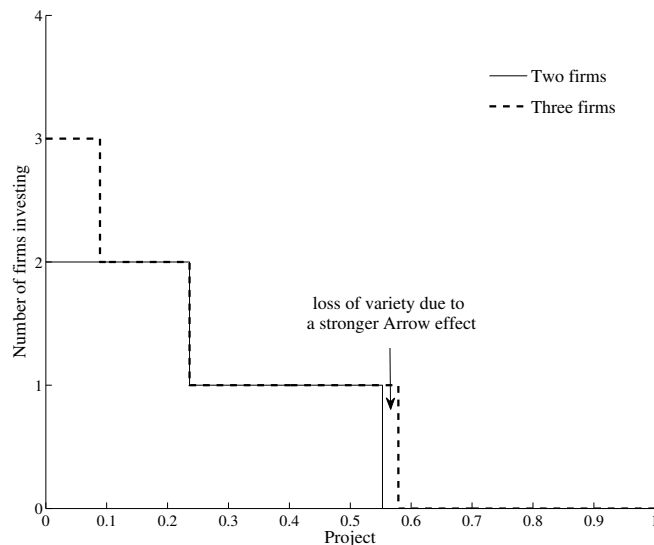


Figure 1.2: Merger in a 3-firm Cournot market.

of an increase in the number of firms on the variety of projects developed, as only one firm will invest in the most expensive projects.

Consider in this context the invariance result of Sah and Stiglitz (1987), which states that the number of research projects is invariant to the number of firms in the market. The invariance can only hold if $r(0, N) = r(0, N + 1)$, that is, only if the Arrow replacement effect is constant. Clearly this will hold under homogeneous goods Bertrand competition as Sah and Stiglitz (1987) have originally assumed, because $r(0, N) = 0$ for any $N \geq 2$. Conversely, it will not hold (in general) under Cournot competition as $r(0, N)$ will be decreasing in N , which is in line with the results derived in Reynolds and Isaac (1992) and Farrell et al. (2003).

Merger analysis

One implication of this result is that a merger in an imperfectly competitive industry will potentially lead to a loss of variety of approaches to innovation (see Figure 1.2). Thus, competition authorities should take this loss of variety of approaches to innovation into account when reviewing merger cases, especially if innovation is important in the industry, as it was in the proposed Lockheed-Northrop merger.¹⁰ As the loss of the variety of approaches to innovation is driven by the Arrow replacement effect, the magnitude of the loss of variety will be proportional to the increase in profits (in the market without the innovation) due to the merger.

This result depends on the assumption that the merger merely reduces the number of active firms in the industry, without changing the production or the innovation cost functions of the merged firms.¹¹ However, if the merged firm is more efficient than the individual firms, then this efficiency gain can outweigh the anti-competitive effects of the merger. As a matter of fact, U.S. Department of Justice and the Federal Trade Commission (2010) *Horizontal Merger Guidelines*

¹⁰For details see Robinson (1999) and Rubinfeld and Hoven (2001).

¹¹I would like to thank an anonymous referee for this point.

explicitly recognize this efficiency defense when analyzing the effects of mergers on innovation.¹²

The result in Proposition 1.3 shows that a merger can lead to a decrease in the variety of approaches to innovation. Two extensions I develop in the Appendix (Section A.2.1) show how this result can be overturned if the merger in question leads to efficiency gains. The first extension (Proposition A.1) supposes that the merged firm can invest in innovation at a lower cost than its competitors. Importantly, the efficiency is not dependent on the identity of the merging firms. Proposition A.1 shows that, for a general specification of cost efficiencies, a pure strategy equilibrium exists, the variety of approaches developed in equilibrium is uniquely determined and the variety does not decrease in the post-merger market if the efficiency is large enough. The second extension (Proposition A.2) considers an alternative form of efficiency gains. There, the efficiencies depend on the identities of merging firms, so that each merger reduces the cost of innovation in a merger-specific interval of approaches to innovation. Proposition A.2 shows that also in this setting a pure strategy equilibrium exists and the variety of approaches developed in equilibrium is uniquely determined. However, in this case it is not sufficient for the efficiency gains to be large enough. In order for such a merger to not decrease the variety of approaches to innovation, the merger has to be of the right kind — namely, the efficiency gains must affect those approaches to innovation which would not have been developed in the absence of the efficiency gains.

These results suggest that a merger in a highly innovative but imperfectly competitive industry might lead to a decrease in the variety of approaches to innovation, as has been suggested in the Lockheed-Northrop case. Thus, the competition authorities should take this potential effect into account when reviewing merger applications. At the same time, if a merger would lead to efficiency gains which are large enough and of the right kind, the merger need not decrease variety and could even lead to an increase in the variety of approaches to innovation. Thus, an efficiency defense should be considered also in the case where the effect of the merger on variety of approaches to innovation is a cause for concern.

Change in the intensity of competition

The competitive structure of the market is not only determined by the number of firms which are active in the market, but also by the intensity of competition among firms. Suppose that there are two sets of reward functions $\{R, r\}$ and $\{R', r'\}$ such that $R(n) > R'(n)$ for every $n > 1$, $R(1) = R'(1)$ and $r(0, N) > r'(0, N)$. Then we can interpret the move from $\{R, r\}$ to $\{R', r'\}$ as an increase in the intensity of competition. Most standard examples of an increase in the intensity of competition correspond to this definition. In particular, in the appendix A.1.10 I will consider a move from Cournot to Bertrand type of competition, but models of differentiated Cournot/Bertrand also correspond to this definition. The next result considers the effect of an increase in the intensity of competition on the market R&D portfolio.

Proposition 1.4 (Increase in the intensity of competition).

Suppose that Assumptions 1.1 and 1.2 hold. Let $N \geq 2$ and suppose the intensity of competition increases so that the PSE investment plan changes from I to I' . Then the variety of research

¹²“When evaluating the effects of a merger on innovation, the Agencies consider the ability of the merged firm to conduct research or development more effectively.” See Section 10 of the *Guidelines*.

projects undertaken and the probability of discovering the innovation increase. That is $\alpha_1 < \alpha'_1$. The amount of duplication of research decreases. That is for each j such that $n(j, I) \geq 2$ we have $n(j, I) \geq n(j, I')$ with $n(j, I) > n(j, I')$ for at least some projects.

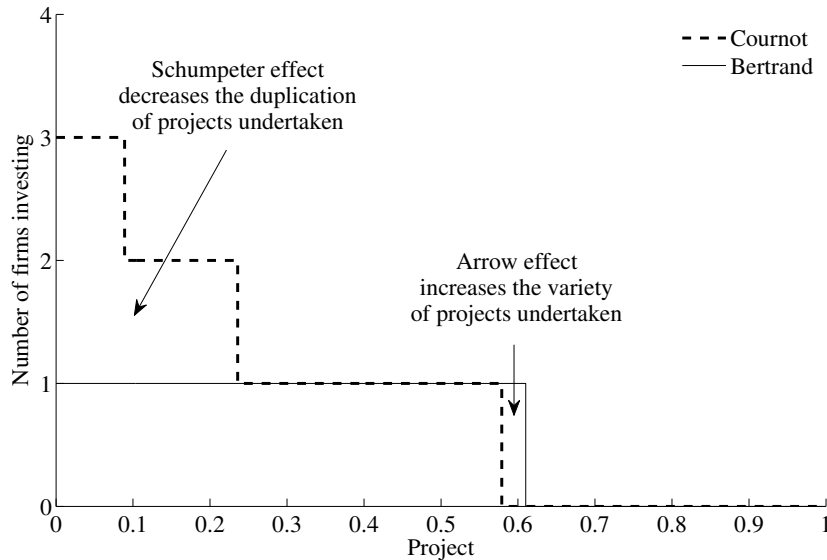


Figure 1.3: An illustration of an increase in intensity of competition.

An increase in the intensity of competition decreases the profits firms receive if no firm successfully innovates, thereby weakening the Arrow replacement effect and leading to an increase in the variety of developed research projects. On the other hand, it also decreases payoffs to firms if there are multiple innovators, leading to a (weakly) decreasing number of firms investing in duplicative research projects. An increase in the intensity of competition “flattens out” the equilibrium research portfolio, reducing the duplication of costs (see Figure 1.3). However, it is not clear that an increase in the intensity of competition will lead to higher social welfare. On the one hand, duplication of costs is reduced and the variety of research projects is increased. On the other, less duplication of costs also implies fewer firms (though competing more vigorously!) in the product market leading to a possible efficiency loss. Which effect prevails will depend on the exact specification of the product market competition and the demand function.

An increase in the number of firms is sometimes used as a way to model an increase in the intensity of competition. The preceding results highlight the difference between an increase in the number of firms and an increase in the intensity of competition as defined here. An increase in the intensity of competition reduces firm profits whenever there are multiple firms competing. That is, it reduces firm profits both ex ante and ex post. The number of firms, due to the endogeneity of the ex post market structure, affects firm profits only ex ante. Thus, the Schumpeter effect is present only in the case of an increase in the intensity of competition and not in the case of an increase in the number of firms.

1.6 Optimal portfolio

There are several reasons to suspect that a market R&D portfolio will not be optimal. When the innovator cannot appropriate the entire surplus because a part of the surplus is captured by the consumers, the incentive to innovate may be too low. On the other hand, if innovation enables firms to become more competitive in the market and steal business from their competitors, the incentive to innovate may be too high.¹³ However, as this article argues, looking solely at the levels of investment in innovation is misleading. Rather, the question to be posed is whether the market invests in the optimal variety of projects and whether it optimally duplicates projects. That is, the question is how the market R&D portfolio compares to the socially optimal portfolio.

The approach here is to ask what is the R&D portfolio that maximizes the expected social welfare. That is, the social planner can determine the R&D portfolio, but given the portfolio the firms will be profit maximizing. In particular, firms do not share the results of research, so some duplication of research will be optimal, as duplication of the successful project implies higher product market efficiency *ex post*.

Recall that $W(n)$ denotes the social welfare generated by the product market if there are n firms with the new technology, for every $n \leq N$. That is, $W(0)$ is welfare if no firm has successfully innovated and $W(N)$ is welfare if all firms have the new technology. The welfare is the sum of consumer surplus and producer surplus, that is $W(n) = CS(n) + nR(n, N) + (N - n)r(n, N)$.

Analogously to Proposition 1.2, the optimal portfolio is characterized:

Proposition 1.5 (Characterization of the optimal portfolio).

Suppose that Assumption 1.4 holds. Denote with m^o the optimal number of firms developing the least expensive project:

$$\begin{aligned} m^o &= \max_{\{1, \dots, N\}} n \\ \text{s.t. } W(n) - W(n-1) - C(0) &> 0 \end{aligned}$$

and with α_k^o for $k \in \{1, 2, \dots, m\}$ the most expensive project in which at most k firms can optimally invest. That is:

$$\begin{aligned} W(1) - W(0) - C(\alpha_1^o) &= \\ W(2) - W(1) - C(\alpha_2^o) &= \\ &\vdots \\ W(m^o) - W(m^o - 1) - C(\alpha_{m^o}^o) &= 0. \end{aligned}$$

Let $\alpha_{m+1}^o = 0$ and $\alpha_0^o = 1$. Then the optimal portfolio $n^o(j)$ is given by

$$n^o(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}^o, \alpha_k^o].$$

¹³For an example of the under- and over-investment in innovation due to the two effects outlined here see Bester and Petrakis (1993).

Market investment in variety

It is now possible to directly compare the market R&D portfolio with the optimal portfolio. In this way it is possible to identify if and how the market portfolio differs from the optimum and to suggest a way in which a policy intervention can improve the market outcome.

The net externality from investing in marginal variety (a research project that is not developed by any other firm) is given by:

$$\sigma = -(N - 1)r(0, N) + [CS(1) - CS(0)].$$

The first expression captures the negative externality imposed on the competitors of the firms making the marginal investment. They lose the profits they would obtain if no firm invested in the marginal project and the marginal project turned out to be successful. The second expression captures the positive externality imposed on the customers — who receive the surplus associated with one firm innovating as opposed to the surplus associated without innovation. Corollary 1.1 states that the optimality of investment variety depends on the sign of the net externality imposed by the marginal variety.

Corollary 1.1 (Market investment in variety).

Suppose that assumptions 1.1, 1.2 and 1.4 hold. Then the market will underinvest in the variety of R&D projects if and only if $\sigma > 0$. The market will invest in the optimal variety of R&D projects if and only if $\sigma = 0$. The market will overinvest in the variety of R&D projects if and only if $\sigma < 0$.

In principle the sign of σ should be checked for each model. However, as will be shown later, in a large class of homogeneous goods models the assumption that the innovation is drastic implies that $CS(1) \geq W(0) \geq CS(0) + (N - 1)r(0, N)$. Thus, the market will in this case underinvest in the variety of R&D projects. The intuition for this is as follows. A process innovation is drastic if the monopolist's price is below the marginal cost of production without the innovation. Hence, consumer surplus with a monopolist (i.e., $CS(1)$), which is equal to the difference between the reservation price and the price paid, is greater than total welfare without the innovation (i.e., $W(0)$), which is equal to the difference between the reservation price and the cost of production.

To illustrate the market underinvestment in variety, consider a simple homogeneous product market similar to the one analyzed in Mankiw and Whinston (1986). Suppose that the inverse market demand function is given by $P(Q)$, where Q is the aggregate output in the market and $P'(Q) < 0$ for all Q . There are N symmetric firms, each of which possesses a technology given by the cost function $\bar{c}(q)$, where $\bar{c}(0) = 0$, $\bar{c}'(\cdot) \geq 0$ for all $q \geq 0$. Firms can invest in R&D to develop a drastic process innovation, in which case their technology is given by the cost function $\underline{c}(q)$, where $\underline{c}(0) = 0$, $\underline{c}'(\cdot) \geq 0$ for all $q \geq 0$. A process innovation is drastic if a monopolist facing the cost function $\underline{c}(q)$ chooses a price which is below the marginal cost of production of a firm with the old technology. Formally, an innovating monopolist would choose a quantity q_1 such that $P(q_1) < \bar{c}'(0)$.

Proposition 1.6 (Underinvestment in homogeneous product markets).

Suppose that Assumptions 1.1, 1.2 and 1.4 hold. Then an industry à la Mankiw and Whinston with a potential drastic process innovation always underinvests in the variety of R&D projects.

As the case of homogeneous product market illustrates, a decentralized market will tend to underinvest in drastic innovations. It should be noted that the critical assumption in this example is not the type of the product market competition. Rather, it is the assumption that the innovation is drastic which drives the result.

Proposition 1.6 offers insights relevant to research policy. Suppose that society cannot affect the market structure or the behavior of firms in the market but can offer subsidies for research. The market will tend to underinvest in the variety of drastic innovation by failing to develop high-cost projects which should optimally be developed. Thus, the research subsidies should be directed toward research projects with (1) high costs; and (2) and high potential payoffs.

Market investment in duplication

Typically, the market R&D portfolio will involve some duplication of research projects. As this is duplication of identical projects, it does not increase the probability that an innovation will be discovered. However, duplication is not entirely wasteful either. If multiple firms develop the same project and this project turns out to be the successful one, then there will be more competitors on the product market. So for the cost of duplicating research the society receives the (weakly) higher product market efficiency. The efficient duplication of R&D projects is captured by the optimal portfolio.

In equilibrium, a firm duplicating a research project imposes both negative externalities on its competitors (in the form of business stealing effect) and positive externalities on the consumers (in the form of the efficiency effect). Define the net externalities effect of the k -th duplication as:

$$\delta(k) = \left[(k-1)(R(k) - R(k-1)) \right] + \left[CS(k) - CS(k-1) \right].$$

The first bracket captures the negative externalities generated by the investment of the k -th innovator, which are the reduction of profits of $k-1$ firms from $R(k-1)$ to $R(k)$. The second bracket captures the positive externalities which accrue to the consumers, and which are captured by the difference between $CS(k)$ and $CS(k-1)$, the consumer surplus when there are k competitors and $k-1$ competitors on the product market, respectively.

Corollary 1.2 (Market investment in duplication).

Suppose that Assumptions 1.1, 1.2 and 1.4 hold. Denote with m the maximum number of firms investing in the market equilibrium and with m^o the maximal number of firms investing in the optimal equilibrium. For $2 \leq k \leq \min\{m, m^o\}$, denote with α_k the k -firm frontiers in the market portfolio and with α_k^o the k -firm frontiers in the optimal portfolio.

If $\delta(k) < 0$ then $\alpha_k^o < \alpha_k$ and the market overinvests in duplication of all projects $j \in (\alpha_k^o, \alpha_k)$. If $\delta(k) > 0$ then $\alpha_k < \alpha_k^o$ and the market underinvests in duplication of all projects $j \in (\alpha_k, \alpha_k^o)$. If $\delta(k) = 0$ then $\alpha_k^o = \alpha_k$ and the market optimally invests in duplication of all projects in the neighborhood of α_k .

If $m \geq m^o$ and $\delta(k) \geq 0$ for all $k \in \{2, \dots, m^o\}$, then the market (weakly) overinvests in duplication of all R&D projects. Conversely, if $m \leq m^o$ and $\delta(k) \leq 0$ for all $k \in \{2, \dots, m\}$, then the market (weakly) underinvests in duplication of all R&D projects. If the net externalities are negative ($\delta(k) < 0$), then it would be optimal to reduce the equilibrium number of firms investing in projects (α_k^o, α_k) from k to $k - 1$. If the externalities are positive then the number of firms should be increased in the interval (α_k, α_k^o) .

From the perspective of a fixed project j , the question of whether the amount of duplication is optimal or not is essentially equivalent to the question whether the free entry in an industry with fixed costs is optimal or not. Here, the question is of an entry in a ‘potential’ industry, fixed costs are the cost of developing this specific project $C(j)$, and the number of firms that can enter is limited by the number of firms which are active in the pre-innovation market. Keeping in mind the upper bound on the number of firms imposed by N , the results derived in Mankiw and Whinston (1986) apply in this setting as well. For the homogeneous product market and ignoring the integer constraint, Mankiw and Whinston find that the free-entry equilibrium number of firms is not less than the socially optimal number of firms (i.e., there is no underinvestment in duplication in our terminology), and furthermore if the equilibrium price is above the marginal costs, then the equilibrium number of firms is strictly greater than the optimal number (i.e. there is overinvestment in duplication).¹⁴ That is, Mankiw and Whinston identify conditions under which an industry equilibrium would tend toward excessive entry. In the context of the present model, this implies that there should be a tendency toward overinvestment in duplication of R&D projects. Taking into account the integer constraint weakens this result somewhat — Mankiw and Whinston establish that the free-entry equilibrium number of firms is not lower than the optimal number of firms less one.¹⁵ In the notation of this article, that would be $n^*(j) \geq n^o(j) - 1$ for appropriate project j . This suggests that even though there might be underinvestment in the duplication of R&D projects, it will be bounded from below.

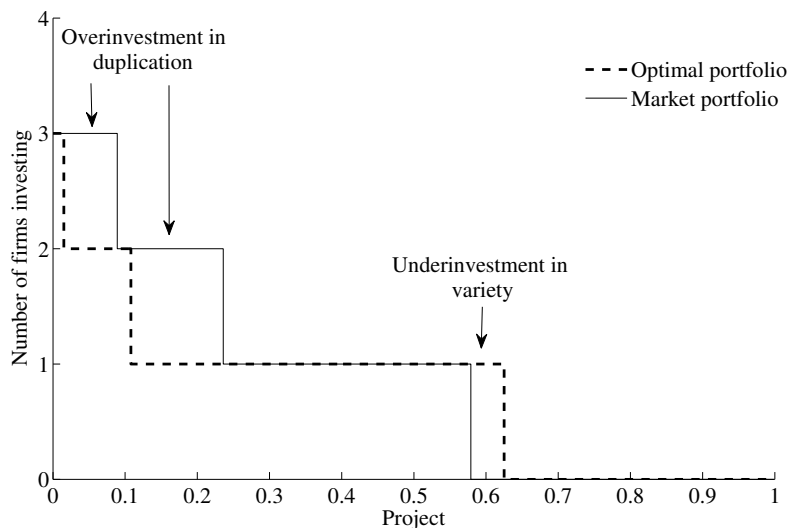


Figure 1.4: Optimal and market portfolios of research projects.

¹⁴Proposition 1 in Mankiw and Whinston (1986).

¹⁵Proposition 2 in Mankiw and Whinston (1986).

1.7 Extensions and robustness

This section relaxes several assumptions made in the model. For simplicity, in all following subsections except the first one, I will assume that there are only two firms in the market.

Non-drastic innovations

The assumption that innovation is drastic significantly simplifies the analysis, as it allows us to ignore all firms which have failed to innovate whenever at least one firm has innovated. However, there are many innovations which are incremental and which give only a slight advantage to the innovating firm over its rivals. This section relaxes Assumption 1.2 and provides a more general characterization of the market equilibrium portfolio.

First observe that Proposition 1.1 does not rely on Assumption 1.2. Hence, an equilibrium of the investment game exists and except in trivial cases an infinite number of equilibria exists. However, the equilibrium market portfolio is unique and any investment plan that generates the equilibrium market portfolio is itself an equilibrium of the investment game. The next result characterizes the equilibrium market portfolio without Assumption 1.2.

Proposition 1.7 (Characterization of equilibrium portfolio).

Suppose that Assumption 1.1 holds. Denote with m the maximum number of firms investing in any project:

$$m = \max_{\{1, \dots, N\}} n$$

$$\text{s.t. } R(n, N) - r(n-1, N) - C(0) > 0$$

and with α_k for $k \in \{1, 2, \dots, m\}$ the most expensive project in which k firms can profitably invest. That is:

$$\begin{aligned} R(1, N) - r(0, N) - C(\alpha_1) &= \\ R(2, N) - r(1, N) - C(\alpha_2) &= \\ &\vdots \\ R(m, N) - r(m-1, N) - C(\alpha_m) &= 0. \end{aligned}$$

Let $\alpha_{m+1} = 0$ and $\alpha_0 = 1$. Then the PSE portfolio $n^(j)$ is given by*

$$n^*(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}, \alpha_k).$$

The basic form of the equilibrium portfolio is the same as in the case with the drastic innovation — it is still a step function with a declining number of firms investing as projects become more expensive. There are two differences however. First, the payoffs with the innovation $R(\cdot, N)$ are now functions of N , because the firms without the innovation can put competitive pressure on the firms with the innovation. Second, firms without the innovation can now obtain positive profits, which decreases the incentive to duplicate research.

As a consequence, the comparative statics results become ambiguous if Assumption 1.2 does not hold. Consider a merger, so that the number of firms in the industry is reduced from N to

$N - 1$. If Assumption 1.2 holds, then Proposition 1.3 holds and the merger leads to a decrease in the variety of projects undertaken. If Assumption 1.2 does not hold, the variety of projects undertaken will (weakly) decrease if and only if

$$r(0, N - 1) - r(0, N) \geq R(1, N - 1) - R(1, N),$$

or, in words, only if the merger increases profits without the innovation more than it does for the single innovator. An analogous condition is required for any other n -firm frontier as well as for the changes in the intensity of competition among firms.

Figure 1.5 illustrates the Cournot duopoly example from the appendix A.1.10 with drastic and with non-drastic innovation and shows that the essential structure of the model does not depend on Assumption 1.2.

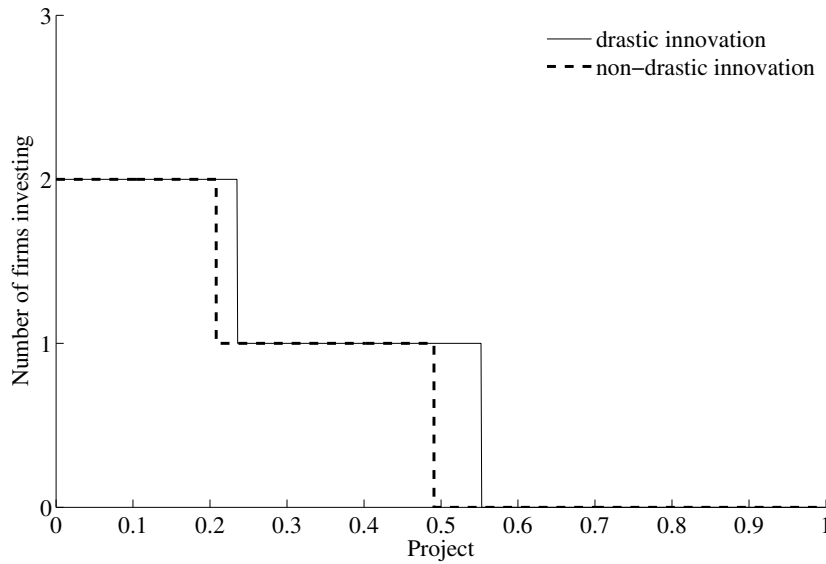


Figure 1.5: Drastic versus non-drastic innovation.

Asymmetric firms

Suppose that two firms are producing homogeneous goods with different technologies, so that one firm has lower marginal costs of production than the other. Call the more efficient firm the leader and denote its marginal production cost with \bar{c}_{lead} . Call the less efficient firm the laggard and denote its marginal production cost with \bar{c}_{lag} . Suppose that the firms are symmetric in all other aspects and furthermore suppose that firms can invest in the development of a new production technology which would lower the production costs of whichever firm develops it to \underline{c} , such that $\bar{c}_{lag} > \bar{c}_{lead} > \underline{c}$. Suppose that Assumptions 1.1 and 1.2 hold.

First observe that if neither firm develops the innovation, firms will continue competing with the old technology and the leader's profits $r_{lead}(0, 2)$ will be greater than the laggard's profits $r_{lag}(0, 2)$. However, as the innovation is drastic, the profits post-innovation will be the same for both firms $R_{lead}(1) = R_{lag}(1)$ and $R_{lead}(2) = R_{lag}(2)$. To simplify exposition, assume

$C(0) < R(2)$. Analogously to before, denote the k -firm frontiers as

$$\begin{aligned}\alpha_{1,lag} &= C^{-1}(R(1) - r_{lag}(0, 2)), \\ \alpha_{1,lead} &= C^{-1}(R(1) - r_{lead}(0, 2)), \\ \alpha_2 &= C^{-1}(R(2)),\end{aligned}$$

where $\alpha_{1,lag}$ is the most expensive project in which the laggard would invest and $\alpha_{1,lead}$ is the most expensive project in which the leader would invest. It is straightforward to see that $\alpha_2 \leq \alpha_{1,lead} < \alpha_{1,lag}$. In equilibrium, both firms will invest in the interval $[0, \alpha_2)$, for any project in the interval $[\alpha_2, \alpha_{1,lead})$ either the leader or the laggard will invest (but only one will), and only the laggard will invest in the interval $[\alpha_{1,lead}, \alpha_{1,lag})$, whereas no firm will invest in the interval $[\alpha_{1,lag}, 1)$. Hence this model predicts that the laggard firms will be more likely to invest in the most expensive projects. Furthermore, if one is willing to assume that where both firms can invest they do so symmetrically, the laggard will be more likely to develop drastic innovations. This prediction is consistent with the results in Akcigit and Kerr (2010), who find that smaller firms tend to have higher R&D expenses per employee and more patents per employee than larger firms.

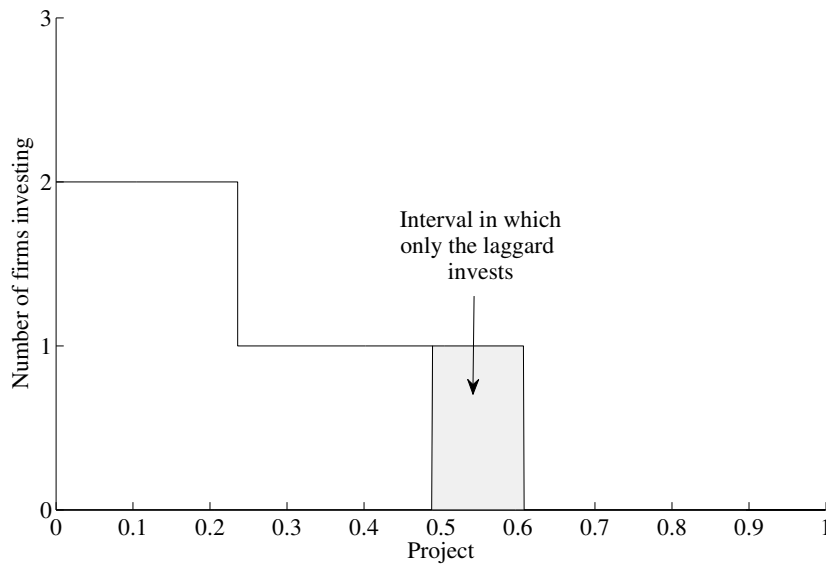


Figure 1.6: Asymmetric firms.

Figure 1.6 illustrates the equilibrium market portfolio of research projects, using the Cournot duopoly from the appendix A.1.10.

Innovations as strategic complements

Consider a case with two firms in the industry and relax Assumption 1.1. Assumption 1.1 will not hold if innovations are sufficiently strong complements, for example in the case of research spillovers. If Assumption 1.1 does not hold then it must be true that:

$$R(2, 2) - r(1, 2) > R(1, 2) - r(0, 2).$$

Assumption 1.2 is immaterial for the following discussion and it is not assumed to hold. Analogous to the use of k -firm frontiers before, let:

$$\begin{aligned}\alpha_1 &= C^{-1}(R(1, 2) - r(0, 2)), \\ \alpha_2 &= C^{-1}(R(2, 2) - r(1, 2)),\end{aligned}$$

where $C^{-1}(\cdot)$ is the inverse of the function $C(\cdot)$. As $C(\cdot)$ is a strictly increasing function, we have $\alpha_2 > \alpha_1$. This introduces ambiguity in the number of firms that will, in equilibrium, invest in the interval $[\alpha_1, \alpha_2)$. A single firm cannot profitably invest in any project in this interval, but two firms can. Hence, in equilibrium, it must hold that in any project in this interval, either no firm invests or both do. In the interval $[0, \alpha_1)$ both firms will invest, whereas in the interval $[\alpha_2, 1)$ neither firm will invest. If Assumption 1.1 does not hold, there will be an infinity of equilibrium market portfolios. In this sense, Assumption 1.1 is essential for the model. Figure 1.7 illustrates the equilibrium market portfolio of research projects.

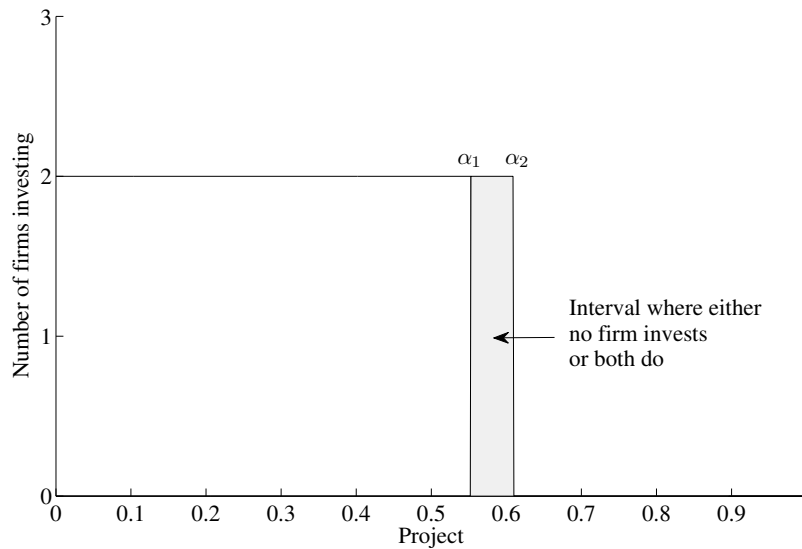


Figure 1.7: Violating Assumption 1.1.

1.8 Conclusion

This article studies how the market structure, that is the number of firms competing on the market and the nature of competition among them, influences the choice of research projects undertaken. The main object of analysis is the R&D portfolio, an object that captures both the variety of research projects undertaken, as well as the amount of duplicative research.

It is shown that, even though the effect of an increase in competition on the total level of investment in innovation is ambiguous, the increase in competition increases the variety of approaches to innovation and increases the probability that an innovation is discovered. The policy recommendation drawn from this conclusion is that competition authorities should take into account this negative effect on the investment in innovation when reviewing merger cases.

Comparing the equilibrium market portfolio with the optimal equilibrium portfolio, it is shown that the market will tend to underinvest in drastic innovation. This underinvestment will be more severe the higher the potential benefit from innovation and the lower the overall intensity of competition in the industry. This suggests that R&D subsidies should be targeted at high cost and high potential benefit projects (so-called blue sky projects) — especially in the industries with few firms and low intensity of competition.

This article presents an innovation model where firms choose R&D projects in which to invest. In this model, the variety of approaches to innovation as well as the duplication of R&D projects can be explicitly analyzed. This opens up at least two avenues for future research. First, a choice-of-R&D-projects model could be embedded into a growth framework, which could be used to analyze how different market structure and government policies could influence variety and duplication of R&D projects and through it long-term economic growth. Second, as eloquently argued in Segal and Whinston (2007), competition policy needs to focus more on the long-term effects such policy has on innovation in any given industry. The present model could be extended into a dynamic framework, so that questions of variety and duplication of R&D projects could be analyzed in a dynamic setting.

Acknowledgments

I am grateful to Ufuk Akcigit, Paul Heidhues, Arnd Heinrich Klein, Nick Netzer, Georg Nöldeke, Daniel L. Rubinfeld, Sabrina Studer, Xavier Vives, David Wettstein, E. Glen Weyl, Fabrizio Zilibotti and to seminar participants for helpful discussions and suggestions. Special thanks are due to Armin Schmutzler for numerous suggestions and comments. Financial support of the Swiss National Science Foundation is gratefully (projects P1ZHP1_155283 and 100014_131854) acknowledged.

2 Inducing Variety: A Theory of Innovation Contests¹

Joint with Armin Schmutzler

2.1 Introduction

The use of contests to procure innovations has a long history, and it is becoming ever more popular. Recently, private buyers have awarded the Netflix Prize, the Ansari X Prize, and the InnoCentive prizes. Public agencies have organized, for instance, the DARPA Grand Challenges, the Lunar Lander Challenge and the EU Vaccine Prize.² Reflecting the increasing importance of these prizes, a literature on contest design has developed. This literature focuses almost exclusively on how incentives for costly innovation effort can best be provided. However, effort is by no means the only important requirement for a successful innovation. A case in point is the 2012 EU Vaccine Prize to improve the so-called cold-chain vaccine technology. The ultimate goal of the prize was to prevent vaccines from spoiling at higher temperatures, which is particularly challenging in developing countries. The rules of the competition contain the following statement:

"It is important to note that approaches to be taken by the participants in the competition are not prescribed and may include alternate formulations, novel packaging and/or transportation techniques, or significant improvements over existing technologies, amongst others."³

This statement explicitly recognizes the fundamental uncertainty of the innovation process: Even an innovator who is pursuing a clear goal (such as finding a way to prevent vaccine spoilage) will often not know the best approach to achieving this goal. He will therefore have to choose between several conceivable approaches without being sure whether they lead to the goal. If innovators pursue different approaches, chances are higher that the best of these approaches yields a particularly valuable (high-quality) innovation. Thus, even if variety of research approaches has no intrinsic value, it has an option value. Our first question is therefore: Can innovation contests be used to incentivize suppliers to diversify their research approaches, thereby generating a high expected value of the innovation?

In addition to efficiency, contest design may also affect distribution. A contest that induces diversity may yield a high expected value of the innovation and thereby foster efficiency, but

¹This paper should be cited as Letina, I. and A. Schmutzler (2016), "Inducing Variety: A Theory of Innovation Contests," *Mimeo*. This paper has been submitted to the *RAND Journal of Economics*.

²See "Innovation: And the winner is...", *The Economist*. Aug 5, 2010.

³European Commission (2012), "Prize Competition Rules." August 28, 2012 (accessed on April 3, 2015). http://ec.europa.eu/research/health/pdf/prize-competition-rules_en.pdf

at the same time leave high rents to the suppliers. Thus, the main question of our paper will be: Which contests are optimal for the buyers, when the expected value (reflecting the induced variety of approaches) as well as the expected payments to the suppliers are taken into account?

The diversity of potential approaches, which is highlighted in the guidelines of the Vaccine Prize cited above, played an important role in many other examples of innovation procurement. First, the often cited Longitude Prize of 1714 for a method to determine a ship's longitude at sea featured two competing approaches.⁴ The lunar method was an attempt to use the position of the moon to calculate the position of the ship. The alternative, ultimately successful, approach relied on a clock which accurately kept Greenwich time at sea, thus allowing estimation of longitude by comparison with the local time (measured by the position of the sun). Second, when the Yom Kippur War in 1973 revealed the vulnerability of US aircraft to Soviet-made radar-guided missiles, General Dynamics sought to resolve the issue through electronic countermeasures, while McDonnell Douglas, Northrop, and eventually Lockheed, attempted to build planes with small radar cross-section.⁵ Third, the announcement of the 2015 Horizon Prize for better use of antibiotics contains a similar statement as the announcement of the vaccine prize.⁶

Architectural contests share some important properties with innovation contests. A buyer who thinks about procuring a new building usually does not know what exactly the ideal building would look like, but once she examines the submitted plans, she can choose the one she prefers. Guidelines for architectural competitions explicitly recognize the need for diversity. For example, the Royal Institute of British Architects states: "Competitions enable a wide variety of approaches to be explored simultaneously with a number of designers."⁷

Motivated by this long list of examples, we focus in the following on the design of contests for innovation, with a view towards the induced variety of research approaches. As we will sketch below, however, our analysis also has interesting implications for the case that suppliers on anonymous markets decide on the introduction of new products.

In line with the examples, we consider innovation contests in a setting where both the buyer (the contest designer) and the suppliers (contestants) are aware that there are multiple conceivable approaches to innovation. Furthermore, none of the participants knows the best approach beforehand. However, after the suppliers have followed a particular approach, it is often possible to assess the quality of innovations, for instance, by looking at prototypes or detailed descriptions of research projects. In such settings, can buyers design contests in such a way that suppliers have incentives to provide variety? And will they benefit from doing so?

A large literature has dealt with the optimal design of innovation contests, reflecting their ever-increasing importance. However, this literature mainly focuses on the problem of providing

⁴See, e.g., Che and Gale (2003) for a discussion of the Longitude Prize.

⁵See Paul Crickmore (2003), *Nighthawk F-117: Stealth Fighter*. Airline Publishing Ltd.

⁶"The rules of the contest specify the targets that need to be met but do not prescribe the methodology or any technical details of the test, thereby giving applicants total freedom to come up with the most promising and effective solution, be it from an established scientist in the field or from an innovative newcomer." European Commission (2015), "Better use of antibiotics." March 24, 2015 (accessed on April 3, 2015). <http://ec.europa.eu/research/horizonprize/index.cfm?prize=better-use-antibiotics>

⁷See Royal Institute of British Architects (2013), "Design competitions guidance for clients." (accessed on Apr 3, 2015); <http://competitions.architecture.com/requestform.aspx>.

incentives for costly innovation effort.⁸ To our knowledge, we are the first to analyze the optimal design of innovation contests with multiple conceivable research approaches.

In our main model, there are two homogeneous suppliers who decide whether to exert costly research efforts and which research approach to choose. The research approach is captured as a point on the unit interval. We assume all approaches are equally costly, so as to focus on suppliers' incentives to diversify. Crucially, the quality of an innovation and thus the value to the buyer depends inversely on the distance between the chosen research approach and an ideal approach that is unknown to all parties. The suppliers and the buyer agree about the distribution of this ideal approach, which has a strictly positive, symmetric and single-peaked density. If different suppliers try different approaches, this creates an option value for the buyer who can choose the preferred innovation once uncertainty is resolved.

In line with the literature on innovation contests, we assume that neither research inputs (approaches) nor research outputs (qualities) are verifiable, because they are both difficult to evaluate and the relation between them is stochastic. The lack of verifiability of research activity precludes any kind of contract that conditions on research inputs or outputs, and it motivates the focus on contests.⁹ The notion of contest design that we use was suggested by Che and Gale (2003). The buyer prescribes a possible set of prices and commits herself to paying the price chosen by the supplier from which the innovation is procured. The class of such contests is very rich.¹⁰ Examples include fixed-prize tournaments (when the price set is a singleton) as well as auctions (when the price set is the set of non-negative real numbers). We also allow the buyer to pay subsidies to the suppliers to induce them to participate in the contest and exert costly research effort. Moreover, we consider the cases with and without participation fees. Contest design in this setting is the choice of the allowable price set and the subsidy (or participation fee).

The sequence of moves in our model is as follows: After the buyer has communicated the rules of the game (and, in particular, the price set), the suppliers choose whether to enter and, if so, which approach to pursue. Then qualities become common knowledge. After having observed qualities, suppliers choose bids from the price set. Finally, the buyer selects the preferred supplier.

We show that, no matter whether participation fees are allowed or not, the optimal contest for the buyer is what we call a bonus tournament. In a bonus tournament, the price set consists of two elements — a low price and a high (“bonus”) price. After qualities have been realized, the suppliers thus can only choose whether to ask for the high price or the low price. The selected supplier will be paid his bid. Anticipating this, the suppliers diversify in the hope that their quality advantage over the competitor will be sufficiently high that they can bid the bonus price and win even so. It will turn out that the amount of diversity implemented in a bonus tournament is determined by the difference between the bonus price and the low price. We show that, with a bonus tournament, the buyer can implement essentially any level of diversity. In particular, a bonus tournament with suitably chosen prices (and possibly a subsidy) implements

⁸Section 2.6 discusses this literature.

⁹For an extensive discussion see Che and Gale (2003) and Taylor (1995).

¹⁰See Che and Gale (2003) for a detailed discussion.

the socially optimal diversity. When sufficiently high participation fees are possible, the buyer implements the social optimum and appropriates the surplus with the participation fees.

When unlimited participation fees are not allowed, the analysis is more subtle. Full rent extraction is not always possible, and the buyer must trade off efficiency against rent extraction. Bonus tournaments are still optimal for the buyer: They induce any desired level of diversity while minimizing rent extraction. We show that the optimal contest leads to just enough diversity that expected supplier revenues are equal to the cost needed to develop the innovation. This will imply less diversity than socially optimal, except when research costs are very high. Thus the buyer resolves the trade-off between efficiency and rent extraction in favor of the latter.

The existing literature on innovation contests has put particular emphasis on auctions and fixed-prize tournaments. We therefore also analyze how these institutions perform in our setting and why they fail to be optimal for the buyer. Unrestricted auctions induce the social optimum, while auctions with price ceilings induce less variety. The price ceiling determines the amount of variety. While auctions can in general implement the same diversity as the optimal bonus tournaments, they always generate higher revenues for the suppliers. Thus the buyer prefers bonus tournaments to auctions. Fixed-prize tournaments do not induce any diversity and are therefore less efficient than auctions and optimal bonus tournaments. Nevertheless, for low research costs, the buyer prefers the inefficient fixed-prize tournaments to the socially efficient unrestricted auctions.

We then extend the analysis, and show that, with some caveats, bonus tournaments perform well even in more general environments. In particular, we study contests with multiple suppliers, and contests with more general distributions and quality functions. In addition, we discuss heterogeneous suppliers, multiple prizes and multiple approaches per supplier. Under very general conditions, bonus tournaments still induce the social optimum. The buyer continues to prefer them to fixed-prize tournaments, even though the latter induce some diversity (but suboptimal amounts) when there are multiple suppliers. However, when research costs are sufficiently high, the buyer may prefer auctions to bonus tournaments. Moreover, she may benefit from inviting a large number of suppliers, which is a straightforward implication of the option value provided by additional suppliers.

As we discuss in more detail in the conclusion, our analysis has potential applications beyond innovation contests organized by a single buyer. Our model can be applied to situations when suppliers in a new market choose products in the face of uncertain demand by a potentially large number of homogeneous buyers. If we interpret the prize as the expected product market profit of a successful innovator, contest design then corresponds to the choice of alternative regulatory frameworks for the new market. Our approach shows that unregulated markets provide incentives for suppliers to choose the socially optimal products, but at the cost of endowing them with ex-post market power. As a result, regulation may yield higher expected consumer surplus, even though it does not induce the optimal expected product quality.

While our main application is to the design of innovation contests, the model is not limited to innovation settings. Our results have important implication for any contest where contestants can choose some measure of correlation of outcomes. In particular, prize rules that award winners based on the margin by which they outperform the second-best contestant (like auctions or bonus

tournaments) will incentivize the contestants to choose less correlated outcomes. Alternatively, fixed-prize contests will cause contestants to choose too correlated outcomes.

In Section 2.2, we introduce the model. Section 2.3 deals with the design of optimal contests for the buyer. Section 2.4 compares several commonly used contests, such as fixed-prize tournaments and auctions with and without price ceilings. Section 2.5 presents extensions of the model. Section 2.6 discusses the relation of our paper to the literature. Section 2.7 concludes, pointing in particular to the above-mentioned re-interpretation of our model for a world with many buyers. Proofs are in the Appendix.

2.2 The Model

A risk-neutral buyer B needs an innovation that two risk-neutral suppliers ($i \in \{1, 2\}$) can provide. Each supplier simultaneously chooses whether to carry out costly research and which approach $v_i \in [0, 1]$ to pursue. Without loss of generality, we assume that $v_1 \leq v_2$; if the ordering of approaches does not matter, we use the generic notation v_i (and $v_j \neq v_i$). The cost of approach v_i is $C(v_i) \equiv C \geq 0$. Thus all approaches are equally costly. The quality q_i of the resulting innovation and thus the value to the buyer depends on a state $\sigma \in [0, 1]$, which is distributed with density $f(\sigma)$, and corresponds to an (ex-post) ideal approach. We thus assume that $q_i = \Psi - \delta(|v_i - \sigma|)$, where $\Psi > 0$ is large enough and δ is an increasing function.¹¹

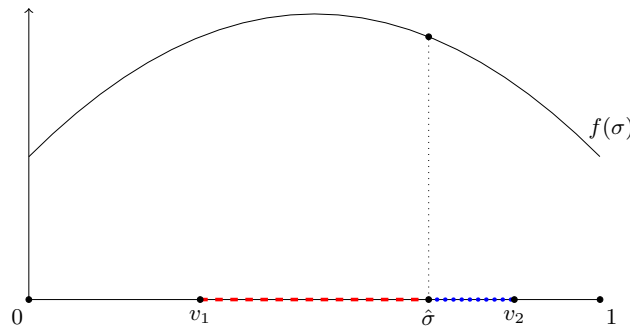


Figure 2.1: Illustration of an outcome given v_1 and v_2 where the ideal approach is $\hat{\sigma}$.

Figure 2.1 illustrates one particular outcome of the model. Suppose that the uncertainty is given by the distribution $f(\sigma)$ and that the suppliers have chosen the approaches $v_1 \leq v_2$.

The quality difference between the ideal approach $\hat{\sigma}$ and v_i is proportional to the distance between v_i and $\hat{\sigma}$ (the dashed line for $i = 1$, and the horizontal dotted line for supplier $i = 2$). Unless specified otherwise, we will maintain assumptions (A1) and (A2) below.

Assumption (A1) *The density function $f(\sigma)$ is (i) symmetric: $f(1/2 - \varepsilon) = f(1/2 + \varepsilon) \forall \varepsilon \in [0, 1/2]$, (ii) single-peaked: $f(\sigma) \leq f(\sigma') \forall \sigma < \sigma' < 1/2$, (iii) has full support: $f(\sigma) > 0 \forall \sigma \in [0, 1]$ and (iv) satisfies $f(x) \leq 2f(y)$ for all $x, y \in [0, 1]$.*

Assumption (A2) $\delta(|v_i - \sigma|) = b|v_i - \sigma|$ with $b \in (0, \Psi]$.

¹¹ Ψ needs to be large enough so that it is worthwhile for the buyer to hold a contest. A simple sufficient condition is $\Psi > \delta(1) + 2C$. This assumption is innocuous as none of our results depend on Ψ .

A wide class of distributions satisfies (A1). For each of these distributions, there is an approach which has the highest expected quality ex ante, namely the median. Furthermore, single-peakedness makes it more difficult to induce diversity: As there is less mass on approaches that are further away from the median, contestants will not choose them without additional incentives. Part (iv) excludes the possibility that some states are much less probable than others, that is, it requires that the amount of uncertainty about the ideal approach is sufficiently high. Using (A2), we denote the quality resulting from approach v_i in state σ as $q(v_i, \sigma) = \Psi - b|v_i - \sigma|$. Thus quality is bounded below by $\Psi - b$ and bounded above by Ψ .

In this setting, the buyer chooses an *innovation contest* determining the procedure for choosing and remunerating suppliers. These contests are closely related to those analyzed by Che and Gale (2003), where suppliers choose efforts rather than approaches. In line with these authors, we assume that neither v_i nor q_i is contractible.¹² The environment (b, Ψ, C) of a contest consists of the utility and cost parameters. The buyer chooses a set \mathcal{P} of allowable prices (bids), where \mathcal{P} is an arbitrary finite union of closed subintervals of \mathbb{R}^+ .¹³ We denote the minimum of \mathcal{P} as \underline{P} and the maximum, if it exists, as \bar{P} . Moreover, the buyer can offer subsidies $t \geq 0$ to the suppliers. An innovation contest is thus the extensive-form game between the buyer and the suppliers given by the buyer's choice of $\{\mathcal{P}, t\}$ and the following rules:

- Period 1:* Suppliers simultaneously choose whether to engage in research and they select approaches $v_i \in [0, 1]$.
- Period 2:* The state is realized. All players observe qualities q_1 and q_2 .
- Period 3:* Suppliers simultaneously choose prices $p_i \in \mathcal{P}$.
- Period 4:* The buyer observes prices; then she chooses a supplier $i \in \{1, 2\}$. She pays $p_i + t$ to the chosen supplier and t to the other supplier.

Importantly, the suppliers receive two types of payments, namely the revenue from the contest (that is paid only to the successful supplier) and the subsidies paid to both suppliers. For ease of exposition, we sharpen the requirement that qualities are observable by assuming that all players observe v_i and σ , as this allows us to apply the subgame perfect equilibrium (SPE). It will be obvious that the observability of v_i and σ plays no role whatsoever; as these variables are payoff-relevant only inasmuch as they affect qualities. As long as all players can observe qualities, all results still hold with the SPE replaced by a Perfect Bayesian Equilibrium with suitably specified beliefs.¹⁴

Moreover, we provide an extensive discussion of the case when not even quality is observable in the working paper (Letina and Schmutzler 2015); we summarize the discussion briefly in Section 2.5.2.2.

¹²For example, Che and Gale (2003) and Taylor (1995) assume that neither inputs nor outputs of innovative activity are verifiable. As an example of the verifiability problem, Che and Gale (2003) point to the protracted battle between John Harrison, the inventor of the marine chronometer, and the Board of Longitude, over whether his invention met the requirements of the 1714 Longitude Prize. See also references in Taylor (1995).

¹³Formally, \mathcal{P} is chosen from $\mathcal{I}(\mathbb{R}^+) := \{\mathcal{P} \subseteq \mathbb{R}^+ : \mathcal{P} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \text{ or } \mathcal{P} = \cup_{k=1}^{\bar{k}} [a_k, b_k] \cup [a_{\bar{k}+1}^-, \infty) \text{ for } a_k \leq b_k \in \mathbb{R}^+, \bar{k} \in \mathbb{N}\}$.

¹⁴Proof available on request.

The following are examples of innovation contests:

1. $\mathcal{P} = \mathbb{R}^+$: an *auction without a price ceiling*.
2. $\mathcal{P} = [0, \bar{P}]$: an *auction with a price ceiling \bar{P}* .
3. $\mathcal{P} = \{A\}$, where $A \geq 0$: a *fixed-prize tournament (FPT)*.
4. $\mathcal{P} = \{A, a\}$, where $A > a \geq 0$: a *bonus tournament*.

The first three examples are well-known. The last example differs from an FPT in that the supplier has to specify whether she accepts a low price a if chosen, or asks for the higher "bonus" price A instead. The bonus tournament will turn out to be the optimal contest for the buyer.

To finish the description of the contests, we require several further conventions. First, we apply the following tie-breaking rules, which can be interpreted as second-order lexicographic preference for winning and for higher quality.

(T1) (Preference for quality) If suppliers offer the same surplus, the buyer prefers the higher quality one. If both have the same quality, the tie is randomly broken.

(T2) (Preference for winning) Given equal monetary payoffs, the suppliers prefer to participate in the contest rather than to stay out and to win the contest rather than not.

(T1) and (T2) guarantee that the outcomes are robust to infinitesimal changes in the reward structure.

Second, we assume that, in cases where only one supplier decides to participate, the contest is called off and players obtain zero overall payoff.

Third, we will confine our analysis to the case of pure-strategy equilibria for simplicity.

2.3 The Optimal Contest for the Buyer

In this section, we characterize the optimal contest for the buyer.¹⁵ We start with some auxiliary results. These results characterize the social optimum, and they deal with the pricing subgames.

2.3.1 Auxiliary Results

We introduce the following terminology which applies when both suppliers participate. For $(v_1, v_2) \in [0, 1] \times [0, 1]$, the (expected) *total surplus* is $S_T(v_1, v_2) \equiv E_\sigma [\max \{q(v_1, \sigma), q(v_2, \sigma)\}] - 2C$. The social optimum is $(v_1^*, v_2^*) \equiv \arg \max_{(v_1, v_2) \in [0, 1]^2} S_T(v_1, v_2)$. Thus, we are focusing here on the optimal choice of approaches for a given number of suppliers (two). In Section 2.5.1, we deal with the optimal number of suppliers.

¹⁵An attentive reader might conjecture that the buyer could implement arbitrary outcomes with a mechanism where he just pays unconditional transfers $t = C$ and sets a singleton prize set $\mathcal{P} = \{0\}$. The suppliers are then indifferent between entering and not entering, and, *in monetary terms*, between all approaches. However, our "preference for winning" assumption (T2) would ensure that such a mechanism would have a unique equilibrium with $v_1 = v_2 = 1/2$. Even if we dispensed with assumption (T2), the equilibrium structure of such a mechanism would not be robust to small changes in the cost of different approaches or to assuming that duplicating an approach is less costly than developing an original one.

For (v_1, v_2) , implemented as an equilibrium of a contest (\mathcal{P}, t) , $S_i^{(\mathcal{P}, t)}(v_1, v_2)$, the (expected) *surplus of supplier* i in an equilibrium, is the sum of the expected revenue and the subsidies, net of research costs. The (expected) *buyer surplus*, $S_B^{(\mathcal{P}, t)}(v_1, v_2)$, is expected maximal quality minus the expected revenues and subsidies of the suppliers. We usually drop the superscript (\mathcal{P}, t) when there is no danger of confusion. For precise definitions of $S_B^{(\mathcal{P}, t)}(v_1, v_2)$ and $S_i^{(\mathcal{P}, t)}(v_1, v_2)$, we refer the reader to Appendix B.1.1.

As the costs of each approach are the same, the social optimum (v_1^*, v_2^*) maximizes the expected maximal quality $E_\sigma [\max \{q(v_1, \sigma), q(v_2, \sigma)\}]$ or, equivalently, minimizes the expected minimal distance to the ideal approach, $E_\sigma [\min \{\delta(|v_1 - \sigma|), \delta(|v_2 - \sigma|)\}]$. With only one potential supplier i , the optimal approach would correspond to $v_i = 1/2$, as this maximizes the expected quality. With two suppliers, the optimization needs to take into account the option value generated by having different choices once qualities have been observed. It is always socially optimal to have at least some diversification. This simple but important observation holds without the restrictions on distributions coming from (A1), as long as there is any uncertainty about the ideal approach. The intuition is simple: Starting from a situation with identical approaches, suppose one of the suppliers chooses an arbitrary alternative approach, whereas the other supplier continues to choose the same one. After this modification, the minimal distance decreases for a set of ideal states with positive measure. There can be no σ for which the expected minimal distance to the best approach increases, as the initial approach is still available. The following result provides a sharper characterization of the social optimum:

Lemma 2.1. *The unique social optimum with $v_1^* \leq v_2^*$ satisfies $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ and thus $v_2^* = 1 - v_1^*$.*

Hence v_1^* and v_2^* are symmetric around $1/2$. The result relies on (A1(iv)), which states that the ideal state distribution is sufficiently dispersed.¹⁶ The social optimum is fully determined by the distribution F , whereas research costs have no influence on the optimal diversity. We now characterize the equilibria of the pricing subgames, using the following notation:

Notation 1. $\bar{p}(\sigma) \equiv \max \{p \in \mathcal{P} \mid p \leq |q(v_1, \sigma) - q(v_2, \sigma)| + \underline{P}\}$.

In words, for any realization of σ , $\bar{p}(\sigma)$ is the maximal allowed price which guarantees that the supplier with higher quality wins the contest, irrespective of the price chosen by the supplier with the lower quality. The following result is closely related to the familiar “asymmetric Bertrand” logic that inefficient firms choose minimal prices, whereas efficient firms translate their efficiency advantage into a price differential.¹⁷

¹⁶The condition guarantees that the expected quality is a strictly concave function of the approaches. It is thus more restrictive than necessary. A simple *necessary* condition for the optimum to satisfy $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ is $f(1/2) < 2f(v_1^*)$; otherwise the objective function is not even locally concave. Moreover, this condition turns out to be necessary for the existence of a social optimum with $v_2^* = 1 - v_1^*$. It is simple to provide examples where $f(1/2) < 2f(v_1^*)$ is violated. For instance, consider the kinked distribution defined by the density

$$f(\sigma) = \begin{cases} 0.6 & \text{if } \sigma \in [0, 0.45) \cup (0.5, 1] \\ 4.6 & \text{if } \sigma \in [0.45, 0.55] \end{cases}.$$

¹⁷The adequacy of pure-strategy equilibria in asymmetric Bertrand games has received some attention, in particular, but not only, because they tend to involve weakly dominated strategies (see Blume 2003 and Kartik

Lemma 2.2. *The subgame of an innovation contest corresponding to (q_i, q_j) has an equilibrium such that $p_i(q_i, q_j) = \bar{p}(\sigma)$ if $q_i \geq q_j$ and $p_i(q_i, q_j) = \underline{P}$ if $q_i < q_j$. In any equilibrium of any contest, $p_i(q_i, q_j) = \bar{p}(\sigma)$ if $q_i \geq q_j$.*

Lemma 2.2 sharpens the Bertrand logic to account for bounded and/or non-convex price sets: The price differential will only fully reflect the quality differential when the corresponding bid of the high-quality supplier is in the price set \mathcal{P} . In many cases, the equilibrium described in Lemma 2.2 is unique.¹⁸ We need further notation:

Notation 2. $\Delta q(v_i, v_j) \equiv |q(v_i, v_i) - q(v_j, v_i)|$ is the maximum quality difference given (v_i, v_j) .

To understand why $\Delta q(v_i, v_j)$ is the maximum quality difference, note that, for $\sigma \in [0, v_1] \cup [v_2, 1]$ the quality difference between the two approaches is equal to $|q(v_i, v_i) - q(v_j, v_i)|$ and thus constant; whereas it is smaller for $\sigma \in (v_1, v_2)$. By Lemma 2.2, in any subgame the successful supplier chooses the highest available price not exceeding the sum of the quality differential and the minimum bid. We now sharpen this result for subgames following equilibrium choices (v_1, v_2) .

Lemma 2.3. *Let $v_1 \leq v_2$. (i) If a contest implements (v_1, v_2) , then $\Delta q(v_1, v_2) + \underline{P} \in \mathcal{P}$. (ii) If $\sigma \in [0, v_1] \cup [v_2, 1]$, the successful supplier bids $p_i(q_i, q_j) = \Delta q(v_i, v_j) + \underline{P}$.*

Lemma 2.3 is a key result. It implies that the amount of diversity that any contest can implement is limited by the highest price that the contest allows. Intuitively, (i) states that, if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, suppliers could increase their chances of winning by small moves towards the approach of the other party, without reducing the price in those cases where they win. (ii) shows that in all states outside the interval (v_1, v_2) the buyer pays a constant price, reflecting the (maximal) quality difference between the two suppliers. Therefore, to implement any (v_1, v_2) , a buyer has to pay at least $\Delta q(v_1, v_2)(F(v_1) + 1 - F(v_2))$ in expectation to the suppliers.

2.3.2 Characterizing the Optimum

We now turn to our main results. Before identifying the optimal contest for the buyer, we first show that bonus tournaments can implement a wide range of allocations.

Proposition 2.1. *Any (v_1, v_2) such that $0 < v_1 \leq 1/2 \leq v_2 < 1$ can be implemented by a bonus tournament with sufficiently high subsidies. In particular, the social optimum can be implemented.*

Thus, the buyer can implement any desired diversity in a bonus tournament. The proof shows that implementation works with $\mathcal{P} = \{A, 0\}$ and $A = \Delta q(v_1, v_2)$, so that A is the corresponding maximal quality difference. For instance, to induce the social optimum, the buyer has to set $A =$

2011). In our setting, these issues are resolved by the appeal to the "preference for quality" (T1) and "preference for winning" (T2). In some of our contests (in particular, in auctions with and without price ceilings), constructions as in Blume (2003) and Kartik (2011) exist, where the low-quality firm mixes over a small interval of prices.

¹⁸If \mathcal{P} is convex and $\sup \mathcal{P} > \bar{p}(\sigma)$ for all σ , then $p_i(q_i, q_j) = \underline{P}$ for $q_i < q_j$ in every equilibrium. To see this, note that, according to Lemma 2.2, $p_j = \bar{p}(\sigma) = \underline{P} + q(v_j, \sigma) - q(v_i, \sigma)$ in any equilibrium for the high-quality supplier j . If $p_i > \underline{P}$, then j can choose a slightly higher prize, and he still wins. Hence, this is a profitable deviation.

$\Delta q(v_i^*, v_j^*)$. The equilibrium pricing strategies turn out to be $p_1(), p_2()$ such that $p_i(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise: The supplier only asks for the bonus A when his quality advantage is maximal ($\sigma \in [0, v_1] \cup [v_2, 1]$); otherwise she accepts the low price. Therefore, the buyer pays the lowest price compatible with Lemma 2.3 for $\sigma \in [0, v_1] \cup [v_2, 1]$. Clearly, the price 0 is also minimal on (v_1, v_2) . The bonus tournament is thus a flexible instrument with which the buyer can fine-tune diversity with low supplier revenues. This suggests that the optimal contest is in this class. However, this intuition is incomplete, as it does not account for subsidies. We now show that it is nevertheless always optimal for the buyer to use bonus tournaments. However, she will not always implement the social optimum.

Theorem 2.1. (i) *The buyer optimum can be implemented with a suitable bonus tournament $(\{A, 0\}, t)$ where the suppliers break even on expectation.*

(ii) *If $C \geq F(v_1^*) \Delta q(v_i^*, v_j^*)$, the optimal contest for the buyer is a bonus tournament that implements the social optimum, with subsidies used to ensure break even.*

(iii) *If $C < F(v_1^*) \Delta q(v_i^*, v_j^*)$, the optimal contest for the buyer is a bonus tournament that implements just enough diversity that the suppliers break even without subsidies.*

Whereas (i) only states the optimality of bonus tournaments, (ii) and (iii) specify the details for the two different parameter regions. When research costs are high enough and quality differences in the social optimum are low, the buyer implements the social optimum. When research costs are low, the buyer induces just enough diversity that the maximal quality differences are sufficiently large that suppliers break even without subsidies.¹⁹ In any event, the suppliers earn zero surplus. Thus, whenever there is a tradeoff between rent extraction and efficiency, the buyer resolves it in favor of rent extraction by reducing variety relative to the social optimum. As C increases, the buyer can implement more diversity without leaving rents to the suppliers. Therefore, we obtain the comparative statics result that an increase in C leads to an increase in the buyer-optimal diversity from 0 to the social optimum.

To understand the desirable properties of bonus tournaments, recall from Lemma 2.3 that in any contest implementing (v_1, v_2) , the price $\Delta q(v_1, v_2) + \underline{P}$ has to be in the price set. This fixes the price that the buyer has to pay in any state of the world when the quality difference is maximal. What contest design can achieve, then, is to reduce prices paid in those states of the world when $\sigma \in (v_1, v_2)$, implying that the quality difference is not maximal. With a bonus tournament (A, a) , the buyer commits herself not to pay prices between a and A in these states: Even when the quality difference is greater than a , she only pays a . Setting $a = 0$ clearly minimizes the revenues of the suppliers. The only remaining question is how much diversity the buyer optimally induces. Through the option value it generates, diversity can increase efficiency. However, it is costly for the buyer to induce. As mentioned before, the theorem shows that whenever there is a tradeoff between efficiency and rent extraction, the buyer sacrifices efficiency.

Even when subsidies are not feasible, the buyer can still implement the same outcome as with subsidies unless research costs are too high. For $C < F(v_1^*) \Delta q(v_i^*, v_j^*)$, this is evident, as the bonus tournament in Theorem 2.1(iii) does not require subsidies. For higher research costs

¹⁹Recall that, to implement (v_1, v_2) , the buyer sets $A = \Delta q(v_1, v_2)$. Thus, for small diversity the bonus price is small.

the buyer can increase the low price a in order to make sure that the participation constraints of the buyers hold.

Corollary 2.1. *Suppose that (A1) and (A2) hold and that the buyer cannot use subsidies. If $F(v_1^*) \Delta q(v_i^*, v_j^*) < C \leq F(v_2^*) \Delta q(v_i^*, v_j^*)$, the buyer surplus is maximized by the bonus tournament implementing (v_1^*, v_2^*) with $\mathcal{P} = \{A, a\}$, where $A = 2C + \Delta q(v_i^*, v_j^*)/2$ and $a = 2C - \Delta q(v_i^*, v_j^*)/2$.*

The low positive price acts as an imperfect substitute for subsidies. In particular, when $C > F(v_2^*) \Delta q(v_i^*, v_j^*)$, the difference $\Delta q(v_i^*, v_j^*)$ between prices A and a is too small relative to the size of the price a . Thus, suppliers are willing to sacrifice the bonus price in order to increase their probability of winning the low price.

The buyer can increase her surplus if she is allowed to charge entry fees $e > 0$. She would charge such fees only if $C < F(v_1^*) \Delta q(v_i^*, v_j^*)$, in which case the optimal fee e^* satisfies $C + e^* = F(v_1^*) \Delta q(v_i^*, v_j^*)$, so that she achieves the first-best.²⁰ With or without entry fees, the buyer thus designs the contest so that the suppliers exactly break even on expectation.

2.4 Auctions and Fixed Prize Tournaments

In Section 2.3.2, we characterized the optimal contest. We now study two other types of contests that are discussed in the literature, namely auctions and fixed prize tournaments. Auctions generally have good incentive properties; for example, auctions are the optimal contest in the setting of Che and Gale (2003). On the other hand, fixed prize tournaments are very common innovation contests. Next, we examine how these contests perform in our setting, where the choice of research approaches is important.

Proposition 2.2. *(i) For any t such that the suppliers' participation constraints are met, the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implements the social optimum. (ii) For any $A \geq 2C$, the unique equilibrium of an FPT ($\mathcal{P} = \{A\}$) implements $(v_1, v_2) = (1/2, 1/2)$. (iii) Whenever $C < F(v_1^*) \Delta q(v_i^*, v_j^*)$, the buyer prefers the inefficient FPT to the efficient auction.*

Proposition 2.2(i) states that the auction induces the efficient amount of diversity. It is intuitively clear that an auction implements some diversity: With identical approaches, no supplier will earn a positive revenue. Any move away from the other supplier will lead to quality advantages in a measurable set of states and thereby to positive expected revenues. Auctions implement the socially efficient outcome because they align the externalities of the choice of an approach v_i with the private benefits. For example, fix some v_2 and consider a marginal change of v_1 . Such a change generates externalities only in the states of the world for which the quality of supplier 2 is greater than the quality of supplier 1. Furthermore, the size of the externality is exactly the change in the quality difference. Since supplier 1 wins the auction only when his quality is higher and he bids exactly the quality difference, the private incentives and the externalities are aligned. While we prove Proposition 2.2(i) directly in Appendix B.4, an analogous result also applies for more general state distributions and quality functions and

²⁰If the buyer is limited to setting fees below e^* , she will charge the maximum allowable fee.

for arbitrary numbers of suppliers. It also holds when suppliers are heterogeneous. This result extends beyond auctions to any type of institution that gives the chosen supplier a positive share of the quality difference to the next-best alternative.

Proposition 2.2(ii) states that an FPT induces no diversity at all. The intuition for the absence of diversity is straightforward. As the size of the prize is independent of quality differences in an FPT, the suppliers care only about maximizing the expected winning probability. By (A1), this requires moving to the center. In particular, there is no diversity.

As to (iii), even though an auction implements the social optimum, it leaves rents to the successful supplier whenever research costs are low enough. Because it avoids such rents, the buyer may prefer to use a suitable FPT. On the other hand, a bonus tournament can achieve the same efficiency as an auction, while reducing supplier rents, as the Figure 2.2 illustrates. If the realized state of the world is $\sigma \in [0, v_1] \cup [v_2, 1]$, the payment is the same in the auctions and in the bonus tournament by Lemma 2.3. However, for $\sigma \in (v_1, v_2)$ the winning supplier captures the entire quality difference in the auction, while in the bonus tournament the winning supplier receives only the low price a .

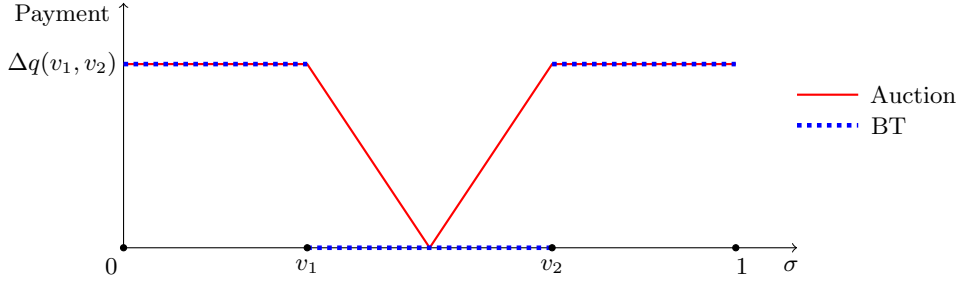


Figure 2.2: Comparison of payments in an auction and a bonus tournament.

The trade-off between efficiency and rent extraction also shows up when analyzing the price ceiling in auctions.

Corollary 2.2. *An outcome (v_i, v_j) that is implemented in an auction with price ceiling \bar{P} satisfies $\Delta q(v_i, v_j) \leq \bar{P}$. Thus diversity is bounded by the price ceiling.*

If the maximal quality difference between the two suppliers were above the maximum feasible bid, the supplier could not charge the buyer for this quality difference. He could thus choose an approach slightly closer to the competitor to increase his chances of winning without reducing the price.

Corollary 2.2 embeds the auction without price ceiling and the FPT as polar cases. In an auction without price ceiling, suppliers are free to choose the bid and thus capture the benefits of diversification. This results in optimal diversity. By Corollary 2.2, price ceilings limit this possibility: They determine an upper bound on equilibrium diversity. A reduction in the price ceiling leads to lower equilibrium diversity. Thus, the choice of the price ceiling involves a trade-off between efficiency-increasing diversity and market power for the suppliers. Consistent with the logic of Theorem 2.1(iii) and Proposition 2.2(ii), the following result shows that the buyer never resolves the trade-off in favor of efficiency when costs are low.

Corollary 2.3. *Let $C = 0$. Among all contests where \mathcal{P} is convex, the buyer's surplus is maximal in an FPT with $A = 0$.*

The proof of Corollary 2.3 relies heavily on the fact that higher quality suppliers bid the sum of the quality differential and the minimum \underline{P} whenever available (Lemma 2.2). Thus the buyer surplus, as the difference between the expected maximal quality and the expected payment, is the difference between the expectation of the minimum quality and the minimum bid. The buyer's best choice is an FPT with $A = 0$, because this maximizes the minimum quality and minimizes the minimum bid.

Remember that the price set in a bonus tournament is of the form $\{A, a\}$. The last result thus clearly underlines the role of non-convex price sets in a bonus tournament for the buyer optimum. A buyer who is confined to the class of contests with convex price sets (including auctions and FPTs) cannot profitably induce diversity.

The discussion in this section needs to be qualified if the buyer can charge entry fees. Obviously, in this case any contest which implements the social optimum will lead to the first-best outcome for the buyer, as the buyer can use the entry fees to extract any surplus from the suppliers. In particular, both a bonus tournament and an auction lead to the first best. However, while an auction is only optimal when entry fees are available, bonus tournaments are optimal both with and without entry fees.

2.5 Extensions

In this section, we extend the model in several directions and study the robustness of our main results. We show that with multiple suppliers bonus tournaments still have desirable properties. Bonus tournaments are usually still preferable to FPTs, and they still implement the social optimum with the lowest revenues. However, they may require higher subsidies than alternative contests. For instance, auctions may implement the social optimum with lower supplier surplus than bonus tournaments when research costs are high or when the number of suppliers is large. We also study more general distributions and quality functions and briefly sketch several other extensions.

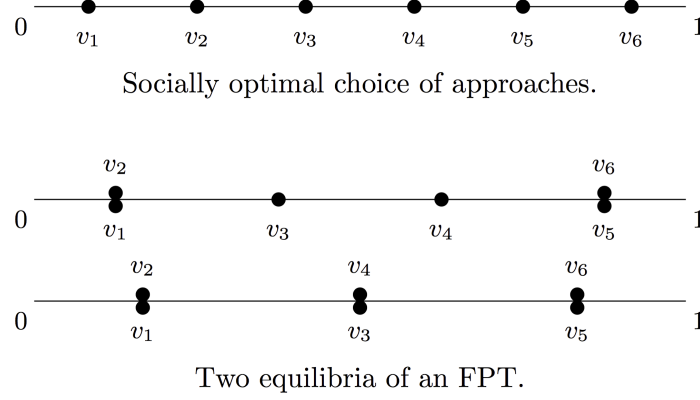
2.5.1 Number of Suppliers

In innovation contests there are usually more than two suppliers. For example, there were 49 registered competitors in the EU Vaccine Prize, 12 of which submitted final designs for evaluation.²¹ We therefore now deal with the possibility that there are many suppliers. For simplicity, we assume that the distribution of ideal states is uniform.

Assumption (A1) $f(\sigma) = 1 \forall \sigma \in [0, 1]$.

With this assumption, we can characterize the social optimum and the equilibria of the main contests previously discussed. Though most results also apply to the case $n = 3$, an FPT does

²¹European Commission (2014), "German company has won the EU's € 2 million vaccine prize." March 10, 2014 (accessed on April 3, 2015). http://ec.europa.eu/research/health/vaccine-prize_en.html

Figure 2.3: Equilibria when $n = 6$.

not have a pure strategy equilibrium in this case. To allow for simple formulations, we confine ourselves to $n > 3$.

Lemma 2.4. *Suppose there are $n > 3$ suppliers and (A1)'' and (A2) hold.*

(i) *The social optimum is $(v_1^*, \dots, v_n^*) = (1/2n, 3/2n, 5/2n, \dots, (2n-1)/2n)$.*

(ii) *The social optimum can be implemented with a bonus tournament where $\mathcal{P} = \{b/n, 0\}$ and $t = C$ or with an auction with appropriate $t \geq 0$.*

(iii) *In any equilibrium of an FPT with n suppliers, there is duplication, and the amount of diversity is inefficiently low. As n increases, the difference between the socially optimal diversity and the minimal diversity in any FPT equilibrium converges to zero.*

Figure 2.3 illustrates the result for $n = 6$. In line with Lemma 2.4(i), there is no duplication in the social optimum, and the approaches are evenly spread. The buyer can implement the social optimum with a bonus tournament or an auction. The two other constellations describing the equilibria of the FPT highlight implications of Lemma 2.4(iii). First, the two most extreme approaches are not as far apart as the most extreme approaches of the social optimum; in this sense, there is less than optimal diversity. Second, there is duplication.²²

Lemma 2.4 allows us to compare different institutions.

Proposition 2.3. *Suppose there are $n > 3$ suppliers and (A1)'' and (A2) hold.*

(i) *The buyer prefers to implement the social optimum with a bonus tournament rather than an auction if and only if $C < (n-1)b/2n^3$.*

(ii) *The buyer strictly prefers the bonus tournament $(b/n, 0)$ to any FPT for $n > 4$; she is indifferent for $n = 4$.*

Proposition 2.3(i) qualifies the result for the case $n = 2$, in which the buyer always prefers bonus tournaments to auctions under assumptions (A1) and (A2), which include uniform state distributions and linear quality functions. Intuitively, with $n > 3$ suppliers bonus tournaments still implement the social optimum with the lowest possible supplier revenue: Mirroring the logic

²²The remaining features of the depicted FPT hold in a class of FPT equilibria given in Lemma B.3 in Appendix B.5: The two most extreme approaches are always chosen by two suppliers. Moreover, depending on the specific equilibrium, there may be additional duplication for intermediate approaches.

for the case of two suppliers, the price is zero except for $\sigma \leq v_1$ and for $\sigma \geq v_n$, when it just compensates for the quality difference to the second-best supplier. This leads to asymmetric revenues of suppliers. In auctions the revenues are more symmetric than in bonus tournaments, so that subsidies for which all suppliers break even involve less rents for the suppliers whose expected revenues are highest.²³ Proposition 2.3(ii) generalizes the corresponding result for the benchmark model, with a small qualification for $n = 4$.

Lemma 2.4 has another simple but important implication: It may be socially optimal to invite a large number of suppliers. This differs from the case of contests that merely influence the suppliers' efforts: Several papers show that, in those settings, the optimal number of participants is typically two.

Corollary 2.4. *Suppose research costs are $C > 0$ and that (A1)'' holds. Define $n_-(C) = \max \{n \in \mathbb{N} \mid 2 \leq n \leq \sqrt{b} / 2\sqrt{C}\}$ and $n_+(C) = n_-(C) + 1$. Auctions or bonus tournament with $n_-(C)$ or $n_+(C)$ suppliers maximize total surplus in the set of all contests with an arbitrary number of suppliers.*

With straightforward additional arguments, Corollary 2.4 is implied by the previous results. Lemma 2.4(i) characterizes the socially optimal allocation for given n , and auctions and bonus tournaments implement this allocation. Corollary 2.4 describes the number of suppliers that optimally balances the gains from higher expected quality against the losses from higher research costs. The result implies that the optimal number of suppliers increases in b and decreases in C . While the corollary is stated for the socially optimal contest, it is simple to show that the buyer can also often benefit from inviting more than two suppliers and that the comparative statics are similar. In particular, in a bonus tournament an increase in n leads not only to an increase in the expected quality (reflecting higher option value), but also to a reduction in rents that suppliers 1 and n obtain (reflecting an increase in competition).

2.5.2 Other Extensions

We now discuss several other extensions. We deal with heterogeneous suppliers, multiple prizes and multiple research approaches of each supplier. In particular, the first issue is treated in much more detail in the working paper (Letina and Schmutzler 2015).

2.5.2.1 Generalized distributions and quality functions.

In this subsection, we still assume that there are only two suppliers, but we generalize the assumptions as follows:

Assumption (A1)' *The density function $f(\sigma)$ is (i) symmetric and (ii) has full support: $f(\sigma) > 0 \forall \sigma \in [0, 1]$.*

Assumption (A2)' *$\delta(|v_i - \sigma|)$ is increasing and continuous.*

Thus, we relax the requirements that the distribution be single-peaked and relatively flat and that the distance function be linear.

²³This issue obviously does not arise when differentiated subsidies are possible.

Lemmas 2.2, 2.3 and Proposition 2.1 also hold under the relaxed assumptions (A1)' and (A2)'. The proofs are analogous and are therefore omitted here. As a result, the main contests that we previously dealt with have the same properties as before:

Corollary 2.5. *Suppose that (A1)' and (A2)' hold. Then, (i) the bonus tournament ($\mathcal{P} = \{\Delta q(v_1^*, v_2^*), 0\}$) and the auction mechanism ($\mathcal{P} = \mathbb{R}^+$) implement the social optimum with appropriate $t \geq 0$. Moreover, (ii) in any FPT ($\mathcal{P} = \{A\}$ for $A \geq 2C$), the unique equilibrium is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

The rankings between the contests are similar to the benchmark model of Section 2.2. However, there are cases where the buyer prefers auctions to bonus tournaments. The intuition is essentially the same as for the case with multiple suppliers: While the bonus tournament implements the social optimum with lower supplier revenues than the auction, it may require higher subsidies. The following result clarifies the circumstances under which bonus tournaments are preferable even so.

Proposition 2.4. *Suppose that (A1)', and (A2)' hold. Then, (i) the buyer strictly prefers a suitable bonus tournament to the FPT whenever $C > 0$. (ii) The buyer weakly prefers a suitable bonus tournament to the auction if at least one of the following conditions holds: (a) $C \leq \min \{F(v_1^*) \Delta q(v_1^*, v_2^*), (1 - F(v_2^*)) \Delta q(v_1^*, v_2^*)\}$, (b) $v_1^* + v_2^* = 1$, or (c) $f(\sigma)$ is single-peaked. Whenever (a) holds, the preference is strict.*

According to (i), a suitable bonus tournament is still always preferable to an FPT in the more general set-up. Together, the conditions in (ii) show that a suitable bonus tournament dominates an auction under quite general conditions: Counterexamples require that research costs are high, that the social optimum is not symmetric and that $f(\sigma)$ is not single-peaked.

2.5.2.2 Heterogeneous Suppliers

The assumption of homogeneous suppliers simplifies the analysis. In many contexts, it is nevertheless natural to allow for exogenous heterogeneity: Suppliers may differ with respect to expertise or research capabilities. Architects may have different and essentially fixed styles. In Letina and Schmutzler (2015), we extend the model to allow for such exogenous heterogeneity. To this end, we consider a two-dimensional state space to capture both exogenous and endogenous heterogeneity. We focus on uniform state distributions and the case $C = 0$. We show that the social optimum only involves diversification if exogenous heterogeneity is not too strong. As in the case of homogeneous suppliers with low research costs, however, fixed-prize tournaments do not induce any diversification, but buyers prefer them to auctions.

The framework with heterogeneous suppliers has an additional advantage: For sufficiently heterogeneous buyers, the modified framework allows us to use the alternative informational assumption that suppliers cannot observe qualities when they submit bids, which is intractable for homogeneous suppliers. We show that there is no diversification in this equilibrium for an auction.

2.5.2.3 Fixed-Prize Tournaments with Multiple Prizes

The US military research agency DARPA carried out various contests to foster the development of unmanned vehicles capable of navigating in rugged terrain. In the 2005 DARPA Grand Challenge, only the winner of the contest was eligible for the prize (\$2 million), while the other contestants received nothing. This corresponds to an FPT as introduced above. However, in the subsequent DARPA contest, known as the 2007 Urban Challenge, rules specified that not only would the winner receive a prize (which was again \$2 million), but the next two participants would also receive prizes (\$1 million and \$0.5 million).²⁴ While a full analysis is beyond the scope of this paper, we can show that a buyer is worse off in an FPT with two prizes than with a single prize.²⁵ The following result shows that the buyer has nothing to gain from using multiple prizes.

Lemma 2.5. *Suppose that $n > 3$ and that (A1)” and (A2) hold. Further, suppose that t is sufficiently large, and that the two prizes are $A_1 > A_2 > 0$. For any equilibrium in an FPT with two prizes, there exists an equilibrium in an FPT with a single prize which makes the buyer strictly better off.*

Clearly, when there are only two suppliers, a second prize has no effect, as the suppliers would consider it as a pure subsidy, and the effective prize would be the difference between the first and the second prize. The proof of Lemma 2.5 shows that any equilibrium of an FPT with two prizes involves more duplication than the chosen equilibrium of an FPT with a single prize, which leads to a lower buyer surplus. This result suggests that multiple prizes do not improve diversity.²⁶

2.5.2.4 Multiple Designs by the Same Supplier

We have assumed so far that each supplier can only develop a single approach. However, in the 2005 DARPA Grand Challenge, vehicles designed by the Red Team from Carnegie Mellon University took the second and third place. By developing multiple designs, a supplier internalizes some of the resulting option value. It is thus natural to allow for multiple approaches of different suppliers. The modified model is analytically intractable, but a numerical analysis suggest that our main results are robust. We study the cases with $n \in \{2, \dots, 5\}$ suppliers, each of which can develop $m = 2$ approaches, and the case with $n = 2$ suppliers, each of which can develop $m = 3$ approaches. We assume that (A1)” and (A2) hold and that $C = 0$. We also fix values of Ψ and b .²⁷

Numerical Result: *If there are n suppliers and each develops m approaches, then: (i) Both a bonus tournament and an auction implement the socially optimum described in Lemma 2.4(i),*

²⁴See Section 1.4 of the DARPA Urban Challenge Rules (2007) (accessed on June 24, 2015). http://archive.darpa.mil/grandchallenge/docs/Urban_Challenge_Rules_102707.pdf

²⁵The results can be extended to more than two prizes.

²⁶Of course, there may be reasons outside of the model which would make multiple prizes a desirable choice for a contest designer. For example, if suppliers are risk averse, providing multiple prizes may be a way of increasing their expected utility.

²⁷For details and the code used to obtain numerical results, see Supplementary Material for Section 5.2.4, available at <https://sites.google.com/site/iletina/research>.

with n replaced by $n \cdot m$. (ii) In an FPT, there exists an equilibrium which is identical to the maximally duplicative equilibrium of an FPT with $n \cdot m$ suppliers, each of which develops one approach.

The notion of a maximally duplicative equilibrium is made precise in Lemma B.3 in Appendix B.5: There, we consider a class of equilibria where maximal duplication occurs when each active research approach is chosen by two suppliers. While the analysis is clearly incomplete, the numerical result suggests that the case where n suppliers each develop m approaches can be analyzed using the framework where $n \cdot m$ suppliers each develop one approach (see Section 2.5.1).

2.6 Relation to the Literature

This paper contributes to the literature on optimal contest design, especially the design of innovation contests. The existing design literature focuses exclusively on effort incentives. In models of fixed-prize tournaments, Taylor (1995) shows that free entry is undesirable, and Fullerton and McAfee (1999) show that the optimal number of participants is two. Fullerton et al. (2002) find that buyers are better off with auctions rather than fixed-prize tournaments. In a very general framework, Che and Gale (2003) show that an auction with two suppliers is the optimal contest. Contrary to the previous literature, our paper focuses on the suppliers' choice of research approaches rather than on effort levels. We characterize the optimal contests in such settings, highlighting in particular the useful role of bonus tournaments.

Letina (2016) also studies the diversity of approaches to innovation, but the objects of analysis and the employed models are very different. He focuses on a market context with anonymous buyers, and he deals with comparative statics rather than optimal design. In particular, the paper finds that a merger decreases the diversity of approaches to innovation.

While we are not aware of any other paper that considers optimal contest design when diversity plays a role, some authors compare contests in related, but different settings. In Ganuza and Hauk (2006), suppliers choose both an approach to innovation and a costly effort.²⁸ However, these authors focus exclusively on fixed-prize tournaments, while we study the optimal contest design. Erat and Krishnan (2012) analyze a fixed-prize tournament where suppliers can choose from a discrete set of approaches.²⁹ The authors find that suppliers cluster on approaches delivering the highest quality. This result is related to our result that there is duplication of approaches in the equilibria of fixed-prize tournaments. In addition to allowing for alternative contests, our model also considers correlated rather than independent qualities; it is thus meaningful to speak of similar approaches.³⁰ Schöttner (2008) considers two contestants who influence quality stochastically by exerting effort. She finds that, for large random shocks, the buyer prefers to hold a fixed-prize tournament rather than an auction to avoid the market power

²⁸In Ganuza and Pechlivanos (2000), Ganuza (2007) and Kaplan (2012), the buyer has to choose the design or alternatively can reveal information about the preferred design.

²⁹See also Terwiesch and Xu (2008) for the effect of number of suppliers when exogenous random shocks are large. For empirical evidence see Boudreau et al. (2011).

³⁰See also Konrad (2014) for a variant of Erat and Krishnan's model where first best is restored if the tie-breaking is decided via costly competition (for example lobbying) as opposed to randomly.

of a lucky seller in an auction. This resembles the trade-off underlying our Proposition 2.2. However, her analysis does not speak to optimal design and the role of bonus tournaments. It also does not address the setting with $n > 3$ suppliers.³¹

Our paper is also related to the literature on innovation contests with exponential-bandit experimentation (see Halac et al. (forthcoming) and references therein). In these models, it is uncertain whether the innovation is feasible. Suppliers participating in the contest expend costly effort to learn the state, and they also learn from the experimentation of their opponents. The goal of the contest is to induce experimentation. However, each supplier experiments in the same way. In our model, experimentation arises at the industry level for suitable contests, as the heterogeneity of approaches allows the buyer to pick the best available choice.

More broadly, our paper is related to the literature on policy experimentation. For instance, Callander and Harstad (2015) show that decentralized policy experimentation yields too much diversity. In their model, there are two heterogeneous political districts which choose whether or not to experiment with policy. In addition, they choose which policy to experiment with. A policy experiment is successful with some probability and a successful experiment increases the value of that policy (for all districts) by a fixed amount. Since experimentation is costly, there is a free riding problem, which is especially severe when the districts want to experiment with similar policies. To reduce the free riding problem, a district will choose to experiment with a policy which is not desirable from the perspective of the other district. Hence, in equilibrium the policy experiments will be inefficiently diverse. Next, they show that centralization of political power can improve the outcome by reducing diversity. Contrary to our model, Callander and Harstad (2015) assume that the success probabilities of different experiments are independent, no matter how similar the policies are. This assumption removes the option value of having different experiments, which is central to our model. If there existed an ideal policy (in terms of quality) as in our model, then the option value would have to be traded off against the benefits of convergence emphasized by Callander and Harstad (2015). It would be interesting to see whether and how centralization would help to resolve this trade off.

In a related paper, Bonatti and Rantakari (2016) consider a setting where two agents choose which project to develop. To successfully develop a project, an agent exerts effort until a success occurs. For a successful project to be adopted (and yield a positive payoff) both agents have to consent to the adoption. By assumption, the agents have opposite preferences over the set of projects. The agents have an incentive to pursue extreme projects (which they like the most) but the veto power of the other agent forces them to compromise. As in Callander and Harstad (2015) the success of one approach is unrelated to the success of any other approach. This removes the option value of diversity that we identify in our paper.

³¹More broadly related is Bajari and Tadelis (2001) who do not deal with innovations, but with construction projects. The issue of the right approach to the problem arises in such settings as well. The supplier obtains new information during the period when the contract is being executed, which allows him to adapt the original approach at some cost. Since the relationship is between a buyer and only one supplier, the question of diversity of approaches does not arise. This is also true for the related work by Arve and Martimort (2016) who study risk-sharing considerations in the design of contracts with ex-post adaptation. Additionally, Ding and Wolfstetter (2011) consider a case where a supplier can choose to bypass the contest and negotiate with the buyer directly in an environment where innovation quality is obtained by expending costly effort.

2.7 Conclusions and Discussion

The ideal approach to solving an innovation problem is usually unknown to suppliers and buyers. Our paper investigates the implications of this uncertainty for contest design. Under very general conditions, it is socially optimal for suppliers to take diverse research approaches, and the social optimum can be obtained with both bonus tournaments and auction mechanisms. Inducing diversity of approaches to innovation is costly for the buyer. To reduce supplier rents, she may therefore want to induce suboptimal diversification. Bonus tournaments are in the set of optimal contests under quite general conditions. The difference between the bonus and the low price provides incentives for suppliers to diversify, which allows the buyer to fine-tune the amount of diversity induced. At the same time, bonus tournaments minimize the power of suppliers to exploit their quality advantage. The non-convexity of the price set is decisive for this feature.

Our results have practical implications for the design of innovation contests. While today most innovation contests feature fixed prizes, our results suggest that a better outcome could be achieved if an additional bonus prize was paid whenever the winner outperformed the second-best contestant by a sufficient margin. Such bonus prizes would be easy to implement and would not make the innovation tournaments significantly more complicated than they are today. Bonus prizes would give incentives to contestants to not only win the contest, but to win with a large margin. Our model suggests that this incentive would lead to an increase in the diversity of approaches to innovation.

Beyond innovation contests, our model can be used to analyze how institutions affect the incentives for experimentation when the optimal approach to solving a given problem is not known. We can think of our model as capturing product choice in markets with a unit mass of homogeneous buyers, each of which has unit demand. We can then interpret the uncertainty about the ideal state in two ways. First, it may reflect uncertainty about the buyers' taste. Second, it may capture an "engineering uncertainty" where the suppliers know what the buyers would like, but are uncertain about how to achieve this. Our results imply that an unregulated market maximizes expected total surplus. The unregulated market gives incentives for firms to optimally diversify, but leaves them with market power. The trade-off resembles the one between ex-ante incentives and ex-post monopoly power in the innovation literature. In our case, however, the higher expected quality from the unregulated market does not result from higher innovation incentives at the individual firm level, but rather from the higher diversification incentives at the market level. Our results point to a novel source of potential inefficiency stemming from price regulation: Firms in industries with regulated competition will be less likely to sufficiently experiment by introducing diverse products. At the same time, this result also points to the importance of vigorous competition. The incentive to diversify would be diminished if firms colluded or divided the market.

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3 Designing Dynamic Research Contests¹

Joint with Jean-Michel Benkert

3.1 Introduction

Research contests² have a long history as mechanisms for inducing innovation. From navigation and food preservation, to aviation,³ research contests have been used to find solutions to some of society's most pressing problems. Recently, the use of research contests by both private and public sector has been expanding rapidly. As in the past, research contests are used in order to foster innovation in some of the most pressing and difficult issues that society is facing. Some examples of problems to which research contests have been applied include vaccine technology, antibiotics overuse, space flight, robotics and AI, as well as environment and energy efficiency.⁴ Given that the 2010 America Competes Reauthorization Act authorized US Federal agencies to use prizes and contests, it can be expected that the importance of research contests will only grow in the coming years.

Mistakes in contest design can waste R&D funds and slow the development of important innovations. While some aspects of contest design, like the effect of the number of competitors or the allocation of prizes, are well studied,⁵ less is known about the dynamic aspects of contest design. At the same time, as Lang et al. (2014) point out, "there are surprisingly few multi-period contest models in which each player's decision problem is dynamic." This paper deals with the (buyer-)optimal design of dynamic research contests in precisely such an environment. We identify a novel design lever that the contest designer can use in order to increase efficiency of the contest — namely the fact that the prizes can differ depending on the time when they are awarded.⁶

We build on the model of Taylor (1995), where N sellers choose in each of the T periods whether to invest in research or not. Investment is costly, but in each period a seller does research he obtains an innovation, the quality of which is a random draw from some distribution

¹This paper should be cited as Benkert, J.-M. and I. Letina (2016), "Designing Dynamic Research Contests," *University of Zurich, Department of Economics, Working Paper No. 235*.

²Research contests are sometimes referred to as innovation contests or inducement prizes.

³1714 Longitude Prize, 1795 Napoleon's Food Preservation Prize and 1919 Orteig Prize, respectively.

⁴2012 EU Vaccine Prize; 2015 Better Use of Antibiotics Prize; 1996 Ansari X Prize, 2006 Northrop Grumman Lunar Lander XCHALLENGE and 2007 Google Lunar X-Prize; 2004 DARPA Grand Challenge, 2007 Urban Challenge, 2014 A.I. presented by TED XPRIZE; 1992 Super-Efficient Refrigerator Program, Progressive Insurance Automotive X PRIZE and 2015 NRG Cosia Carbon XPRIZE.

⁵See Che and Gale (2003), Moldovanu and Sela (2001) and Moldovanu and Sela (2006).

⁶To the best of our knowledge dynamic prizes have not been studied in the context of contest design before. Of course, dynamic payoffs have been used in other context, for example in bandit models like in Green and Taylor (forthcoming).

F. The investment decisions, and the current highest quality of their innovations are private information to the sellers. If a seller reveals the innovation to the buyer, the buyer can costlessly and accurately determine its quality. However, the quality of innovations is not verifiable by outside parties. In particular, a contract which conditions on the innovation quality is not enforceable by courts. These assumptions are standard in the literature.⁷ In order to incentivize the sellers to invest in innovation, the buyer commits to a scheme that will, in some enforceable way, pay out a prize to the winning supplier(s). For example in Taylor (1995), the seller commits to paying a prize P at the end of the T periods. Following the literature, we call this institution a *research tournament*.

We deviate from Taylor’s model in two important ways. First, we assume that the buyer can commit to a time-dependent prize scheme. This assumption only requires that the contracts can be time-dependent. There are many examples of such contracts — bills have to be paid by a certain date, with penalties for late payment, savings can be deposited for a fixed period of time, and delivery of parcels can be guaranteed within a specified time period. Even in the context of research contests we find examples of time-dependent contracts: in the 2006 Netflix prize in addition to the final Grand Prize, a \$50,000 Progress Prize could be awarded each year before the contest ended.

Second, in the main section we assume that the innovations have what we will call a *breakthrough structure*. That means that each innovation falls broadly into two categories — it is either a breakthrough or not. All breakthrough innovations are worth approximately the same to the buyer and all non-breakthrough innovations are worth approximately the same. As the name suggests, breakthrough innovations are worth substantially more. This does not imply that there are only two quality levels — there may be many. What is important is that there is a distinction between the very valuable breakthrough innovations and relatively less valuable other innovations. For example, many contests explicitly state the goal of the contest. Then a breakthrough is any innovation that reaches the goal. In the Ansari X Prize, the sponsors used the objectively verifiable goal of “build[ing] and launch[ing] a spacecraft capable of carrying three people to 100 kilometers above the Earth’s surface, twice within two weeks”.⁸ While the spacecraft that met the proxy could be better or worse, the difference to the organizer did probably not matter as much as achieving that publicly stated goal. Thus, we believe that the breakthrough innovation structure fits well with many research contests.

Given the breakthrough innovation structure, we show that in the first-best⁹ all suppliers invest and conduct research until at least one of them achieves a breakthrough, at which point all research stops. This is an example of a global stopping rule. However, giving just a single prize at the end of the T periods of the contest cannot implement a global stopping rule. Indeed, it uniquely implements an individual stopping rule, where each seller performs research until he achieves some threshold value which is determined by the size of the prize (Taylor, 1995). In order to implement a global stopping rule, the sellers must be incentivized to conduct research in every period and to truthfully reveal that they have achieved an innovation, while the buyer

⁷See for example Taylor (1995) and Che and Gale (2003).

⁸See the Ansari X Prize website <http://ansari.xprize.org>.

⁹By first-best we mean maximizing total surplus in the absence of informational barriers.

has to be incentivized to stop the other firms from conducting further research. We show that a *dynamic prize tournament*, which is essentially a research tournament with the value of the prize potentially changing over time, can achieve this. By appropriately choosing a prize schedule, so that the prizes increase in each period the contest continues, the buyer can be incentivized to stop the tournament as soon as a seller submits a breakthrough. This is done by setting the slope of the increase in prizes such that it is exactly equal to the marginal benefit to the buyer of one more round of research. At the same time, if the intercept of the prize schedule is high enough, the sellers will have an incentive to conduct research in every period and to truthfully reveal their breakthroughs as soon as they have them, because otherwise they would risk losing the tournament entirely. Thus, by appropriately choosing the slope and the intercept of the prize schedule the seller can implement any global stopping rule, and the first-best stopping rule in particular.

Next, we relax the assumption on breakthrough innovations. First we characterize the optimal global stopping rule with the more general innovation structure and show that it satisfies the property that the marginal benefit of one more round of research equals the marginal cost. We then show that any global stopping rule can be implemented with an appropriately designed schedule of prices. Thus, our implementation result is robust to the specification of the innovation structure. Moreover, we show that any global stopping rule can be implemented even if the number of sellers doing research changes from period to period, as long as the sequence is fixed *ex ante*. That is, our implementation result does not depend on the assumption that the set of sellers is constant for the entire duration of the contest. This is especially important as some contests proceed in stages, so that some participants are eliminated in every stage.¹⁰ Finally, we consider the effect of a change in the horizon T of the contest. We show that, with a global stopping rule, an increase in T always increases the payoff of the buyer. The intuition is that, with a global stopping rule, the contest is stopped whenever an innovation of high enough quality is realized. Thus, the contest only continues past the time T if it was beneficial to do so. Whereas in a research tournament, an increase in the horizon T can lead to a decrease in buyer payoff. The reason for this is that with individual stopping rules the longer the time horizon is, the higher the chance of wasteful duplication.

Generally, research contests are classified into research tournaments and innovation races.¹¹ To win a research tournament a seller needs to have the best innovation at some specific date, whereas a seller needs to have a specific innovation as quickly as possible to win an innovation race. Thus, for an innovation race to be feasible, verifiability is necessary in order to determine whether some proposed innovation is indeed the innovation required to win the race. When innovation races are implemented in practice, a verifiable proxy is commonly used to determine whether an innovation meets the buyer's requirements. Recall that in the case of the 1996 Ansari X Prize, the objectively verifiable proxy was to have two manned space flights within two weeks using the same spacecraft. The larger objective of the organizer, however, was to "incentivize the creation of a safe, reliable, reusable, privately-financed manned space ship to demonstrate that private space travel is commercially viable". The advantage of a race is that it

¹⁰See for instance the IBM Watson AI XPRIZE.

¹¹See the discussion in Taylor (1995).

proceeds until an appropriate innovation has been developed and that it minimizes the wasteful duplication. When implementing research tournaments, the buyer in general does not have to use a proxy. However, other problems arise. In contrast to innovation races the sponsor of a research tournament needs to announce an end date at which the submissions will be judged. If the competitors are not given enough time, they may fail to produce a good enough innovation.¹² If the deadline is very late, however, there is a risk of wasteful duplication.

Our dynamic prize tournament offers a solution to these problems. We show that when implementing a global stopping rule the buyer benefits from increasing the duration of the contest, thereby allowing for very late deadlines which increase the chance of getting a sufficiently good innovation. At the same time, the nature of the global stopping rule ensures that throughout the contest an innovation race is taking place, thus avoiding the risk of duplication arising in research tournaments. Moreover, this innovation race is implemented without requiring a proxy, hence eliminating this source of inefficiency. Effectively, a dynamic prize tournament inherits the best properties of both innovation races and research tournaments. Overall, our results indicate that using dynamic prizes could result in substantially more efficient research contests.

The structure of the paper is as follows. Section 3.2 reviews the relevant literature. Section 3.3 presents the model. Section 3.4 characterizes the first-best and shows that it can be implemented with a dynamic prize tournament. Section 3.5 considers extensions to the main model and in particular relaxes the assumption of breakthrough innovation. Section 3.6 concludes. All proofs are relegated to the appendix.

3.2 Related Literature

The seminal paper on dynamic research contests is Taylor (1995) on which we build our model.¹³ He shows that a T -period research tournament with N sellers, that is, a fixed prize is awarded at the end of the contest, uniquely implements an individual stopping rule among the sellers. Further, Taylor shows that it is optimal to limit the number of sellers in the contest and that the buyer can extract the entire ex ante surplus using appropriate entry fees. As Taylor himself notes, however, his contest generally fails to implement the first-best, which entails a global stopping rule instead of an individual stopping rule (Gal et al., 1981). Moreover, Morgan (1983) shows that holding the number of sellers conducting research fixed across time is generally not optimal either, as the optimal number should vary over time depending on the currently highest quality among the sellers and the number of periods left.¹⁴ Fullerton et al. (2002) compare Taylor's research tournament to an auction in the same setting. That is, the only change to Taylor's framework is that at the end of the contest an auction is used to allocate the prize among the sellers. The authors argue that this lowers the buyer's informational requirements

¹²The objective of the 2004 DARPA Grand Challenge was to "accelerate the development of autonomous vehicle technologies that can be applied to military requirements" but none of the competitors managed to fulfill the requirements of the tournament. Eventually, the requirements were matched in the 2005 DARPA Grand Challenge, suggesting that more time was needed to be successful. See the official website on <http://archive.darpa.mil/grandchallenge04/>.

¹³Konrad (2009) provides an excellent overview of the literature on contests. See also Siegel (2009) for general results on all-pay auctions.

¹⁴See Morgan and Manning (1985) for general results on the first-best search rule in this environment.

and they provide experimental evidence that this is more cost-effective when the buyer cannot charge entry fees. It is noteworthy that the buyer continues to employ an individual stopping rule when using an auction and therefore still fails to implement the first-best. Rieck (2010) considers a variation of Taylor's framework which enables him to study the role of information revelation. He shows that when the sellers' research outcomes are publicly revealed there are essentially two thresholds instead of one. If the highest quality among the sellers is above the upper threshold all sellers stop research, if the highest quality is between the two thresholds only the leading seller stops research and if all qualities are below the lower threshold all sellers continue to do research. Depending on the parameters the buyer may be better off with or without information revelation.

Recently a number of papers have used bandit models to study the problem of incentive provision for dynamic research activity. Halac et al. (forthcoming) consider the optimal design of contests for innovation when it is unclear *ex ante* whether or not the innovation in question can actually be successfully realized. Thus, in contrast to our setting the reason for research activity is not to get as good an innovation as possible, but rather to determine an innovation's feasibility. The buyer who designs the contest can choose the prize-sharing scheme and a disclosure policy which determines what information is revealed to the sellers about their respective outcomes. Similarly to our setting, the first-best features a global stopping rule. However, Halac et al. (forthcoming) find that a contest which does not entail a global stopping rule can be optimal in the presence of private effort provision by the sellers. More generally, they show that in a broad class of contests it is optimal to stop the contest only once a certain number of sellers had a success and to share the prize between them. Bimpikis et al. (2014) study a closely related question to Halac et al. (forthcoming) but allow for partial progress, i.e., in order for an innovation to be feasible a milestone or breakthrough is necessary before the potential success is realized, and the buyer may set a prize for the breakthrough and the eventual success each. While they do not consider the question of optimal design, they show that the buyer may benefit from not revealing a partial success. Along similar lines Green and Taylor (forthcoming) consider the role of breakthroughs in a single-agent contracting environment. In contrast to our framework, the research outcome can be contracted upon and the problem the buyer faces is how to optimally induce effort over time using a first deadline for the breakthrough, a second deadline for the final outcome and a monetary transfer. In their paper the monetary transfer is decreasing over time, which induces the agent to aim for an early success. Thus, the slope of the prize schedule is used to affect the seller's incentives. In contrast, in our paper the increase in prizes over time serves to align the buyer's incentives.

The seminal paper in the literature on optimal design of research contests in the static context is Che and Gale (2003). The authors consider a model where the innovation technology is deterministic. Once the innovations are developed, the sellers bid for a price at which the buyer can obtain their innovation. The buyer then chooses the bid which offers her the highest surplus, that is, the highest difference between the value of the information and the bid. The contest design consists of the choice of the set from which the sellers can choose their bids. This set of mechanisms turns out to be very flexible and includes the fixed-prize structure of the research tournament (when the set of allowable bids is a singleton) and the auction

(when the set is \mathbb{R}^+). The authors show that with symmetric sellers the optimal contest is an auction and the optimal number of sellers is two. When sellers are asymmetric, the optimal contest is still an auction with two sellers, but the optimal auction handicaps the more efficient sellers.¹⁵ The major difference between Che and Gale (2003) and our paper is that we focus on the dynamic aspects of contest design. Thus, the question of wasteful duplication of effort, which is the central issue we address with the dynamic prize tournament, does not show up in Che and Gale (2003). Another difference is the choice of innovation technology — when innovation is deterministic, as in Che and Gale (2003), there is no sampling benefit from having more than two sellers.¹⁶ Additionally, an auction gives market power to the sellers, and when the innovation technology is sufficiently random, an auction might perform badly as the seller profits from the good realizations.¹⁷ Several other directions have been explored in the static setting. Letina and Schmutzler (2016) consider the optimal contest design when the sellers can choose their approach to innovation and the buyer attempts to give them incentives to diversify their approaches because of the resulting option value. They find that the optimal contest is what they call a bonus tournament, where a winner gets a fixed prize, plus a bonus if he outperforms the second best seller with a high enough margin. Moldovanu and Sela (2001) consider how a total prize sum should be split by a buyer who is maximizing the expected effort. They find that if the cost functions are concave or linear in effort, then it is optimal to allocate the entire prize sum to the winner. If, on the other hand, cost functions are convex, it may be optimal to offer multiple prizes.

Lang et al. (2014) consider a two-player, fixed-price contest where sellers exert effort over time and breakthroughs arrive according to a Poisson process. The seller with the most breakthroughs wins. The authors consider the effect of changing the time T when the contest ends. They find that the buyer can be better off with a shorter deadline, which is exactly our finding in Proposition 3.6 for a research tournament. However, we also show that this result does not hold for arbitrary contests — in our setting, a buyer is always better off with a longer deadline when implementing a global stopping rule.

More broadly related are papers examining if buyers should be split into several subcontests or eliminated over time. Moldovanu and Sela (2006) find that if the buyer is maximizing the expected value of the highest effort it is beneficial to split the sellers into preliminary subcontests and to have the finalists compete against each other. In a different setting,¹⁸ Fu and Lu (2012) find that maximizing expected effort involves eliminating one seller in each round. An additional question that has been examined in the literature on dynamic contests is the question of how the information about the relative performance during the contest should be used. Gershkov and Perry (2009) study a contest with two agents and two stages and ask if a review should be conducted after stage one. They show that, assuming reviews are aggregated optimally, it is always beneficial to conduct a midterm review. In a related setting, Klein and Schmutzler (forthcoming) consider how a prize sum should be allocated for the first and second period

¹⁵Discrimination in contests is also studied in Pérez-Castrillo and Wettstein (2016).

¹⁶See for example Terwiesch and Xu (2008) and Letina (2016).

¹⁷This is the case in Schöttner (2008) who shows that when innovation technology is sufficiently random, a research tournament can outperform an auction.

¹⁸They use nested contest as in Clark and Riis (1996).

performance, and how information about it should be used. They show that for large parameter regions, entire prize sum should be paid to the winner but both first-period and second-period performance should be weighted when determining the winner.

There is relatively little empirical work on dynamic research contests. Using data on software contests Boudreau et al. (2011) find that increasing the number of participants reduces average effort but increases the chance of getting a very high quality innovation. Also using data on software contests Boudreau et al. (2016) find that the results derived in Moldovanu and Sela (2001) generally perform quite well. In particular, they find that the response of participants to an increase in the number of competitors yields heterogeneous responses. Namely, low ability agents respond weakly, medium ability agents decrease their efforts while high ability agents increase their efforts. We refer to the recent survey Dechenaux et al. (2015) for experimental work on contests.

3.3 The Model

There is a risk-neutral buyer who wants to procure an innovation and $N \geq 2$ ex ante identical risk-neutral sellers who can potentially produce the innovation by conducting research. If the buyer obtains the innovation in any period $t \in \{1, \dots, T\}$ with $T < \infty$, her payoff is $\theta - p$, where θ is the quality of the innovation and p is the sum of any transfers. A seller's payoff is $m - c$, where m is the sum of any transfers received and c is the total cost incurred through research activities. The innovations are of no intrinsic value to the sellers.

A seller can conduct research in any period t at per-period cost $C > 0$. In each period in which the seller performs research he obtains an innovation of value $\theta \in \Theta$. The innovation value obtained is an independent draw from some distribution F with full and finite support where $\Theta = \{\theta^1, \theta^2, \dots, \theta^K\}$. Suppose without loss that $\theta^{k+1} > \theta^k$ and we normalize $\theta^1 = 0$. Sellers can repeatedly conduct research and have perfect recall, that is, they can access all their own previous innovations at any point in time. Initially, every seller is endowed with a worthless innovation and in each period a seller does not perform research he receives a worthless innovation.

We say a research contest features a *breakthrough innovation structure* if there exists some innovation $\theta^b \in \Theta$ such that all innovations below θ^b are worth very little to the buyer, while all innovations at or above θ^b are worth approximately the same. Thus, there are essentially two levels of innovation — breakthroughs and low-value innovations. In Section 3.5 we will relax the assumptions on Θ . Formally, we will say that the innovation process has a breakthrough innovation structure if the following assumption is satisfied.

Assumption 3.1 (breakthrough innovation structure). *There exists $\theta^b \in \Theta$ such that (i) $\theta^K - \theta^b < C$ and (ii) $\theta^b \geq \bar{\theta}$, for some threshold value $\bar{\theta}$.¹⁹*

This assumption captures the intuition that (i) all innovations in which a breakthrough was realized are of roughly the same value to the buyer, and (ii) reflects that all breakthroughs are of sufficiently high value. A very simple example of breakthrough innovation structure would be

¹⁹See the proof of Proposition 3.1 for the precise statement of the threshold $\bar{\theta}$.

$\Theta = \{0, \varepsilon, B, B + \varepsilon\}$ for sufficiently large B and small ε . Then B and $B + \varepsilon$ are breakthrough innovations.

The sellers' research activity (whether or not they conduct research in any given period) and research outcomes (the value of an innovation obtained in any given period) are private information. If a seller submits an innovation to the buyer, the buyer can determine the value of the innovation at no cost. However, the value of an innovation is not verifiable by a court. Thus, contracts conditioning on the value of innovation are not credible.²⁰ To overcome the hold-up problem, the buyer commits to holding a contest and to paying a prize p to the winner of the contest. We assume that the buyer can commit to paying a prize p_t if the tournament ends in period t . This requires that the courts can verify (i) when the contest was declared over, and (ii) if the correct prize was paid.

In period 0 the buyer announces the contests $\langle E, \mathbf{p}, n \rangle$ which consists of an entry fee E , a prize schedule $\mathbf{p} = [p_1, p_2, \dots, p_T]$ and the maximal number of participants n . Sellers observe $\langle E, \mathbf{p}, n \rangle$ and decide whether to pay the (possibly negative) entry fee E .²¹ If less than two sellers decide to participate, the contest is canceled. If more than n sellers wish to participate, n are selected randomly. In each period $t \in \{1, \dots, T\}$ the participants in the contest can simultaneously conduct research and subsequently decide whether or not to submit an innovation to the buyer. At the end of the period the buyer can declare any of the submissions the winner in which case the contest ends and the seller who submitted the winning innovation receives the prize p_t .²² If no winner is declared in period t or there have been no submissions, the contest proceeds to period $t + 1$ unless it was already the last period of the contest. In this case, all sellers submit an innovation and the buyer must declare a winner.

This model of dynamic research contests is essentially the same as the one proposed in Taylor (1995). We make only two changes. First, we assume that the buyer can commit to a schedule of period-specific prizes \mathbf{p} . This assumption only requires that the contracts can be time-dependent, which we view as uncontroversial. Second, to derive our main results we impose some structure on the set of innovations Θ . Namely, we assume that innovations have a breakthrough structure as discussed above and that the set Θ is a discrete set.²³

The contest $\langle E, \mathbf{p}, n \rangle$ induces a T -period dynamic game of incomplete information with the set of players being the buyer and the sellers who participate in contest. The set of players, their payoff functions, the research technology and the contest structure are common knowledge. The seller's research activity and outcomes are private information. The timing of the game is as follows.

²⁰Non-observability and non-verifiability is a typical feature of research activity. As Taylor (1995, p. 873) notes "research inputs are notoriously difficult to monitor" and "courts seldom possess the ability or expertise necessary to evaluate technical research projects".

²¹The assumption that the buyer can charge an entry fee is taken from Taylor (1995). It is essentially an assumption that the sellers are not liquidity constrained. Alternatively, the results would remain unchanged if we assumed that the buyer cannot charge an entry fee, but instead maximizes social welfare. Many research contests are motivated exactly by social welfare and not by the private profit of the contest organizer.

²²In principle, the buyer could commit to not end the contest early. We assume that this is not possible in order to reduce the dimension of the buyer's strategy space. The assumption is without loss, as setting very high p_t essentially commits the buyer to not end the contest in period t .

²³The discreteness assumption is made to avoid the technicalities of defining beliefs in a dynamic game with continuous spaces. As already noted we will relax the breakthrough structure in Section 3.5.

Period 0:

- All N invited sellers decide whether to enter or not. If they enter they pay the entry fee E .

Period $t < T$:

- Stage 1: Each seller simultaneously decides whether to perform research at cost C . Sellers do not observe the actions taken by their competitors.
- Stage 2: Each seller i who conducted research receives quality which is a random draw from F . All other sellers receive quality 0.
- Stage 3: Having privately observed the value of their innovation, sellers simultaneously decide whether to privately submit their best innovation.
- Stage 4: The buyer observes the set of submissions. If it is empty the contest continues. If not, the buyer decides whether to declare a winner or not. If a winner is declared the contest stops, the buyer obtains the winning innovation and the seller who submitted the winning innovation receives the prize p_t .

Period T :

- Stages 1-3: As above.
- Stage 4: The contest stops and the buyer has to declare a winner whose submissions the buyer then obtains in exchange for the prize p_T . If no seller submitted, the prize is randomly allocated.

3.4 Optimal Contest

In this section we characterize the optimal research contest. We first show that given our assumptions on the breakthrough innovation structure, the first-best is equivalent to a global stopping rule with all N firms conducting research — that is, all firms do research in every period until at least one firm achieves a breakthrough. As soon as a breakthrough is achieved, all sellers stop doing research. Next, we show that using an appropriately designed dynamic prize tournament, where the prize that the buyer awards is increasing in t , any global stopping rule can be implemented. Finally, combining these two results, we show that in the optimal contest the buyer implements the first-best outcome. The optimal contest features a dynamic prize structure which gives the buyer the incentives to stop the contest as soon as at least one seller has had a breakthrough, and to the sellers the incentives to do research in every period and to truthfully report as soon as they have a breakthrough.

Proposition 3.1. *Consider a breakthrough innovation structure such that Assumption 3.1 holds. Then, in the first-best, all N firms conduct research in every period until a breakthrough is achieved after which research is stopped completely.*

In general, the first-best is characterized by a function $n(\theta, t)$, which specifies the number of sellers which optimally do research as a function of time and the current highest quality θ . Gal et al. (1981) and Morgan (1983) have shown that in the first-best there exists a global stopping quality θ^g such that $n(\theta, t) = 0$ for $\theta \geq \theta^g$ and that the first-best number $n(\theta, t)$ is decreasing in θ and increasing in t . In particular, the optimal number of sellers doing research will generally change non-monotonically over time.²⁴ However, given our assumption about the breakthrough innovation structure, the first-best plan takes a very simple form. Since the breakthrough is very valuable, it is optimal to have all firms perform research as long as no breakthrough has been achieved. However, as all innovations with a successful breakthrough have roughly the same value, continuing the research effort is not worth its cost once a breakthrough was achieved and it is optimal to stop all research activities.

Taylor (1995) shows that any research tournament, that is, a contest with a fixed prize paid out at the end of the contest, uniquely implements an individual stopping rule, where each firm does research until it has reached its individual threshold level, irrespective of the qualities discovered by other sellers. Such an individual stopping rule consequently entails a fairly large risk of wasteful duplication across sellers. Intuitively, a global stopping rule seems better than any individual stopping rule, as it reduces the amount of wasteful research by stopping research once any seller has discovered an innovation of high enough quality. However, in general a global stopping rule can not be implemented with a research tournament. In a research tournament as in Taylor (1995) with a fixed prize, the buyer cannot credibly commit to stop the contest after the threshold quality has been achieved because she does not bear the marginal cost of continued research, while she stands to benefit from any marginal increase in quality. In contrast, our next proposition shows that if it is possible to commit to a dynamic prize schedule, then any global stopping rule can be implemented using a dynamic prize tournament.

Proposition 3.2. *Any global stopping rule $\theta^g \in \Theta$ with N sellers can be implemented using a dynamic prize tournament $\langle E, \mathbf{p}, N \rangle$ with a prize schedule $\mathbf{p} = [p_1, p_2, \dots, p_T]$ such that*

$$p_t = p_1 + (t - 1)\Delta(\theta^g, N),$$

where

$$\Delta(\theta^g, n) = F(\theta^g)^n \theta^g + \sum_{j=g+1}^K \left(F(\theta^j)^n - F(\theta^{j-1})^n \right) \theta^j - \theta^g,$$

for $t \in \{2, \dots, T\}$ and $p_1 \geq \bar{p}$, where \bar{p} is some cutoff value and E is sufficiently low to induce entry of all N sellers.

The proposition shows that for any $\theta^g \in \Theta$, there exists a dynamic prize tournament $\langle E, \mathbf{p}, N \rangle$ with a prize schedule \mathbf{p} , and an entry fee E , such that in equilibrium all N sellers enter. Fur-

²⁴Given the finite time-horizon there is the basic trade-off between increasing the chance of getting a high quality innovation by having many sellers do research in a given period and risking wasteful duplication. As the deadline approaches the buyer becomes more willing to risk duplication for a given quality level as there are less research opportunities in the future. Having a relatively high quality innovation early on reduces the pressure to get a better innovation in the future and the buyer is therefore less willing to risk duplication by having many sellers do research simultaneously.

thermore, each seller performs research until it reaches quality θ^g . Once this occurs, the seller reports the discovered innovation to the buyer, who immediately stops the contest, declares a winner and pays out the prize corresponding to the period.²⁵ The dynamic prize schedule in Proposition 3.2 increases by $\Delta(\theta^g, N)$ in every period, which is exactly the marginal benefit of one additional round of research when the current highest quality is θ^g and N sellers will be conducting research in the next period. Thus, the marginal cost of an additional round equals its marginal benefit, which makes this commitment credible, as the buyer is indifferent between continuing and stopping the tournament when the current highest quality is θ^g . Since the marginal benefit of research is decreasing in θ , the buyer strictly prefers to continue the tournament whenever the highest quality is below θ^g and strictly prefers to stop it whenever it is above. Thus, it is the slope of the prize schedule that provides incentives to the buyer necessary to implement a global stopping rule.

On the other hand, the intercept of the prize schedule gives the incentives to the sellers to perform research in every period and to report their outcomes truthfully. Intuitively, as in the research tournament, each seller pursues an individual stopping rule which is determined by the expected prize. Increasing the expected prize increases the individual stopping threshold. If the individual stopping threshold is above the global stopping threshold, the sellers will conduct research as long as the tournament is ongoing. Similarly, it is the size of the prizes that induces the sellers to truthfully report their research outcomes. By not reporting an innovation above the threshold, a seller could win a higher prize in the future. However, not reporting exposes the seller to the risk that another seller will win in the current period and end the tournament. As long as the increase of the prize in the next period is sufficiently small relative to the current-period prize, the seller will report truthfully. Thus, incentives of the sellers can be satisfied by making the average prize large enough by shifting up the prize schedule sufficiently. Since such a shift in the prize schedule does not affect its slope, which is the sole determinant of the buyer's incentives across periods, both buyer and seller incentives can be satisfied simultaneously, such that a global stopping rule results in equilibrium. Finally, the entry fee E can be chosen such that N sellers indeed want to participate in such a tournament by making it an entry subsidy if necessary.

The above intuition also serves as a sketch of the proof. Alternatively, we could prove the first part of the result regarding the buyer's credibility using the results in Kruse and Strack (2015). They prove in a very general setting that a stopping rule can be implemented using a transfer which depends only on the stopping decision (i.e., the prize paid out in the period in which the tournament ends) if and only if it is a cut-off rule (i.e., stopping the tournament once the threshold θ^g is reached). They consider a choice problem and not a game as we do here. We can nevertheless apply their result to prove the credibility of the buyer to stop the tournament once the threshold is reached and to derive the prize sequence. The second part of the proof which relates to the sellers' incentives can then be proved following the intuition above, that is, by choosing sufficiently high average prizes.

Note that the Proposition 3.2 does not rely on the breakthrough innovation structure. That

²⁵In case of multiple breakthroughs of equal quality being reported simultaneously, the buyer randomly declares a winner among these innovations.

is, it holds for any vector of feasible innovation qualities Θ . In fact, the only strengthening of the assumptions from Taylor (1995) required to implement a global stopping rule instead of an individual stopping rule is that the buyer can commit to a dynamic prize schedule instead of only a fixed prize at the end of the tournament. We view this assumption as uncontroversial as time-dependent contracts are pervasive.

Proposition 3.2 establishes that N participants would wish to take part in such a tournament and behave such that a global stopping rule is implemented. However, we still need to show if the buyer would wish to announce this dynamic prize tournament in the first place. The next proposition answers this question in the case of a breakthrough innovation structure.

Proposition 3.3. *Suppose that Assumption 3.1 is satisfied. Then the optimal contest is a dynamic prize tournament and it implements the first-best.*

The proof of this result is straightforward. We know from Proposition 3.1 that under Assumption 3.1 the first-best is a global stopping rule in which all N sellers conduct research in every period until a breakthrough was had and then stop. Thus, the first-best is characterized by a constant number of sellers doing research until a global threshold is reached. However, this is precisely what Proposition 3.2 tells us we can achieve using an appropriate dynamic prize tournament. Moreover, with the contest implementing the first-best the buyer would obviously want to announce it.

Taylor (1995) notes that the first-best could be achieved if the buyer, instead of holding one multi-period contest, held a series of one-shot contests. In this case the buyer could choose the optimal number of sellers in each period given the current highest quality and compensate them for the one-period effort. However, if inspecting the sellers' submissions is costly, running a sequence of one-period tournaments and inspecting submissions after every period may be very costly. This points to another advantage of the dynamic prize tournament. Namely, the buyer only has to inspect submissions once and only from the sellers who have developed an innovation of high enough quality. As mentioned in the introduction, we commonly distinguish between innovation races and research tournaments. Interestingly, our dynamic prize tournament acts like an innovation race as long as the tournament runs, as the first seller who makes a breakthrough wins and eventually turns into a classic research tournament with the best innovation available in the last period winning. Moreover, this innovation race is implemented without requiring verifiability, a sharp contrast to regular innovation races.

3.5 Robustness

In this section we show that we can implement global stopping rules more generally using dynamic prize tournaments and we compare their performance to research tournaments, which have a fixed prize. Throughout this section we do not impose any structure on $\Theta = \{\theta^1, \theta^2, \dots, \theta^K\}$ except $\theta^{k+1} > \theta^k$ and the normalization $\theta^1 = 0$.

We stated in Proposition 3.1 that it is optimal to stop research once a breakthrough has been achieved. The next result characterizes the threshold at which research should stop in the absence of the breakthrough structure, i.e., it gives the optimal stopping rule for N sellers.

Proposition 3.4. *The optimal global stopping rule with N sellers is given by the smallest $\theta^k \in \Theta$ such that*

$$F^N(\theta^k) \theta^k + \sum_{j=k+1}^K (F^N(\theta^j) - F^N(\theta^{j-1})) \theta^j - \theta^k \leq NC. \quad (3.1)$$

When the number of sellers is not fixed the global stopping level is determined by the point at which the marginal gain of doing another round of research with exactly one seller equals its marginal cost. Essentially, there is a quality level at which another round of research with even only a single seller is not worth the cost, irrespective of how many opportunities to do research are yet to arrive. If the buyer could reduce the number of sellers further, another round of research might be optimal. However, as she has hit the lower bound, there is no room to reduce the number further and the research optimally stops. Inspecting equation (3.1) we see that this intuition is also present in the case where the number of sellers is fixed across time. In contrast to the case with a flexible number of sellers, the buyer compares the marginal gain of doing another round of research to its marginal cost with N sellers instead of only one. But given that the number of sellers is fixed, there is no possibility to continue doing research with less sellers and, hence, she has hit the lower bound and research optimally stops.

In practice we observe that the number of participants in a tournament may change over time. Typically, participants are being eliminated as the end of the contest draws closer.²⁶ Moreover, we noted earlier that the first-best may include both elimination and addition of participants over time. We showed in Proposition 3.2 that we can implement any global stopping rule with a fixed number of participants using a dynamic prize schedule. However, this result can be generalized considerably. The following result shows that a dynamic prize schedule allows us to implement any global stopping rule with any, arbitrarily changing number of participants. Formally, a dynamic prize tournament is then described by $\langle \mathbf{E}, \mathbf{p}, \mathbf{N} \rangle$, where $\mathbf{N} = [N_1, \dots, N_T]$ is the fixed number of participants in each in period. Note that the numbers N_t are fixed ex ante and do not depend on the actual research outcomes of the sellers. However, the identity of the sellers who continue can depend on their research outcomes. Given this environment, we obtain the following result.

Proposition 3.5. *Any global stopping rule $\theta^g \in \Theta$ with a sequence of $\mathbf{N} = [N_1, \dots, N_T]$ sellers participating in each period can be implemented using a dynamic prize tournament $\langle \mathbf{E}, \mathbf{p}, \mathbf{N} \rangle$ with some increasing prize schedule $\mathbf{p} = [p_1, \dots, p_T]$ and some sequence of entry fees $\mathbf{E} = [E_1, \dots, E_T]$.*

The intuition for the result is analogous to the result in Proposition 3.2. Namely, the slope of the prize schedule takes care of the buyer's incentives while the size of the prizes incentivizes the sellers to conduct research, truthfully reveal their innovations and to stay in the tournament for as long as possible. The difference is that the slope of the prize schedule is no longer constant but changing over time. Recall that the prizes increase from period to period to equate the marginal benefit of an additional round of research to the buyer with its marginal cost when the

²⁶For example, the 2015 NRG COSIA Carbon XPRIZE consists of three rounds. Only up to 15 participants will proceed to the second round and only up to 5 five will proceed to the third round. See <http://carbon.xprize.org/about/overview>.

threshold is reached. Since the marginal benefit changes when the number of sellers is changed, the prize schedule has to increase less strongly following a round of elimination and more strongly following an addition of new sellers. Moreover, it is easier to incentivize the sellers to conduct research and report their innovations when there is a round of elimination ahead, because the threat of elimination makes truthful reporting more attractive and increases the rewards of research, as this increases the chances of not being eliminated and therefore retaining a chance at getting the prize. Similarly, the prospect of increasing the number of sellers in the next period increases the incentives to report truthfully and conduct research, as more competition in the future makes trying to win in the current period more attractive.

We noted earlier that the finite time-horizon of the contest induces a fundamental trade-off between increasing the chance of getting a high quality innovation and risking wasteful duplication when increasing the number of sellers. What if the buyer instead had the possibility to relax the time constraint, i.e., what if the buyer could choose a later deadline $T' > T$? Intuitively, a longer time-horizon should be beneficial to the buyer, as it should allow the buyer to lessen the risk of duplication while keeping the chance of getting a high innovation. It turns out that this is not necessarily the case as our final result in this section shows.

Proposition 3.6. *A buyer who implements a global stopping rule is strictly better off by increasing the duration of the contest. A buyer who implements an individual stopping rule may be worse off by increasing the duration of the contest.*

There are two opposing effects at play when the length of the contest is increased. First, the buyer benefits because it increases the expected quality of the innovation she will eventually obtain. Second, it increases the risk of wasteful research in the form of duplication. The beneficial effect is clearly present in both a research and a dynamic prize tournament. However, in contrast to the research tournament, there is no change in the amount of duplication when a global stopping rule is implemented, as it ensures that research stops conditional on reaching the threshold. This effect is not present with an individual stopping rule. Thus, depending on which effect dominates, increasing the duration of the contest may be harmful for the buyer under an individual stopping rule, whereas the buyer is unambiguously better off under a global stopping rule.

Moreover, suppose that Assumption 3.1 was satisfied and that the buyer could choose $T \in \mathbb{N}$, the ending of the contest. By Proposition 3.3 we know that the optimal contest is a dynamic prize tournament with a global stopping rule. From Proposition 3.6, we know that the buyer would optimally set the highest feasible T .

3.6 Conclusion

The goal of the present paper is to increase our understanding of the optimal design of research contests, which have recently seen a rapid expansion in practice. Research contests are inherently dynamic by the nature of research itself and by virtue of the contest taking place over a longer period of time. It turns out that taking into account the dynamic nature of research improves contest design. Indeed we show that using a prize schedule with increasing prizes over time instead of a fixed prize yields a strict improvement for the buyer in the face of a breakthrough

structure. In particular, it gives rise to the optimal contest and allows the buyer to implement the first-best. More generally, we show that a dynamic prize tournament with an increasing prize schedule allows us to implement any global stopping rule. The great appeal of this finding lies in the fact that the first-best features a global stopping rule. Moreover, the channels through which this is achieved are strikingly simple: the slope of the prize schedule and its intercept align the buyer's and the sellers' incentives, respectively. Hence, an intuitive and simple departure from a research tournament which has a fixed prize delivers the ability to implement a central feature of the first-best solution.

Our results have important implications for the design of research contests. As mentioned before, we can usually distinguish between innovation races and research tournaments. Recall that in an innovation race the winner is the first seller to achieve a pre-determined quality, whereas in a research tournament the winner is the seller with the best quality at some pre-determined date. Naturally, both innovation races and research tournaments have their advantages and disadvantages. An innovation race avoids wasteful duplication as research stops once the goal has been reached. Moreover, it does not end until the goal is reached. However, it requires a verifiable outcome, so often an imperfect proxy of innovation has to be employed. On the other hand, a research tournament does not have to rely on proxies but there is a risk of wasteful duplication and premature ending. Quite remarkably, a dynamic prize tournament with a global stopping rule allows us to implement a hybrid of the two. Namely, throughout the duration of the tournament an innovation race is taking place, as the tournament ends as soon as an innovation above the stopping threshold is realized. However, there is an end date at which the race ends and turns into a classic research tournament in which the best innovation wins without requiring verifiability. Hence, a dynamic prize tournament allows us to get the best out of the innovation race (long horizon with no wasteful duplication) and the research tournament (no proxy required).

We derive our results under assumptions that are barely stronger than those in the seminal work by Taylor (1995). Yet, even within this rich framework there are further avenues to pursue. Comparative static results along the lines of Proposition 3.6 which considered extending the length of the contest would allow us to increase our understanding of the optimal mechanism in the absence of the breakthrough innovation structure. Further, one can study the scope for beneficial information revelation between the sellers, both from the sellers' and the buyer's perspective. Moreover, although quite ambitious, deriving the optimal contest in the absence of the breakthrough structure would be valuable and an interesting direction for future work.

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4 Delegating Performance Evaluation¹

Joint with Shuo Liu and Nick Netzer

4.1 Introduction

Principals often lack the information or expertise needed to make appropriate decisions. A common response to this problem is to delegate the decision to a better informed party. For example, funding agencies delegate the choice of research projects which will be funded to an expert committee. Within a firm, the CEO usually delegates to a mid-level manager the decision regarding the assignment of bonuses to subordinates. Humanitarian aid is distributed by specialized agencies on behalf of the donor countries.

If the preferences of the principal and the expert who makes the decision are not aligned, then delegation can lead to distorted decisions. The principal can attempt to influence the decision taken by the expert by limiting the set of outcomes from which the expert can select. The literature on optimal delegation studies how this *delegation set* should be designed.² In these papers, the principal wants to base her decision on some stochastic state of nature, the value of which is known only to the expert. Crucially, this state of nature is assumed to be exogenous. However, in the examples above the state of nature (the quality of research projects, the performance of employees, the cooperativeness of receiving countries) is determined in part in anticipation of the decision that the expert will make. As a matter of fact, the goal of the principal is exactly to incentivize the agents to exert effort. For example, the goal of the funding agencies is to stimulate creation of high quality research. Similarly, bonuses in firms are instruments that incentivize employees to work hard, and aid is in part allocated to bring about reforms.

In this paper, we study the optimal delegation problem for performance evaluation. A *principal* wishes to incentivize *agents* to exert costly effort. The efforts are not observable to the principal. However, an expert, which we will from now on refer to as the *reviewer*, can costlessly observe the exerted efforts.³ The principal thus delegates the decision on how to reward the agents to the reviewer, but possibly restricts the set of allowable decisions. The reviewer's preferences may not be perfectly aligned with the principal. While the reviewer takes into account the effect of his actions on the principal's payoff, maybe because he owns shares

¹This paper should be cited as Letina I., S. Liu and N. Netzer (2016), "Delegating Performance Evaluation," *Mimeo*.

²See, for example, Holmström (1977, 1984), Melumad and Shibano (1991), Alonso and Matouschek (2008), Armstrong and Vickers (2010), Amador and Bagwell (2013), and Frankel (2014).

³We do not consider the problem of incentivizing the reviewer to exert costly effort in order to learn the state of nature. This is an interesting but distinct incentive problem which is studied in Aghion and Tirole (1997), Szalay (2005), Rahman (2012), and Pei (2015b).

of the company or he cares intrinsically, he may also care about the agents. For instance, as we will discuss below, there is ample evidence that managers care about the payoffs of their subordinates. Exactly how much the reviewer cares about the agents is the reviewer's private information. Importantly, a reviewer who cares sufficiently much about the agents will be reluctant to punish them even if they do not exert sufficient effort. Anticipating this, the agents will exert less effort. The principal thus has to design the delegation set in a way that restricts the scope of possible leniency of the reviewer.

One could also imagine that the principal tries to correct the distortions by paying transfers to the reviewer conditional on the action that he takes. However, contingent transfers are often not observed in reality. Committees deciding which research projects get funded do not get paid conditional on how many projects they approve or reject. Mid-level managers do not get paid differently depending on how they allocate bonuses among their subordinates. In fact, paying an expert for performing a particular evaluation is often referred to as a conflict of interest and is explicitly forbidden. The delegation approach, which rules out direct monetary incentives, is therefore particularly plausible for our setting of performance evaluation.

Our first main result is that a contest among the agents is an optimal mechanism. That is, the principal defines a set of prizes and the reviewer only decides how to allocate these prizes to the agents. The reviewer does not have the additional freedom to choose the overall size or the split of the agents' compensation. This strongly limits the degree of leniency he can exercise. In particular, the reviewer is always forced to punish some agents by assigning them a small prize, which is crucial for the preservation of incentives. Without this commitment, the reviewer would be lenient and the agents would shirk. The downside of the contest mechanism is that someone needs to be punished (at random) even when all agents provide a sufficient level of effort. Somewhat counter-intuitively, if the reviewer was not averse to punishing, then no agent would have to be punished.

This result is interesting for several reasons. First, while contests are a commonly used and often-studied incentive scheme,⁴ there is not much work on the question whether and under which conditions contests are actually optimal mechanisms.⁵ Exceptions are the seminal paper of Lazear and Rosen (1981), as well as some papers which stress that contests can filter out common shocks when agents are risk-averse (Green and Stokey, 1983; Nalebuff and Stiglitz, 1983) or ambiguity-averse (Kellner, 2015). In our model, contests are optimal because they act as a commitment device. A contest provides two types of commitment. It commits the principal to the announced prizes and thus prevents any manipulation of the sum of payments to the agents. The literature has observed previously that this "commitment to pay" can be beneficial when the agents' efforts are not verifiable. For instance, Malcomson (1984, 1986) argues that piece-rate contracts are not credible in that case, as the principal would always claim low performance ex post in order to reduce payments, while a contest remains credible.⁶ However, this credibility

⁴See e.g. Prendergast (1999) and De Varo (2006), and the references therein.

⁵Prendergast (1999, p. m36) writes: "Rather surprisingly, there is very little work devoted to understanding why this is the case, i.e., why the optimal means of providing incentives within large firms (at least for white-collar workers) seems to be tournaments rather than the other means suggested in the previous sections." We find this still to be the case in the years since Prendergast published his paper.

⁶Similarly, Carmichael (1983) considers a setting where the final output is verifiable but depends on the efforts of both the principal and the agents. With a contract that pays agents based on total output, the principal has

can also be achieved by simply committing to a total sum of payments without setting fixed prizes. In fact, as we will show by example, such a scheme would outperform a contest when the agents are risk-averse, by removing uncertainty from equilibrium payments. Hence the second type of commitment, the above described “commitment to punish,” is crucial in explaining the optimality of contests with fixed prizes.⁷ Second, a contest is a remarkably simple mechanism. Even though we allow for arbitrary stochastic delegation mechanisms with possibly sophisticated transfer rules, the optimum can be achieved by a simple mechanism characterized by a prize profile and a suggestion how to distribute the prizes in response to the agents’ efforts. The principal does not attempt to screen reviewer types, in spite of the fact that the first-best may be achievable if the principal knew the reviewer’s type. This makes the strategic considerations of the agents simple. In particular, their behavior does not depend on beliefs regarding the reviewer type. This robustness property is important because principal and agents may well have different beliefs (for instance, in the example with managers allocating bonuses, it seems reasonable to assume that the employees working directly with a manager have more precise information about their manager’s type than a CEO does).

Our second result characterizes the prize structure of an optimal contest. Given n agents, an optimal contest will have $n - 1$ equal positive prizes and one zero prize. Thus, while the contest acts as a commitment to punish, the punishment is kept at the minimum required to incentivize effort. The delegation set forces the reviewer to punish only one agent, such that the optimal contest exhibits a “loser-takes-nothing” rather than a “winner-takes-all” structure. In equilibrium, when all agents have provided sufficient effort, the reviewer randomly chooses the agent to be punished. Thus all agents are facing the risk of punishment in equilibrium. If agents are risk-averse, they respond to this risk by reducing the amount of effort they are willing to exert. A corollary of this result is that the first-best is implementable if and only if the agents are risk-neutral. We also show that an optimal contest implements an outcome close to the first-best if the agents’ risk-aversion is moderate or the number of agents is large.

Our third result shows that a familiar all-pay auction with a slight twist also implements the optimum, and it does so in unique equilibrium.⁸ As in a standard all-pay auction (with $n - 1$ identical prizes), the agent with the lowest effort receives the zero prize. However, efforts at or above the desired equilibrium level are not differentiated, and ties are broken randomly. This removes the incentive of the agents to exert slightly more than the equilibrium effort in order to guarantee themselves a positive prize with probability one. We refer to this mechanism as an all-pay auction with censoring.

In addition to the all-pay auction, we show as our fourth result that the optimum can also

an incentive to reduce own effort in order to reduce the payments to agents.

⁷The problem of committing to punishment is related to Konrad (2001) and Netzer and Scheuer (2010). They study the problem of a planner who would like to implement redistribution after agents have chosen their actions, the anticipation of which may destroy incentives to choose costly but socially desirable actions. In the context of optimal income taxation, Konrad (2001) shows that private information about labor productivity provides a commitment against excessive redistribution. In the context of insurance and labor markets, Netzer and Scheuer (2010) show that adverse selection provides a commitment by generating separating market equilibria. In both cases, agents who choose socially less desirable actions are punished by having to forego information rents.

⁸As we will explain in Section 4.3, uniqueness only refers to the agents’ choice of efforts in the given contest. The reviewer will be indifferent among several actions, and in particular a “babbling” equilibrium exists where his assignment of prizes is unresponsive to the agents’ efforts and they exert zero effort.

be achieved with an imperfectly discriminating contest, such as the well-known Tullock contest. The Tullock-type contest success function arises endogenously as part of an optimal contract in our analysis. In summary, the optimum can be achieved with all of the commonly studied formats of contests (see Konrad, 2009). This shows that the essential feature of our main result is the fixed profile of prizes, and not the exact procedure how these prizes are awarded.

We then consider extensions where the reviewer observes only effort differences between the agents or noisy signals of individual efforts. We show that, by using stochastic allocation rules, the principal can often still implement the optimal allocation with a contest. Next, we consider a model of cheap talk where the reviewer does not make the allocation decision himself but communicates the observed effort levels to the principal. We show that our results for the delegation model continue to hold in the cheap talk model. This adds robustness to our results, especially in the view of experimental findings which show that principals may be reluctant to delegate authority even when it is in their interest to do so.⁹ Finally, we discuss how our analysis can be extended to settings with non-separable and/or asymmetric preferences.

Our contribution is related to three strands of literature: on the optimality of contests, on biased reviewers, and on optimal delegation. A more detailed discussion of the literature is postponed to Section 4.5. Another paper that also contributes to all three strands is Frankel (2014). Like in our paper, he considers a multidimensional delegation problem with uncertainty about the expert's preferences. He assumes that the state of the world is exogenous and is not affected by the choice of the delegation mechanism. In contrast, our prime concern is how the delegation mechanism affects the state of the world, i.e., how it provides incentives for agents to exert effort. Another difference is that Frankel (2014) derives max-min mechanisms, which are optimal for the worst possible realization of the expert's bias, while we are interested in mechanisms that maximize the principal's expected payoff given her beliefs about the expert's type. Frankel (2014) shows that, when the set of possible preferences of the expert is rich enough, a specific contest is max-min optimal, because it is a very robust mechanism.¹⁰ We show that contests maximize the principal's expected payoff because they provide optimal incentives to the agents. Furthermore, since the principal's payoff turns out to be independent of the reviewer's type in our optimal contests, they not only maximize expected payoffs but are also max-min optimal.

Our model applies to many situations where a principal wants to incentivize agents but cannot directly supervise them. Here we will discuss two possible applications, which are meant to illustrate the range and scale of our model. One application is the design of performance evaluation schemes in firms. The performance evaluation scheme is designed by the CEO, but the CEO does not observe the individual efforts of the employees to which the scheme applies. Hence, the actual performance evaluation is delegated to the employees' supervisor. By virtue of working closely with the employees, the supervisor observes their efforts but also cares about their payoffs. There is ample evidence (both empirical and experimental) that supervisors tend to be too lenient when judging the performance of their subordinates, and

⁹See Fehr et al. (2013) and Bartling et al. (2014).

¹⁰Richness requires that the set of preferences includes all (concave) utility functions over states and actions which exhibit increasing differences. The resulting max-min optimal mechanism corresponds to a standard all-pay auction. For a general treatment of contests with all-pay structure see Siegel (2009).

that the degree of leniency varies and depends on (among other things) social ties between the supervisor and the team.¹¹ Our results have direct implications for the controversial debate over the use of the so-called “forced rankings,” a review system which was most famously used by General Electric under Jack Welch during their fast growth in the 1980s and 90s.¹² Our contribution to this discussion is (i) to show that forced rankings are optimal for motivating effort under the assumptions of our model, (ii) to show how optimally forced rankings should be constructed, and (iii) to show that some elements of forced rankings which are usually criticized are actually necessary for incentivizing effort. In particular, forced rankings are criticized for forcing managers to assign low rankings even when all workers are performing well: “What happens if you’re working with a superstar team? You’ve just forced a distribution that doesn’t exist. You create this stupid world where [great] people are punished.”¹³ Similarly, Brad Smart who worked with Jack Welch on developing GE’s forced ranking system criticized GE’s decision to assign 10% of the workers a low evaluation: “To force those distributions when the percentages don’t meet the reality is nuts.”¹⁴ Our results show that, far from being “stupid” or “nuts,” punishing some workers even when they perform well is necessary, since if the managers were given an option not to punish, they may choose it irrespective of the actual performance, which would destroy any incentive effect of the evaluation system.

A very different situation for which our model offers insights is foreign aid. Donors have been trying for decades to use foreign aid to incentivize reforms in recipient countries, but there is little empirical evidence that it has been effective (see e.g. Easterly, 2003; Rajan and Subramanian, 2008). In response, funding agencies and governments have tried to improve mechanisms for the allocation of foreign aid in ways that link aid to improvement in governance and other policy reforms. One early approach has been the so-called “conditional aid,” where donors promise to withdraw future aid if the agreed policy reforms have not been achieved. However, the donors’ threats to withdraw aid were not credible and, unsurprisingly, reforms were usually not carried out. As Easterly (2009) somewhat amusingly points out, the World Bank conditioned aid on the same agricultural policy reform in Kenya five separate times – and the conditions were violated each time. In a very interesting paper, Svensson (2003) proposes a solution to this problem. Instead of allocating the budget for each country to a different aid officer, similar countries could be pooled together and the total budget for all these countries could be allocated to a single aid officer. This way, if one country does not reform, the aid officer has the option of reallocating the aid from that country to another. Our paper points to a potential problem with this approach and offers a solution. A benevolent aid officer may still be tempted to split the aid more or less equally among the countries, their efforts towards reform notwithstanding. Our paper suggests

¹¹For example, Bol (2011) and Breuer et al. (2013) find evidence of leniency bias which depends on the strength of the employee-manager relationship. Bol (2011) cites studies documenting leniency bias going back to the 1920s, while citations to similar findings in the 1940s can be found in Prendergast (1999). Berger et al. (2013) find experimental evidence of leniency bias, and Bernardin et al. (2000) document that the degree of leniency bias in an experiment depends on personality traits of the reviewer. More generally, Cappelen et al. (2007) show experimentally that individuals exhibit a variety of different fairness preferences.

¹²For example, see “‘Rank and Yank’ Retains Vocal Fans” (L. Kwoh, *The Wall Street Journal*, January 31, 2012) and “For Whom the Bell Curve Tolls” (J. McGregor, *The Washington Post*, November 20, 2013).

¹³Quote of a management adviser in “For Whom the Bell Curve Tolls” (J. McGregor, *The Washington Post*, November 20, 2013).

¹⁴In “‘Rank and Yank’ Retains Vocal Fans” (L. Kwoh, *The Wall Street Journal*, January 31, 2012).

that holding a contest among recipient countries can overcome this problem. That is, instead of giving the aid officer full discretion over the total budget for multiple countries, the budget could be partitioned into fixed “prizes” that the officer allocates to the countries. Our results show that this would indeed be an optimal mechanism. Obviously, it may be politically difficult to implement a contest where a country receives zero aid even if it invested effort in reforms. However, some variant of our mechanism, where all countries receive aid but some countries receive “bonus aid” through a contest might be both politically feasible and desirable from the incentive point of view.

The remainder of the paper is organized as follows. Section 4.2 describes the model. In Section 4.3 we show that the set of optimal contracts contains a contest, we characterize all optimal contests, and we deal with unique implementation of the second-best outcome. In Section 4.4 we develop several extensions of the baseline model. Section 4.5 contains a discussion of the related literature, and Section 4.6 concludes.

4.2 The Model

4.2.1 Environment

A principal contracts with a set of agents $I = \{1, \dots, n\}$ where $n \geq 2$. Each agent $i \in I$ chooses an effort level $e_i \geq 0$ and obtains a monetary transfer $t_i \geq 0$. The agents have an outside option of zero. The payoff of agent i is given by

$$\pi_i(e_i, t_i) = u(t_i) - c(e_i),$$

where $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing, weakly concave and satisfies $u(0) = 0$, and $c : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice differentiable, strictly increasing, strictly convex and satisfies $c(0) = 0$ and $c'(0) = 0$. The assumption of additive separability of transfers and efforts is standard in contract theory, mechanism design, and contest theory. Some of our results depend on this assumption, so we will discuss robustness with respect to non-separable and also asymmetric preferences in Section 4.4.3. We denote effort profiles by $e = (e_1, \dots, e_n) \in E$ and transfer profiles by $t = (t_1, \dots, t_n) \in T$. We assume that $E = \mathbb{R}_+^n$ and $T = \{t \in \mathbb{R}_+^n \mid \sum_{i=1}^n t_i \leq \bar{T}\}$, where $\bar{T} > 0$ can be arbitrarily large. Our results hold no matter whether \bar{T} is binding or not. The payoff of the principal from an allocation (e, t) is

$$\pi_P(e, t) = z(e) - \sum_{i=1}^n t_i,$$

where $z : E \rightarrow \mathbb{R}_+$ is interpreted as the production function that converts efforts into output. For clarity of exposition we will focus only on the case where $z(e) = \sum_{i=1}^n e_i$. Our main results continue to hold if we assume more generally that z is symmetric, weakly concave, and strictly increasing in each of its arguments.

Example. We will use a parameterized example to illustrate our results throughout the paper.

In this example, each of the n agents has the payoff function

$$\pi_i(e_i, t_i) = t_i^\alpha - \gamma e_i^\beta,$$

where $\alpha \leq 1$ parameterizes risk-aversion, $\beta > 1$ describes the degree of cost convexity, and $\gamma > 0$ determines the relative weight of effort costs. We will always assume that \bar{T} is large enough to be non-binding in the example. The first-best effort level e^{FB} is what the principal would demand from each agent if she could perfectly control effort and would only have to compensate the agent for his cost, thus paying $t^{FB} = u^{-1}(c(e^{FB}))$. In our example, maximization of $e - u^{-1}(c(e))$ yields

$$e^{FB} = \left(\frac{\alpha}{\beta \gamma^{1/\alpha}} \right)^{\frac{\alpha}{\beta-\alpha}} \quad \text{and} \quad t^{FB} = \left(\frac{\alpha}{\beta \gamma^{1/\beta}} \right)^{\frac{\beta}{\beta-\alpha}}.$$

The principal's first-best profit is $n(e^{FB} - t^{FB})$. □

The effort exerted by the agents is not verifiable to outside parties (e.g. a court) and is not observable to the principal. However, the efforts can be observed by a reviewer. Consequently, the evaluation of the agents' performance and the decision on how to reward the agents are delegated to the reviewer. In line with the literature on delegation, we assume that the principal cannot pay the reviewer based on the decision made (but can in principle pay a fixed fee to the reviewer, which we normalize to zero). Specifically, the payoff of the reviewer from an allocation (e, t) is given by

$$\pi_R(e, t, \theta) = \pi_P(e, t) + \theta \sum_{i=1}^n \pi_i(e, t),$$

where θ is a parameter that captures how much the reviewer cares about the well-being of the agents, and thus by how much the reviewer's preferences are misaligned with those of the principal. The parameter θ can be thought of as a fundamental preference or as a reduced-form representation of concerns due to other interactions with the agents. We assume that θ is private information of the reviewer, observable neither to the principal nor to the agents. It is drawn according to a commonly known continuous distribution with full support on $\Theta = [\underline{\theta}, \bar{\theta}]$, where $\underline{\theta} < \bar{\theta}$. We describe this distribution by an (absolutely continuous) probability measure τ over Θ . Our results will be independent of the shape of this distribution. In particular, τ could be arbitrarily close to a probability measure with atoms.

4.2.2 Implementation with Credible Contracts

The timing is as follows. First, the principal delegates the evaluation and remuneration of the agents to the reviewer, by endowing the reviewer with a set D of possible actions. An action is a probability measure $\mu \in \Delta T$ on the set of transfer profiles, describing the potentially stochastic payments made to the agents. Next, the agents choose their efforts simultaneously. The reviewer then observes the efforts and chooses an action from D to reward or punish the agents.

Since e and θ are observable only to the reviewer, he is always free to choose any action that

he prefers. We model this by defining a *contract* $\Phi = (\mu^{e,\theta})_{(e,\theta) \in E \times \Theta}$ as a collection of probability measures $\mu^{e,\theta} \in \Delta T$, one for each $(e, \theta) \in E \times \Theta$. The interpretation is that the principal suggests that a reviewer of type θ should reward an effort profile e by transfers according to $\mu^{e,\theta}$.¹⁵ The following incentive constraint makes sure that the reviewer indeed has an incentive to follow this suggestion:

$$\Pi_R(e, \mu^{e,\theta}, \theta) \geq \Pi_R(e, \mu^{e',\theta'}, \theta) \quad \forall (e, \theta), (e', \theta') \in E \times \Theta, \quad (\text{IC-R})$$

where

$$\Pi_R(e, \mu^{e',\theta'}, \theta) = \mathbb{E}_{\mu^{e',\theta'}} \left[\pi_P(e, t) + \theta \sum_{i=1}^n \pi_i(e_i, t_i) \right].$$

We say that a contract Φ is *credible* if it satisfies (IC-R). Given a credible contract, the delegation set is implicitly given by $D = \{\mu \in \Delta T \mid \exists (e, \theta) \text{ s.t. } \mu = \mu^{e,\theta}\}$.

Denote by $\sigma_i \in \Delta \mathbb{R}_+$ agent i 's mixed strategy for his effort provision. We also write $e_i \in \Delta \mathbb{R}_+$ for Dirac measures that represent pure strategies. Strategy profiles are given by $\sigma = (\sigma_1, \dots, \sigma_n) \in (\Delta \mathbb{R}_+)^n$. We also use σ to denote the induced product measure in ΔE . We say a contract Φ implements a strategy profile σ if it is credible and satisfies

$$\Pi_i((\sigma_i, \sigma_{-i}), \Phi) \geq \Pi_i((\sigma'_i, \sigma_{-i}), \Phi) \quad \forall \sigma'_i \in \Delta \mathbb{R}_+, \forall i \in I, \quad (\text{IC-A})$$

where

$$\Pi_i(\sigma, \Phi) = \mathbb{E}_\sigma \left[\mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e,\theta}} [u(t_i)] \right] \right] - \mathbb{E}_{\sigma_i} [c(e_i)].$$

Since a deviation to an effort of zero always guarantees each agent a payoff of at least zero, the agents' participation constraints can henceforth be ignored.¹⁶

The principal maximizes her expected payoff by choosing a contract Φ to implement some strategy profile σ . Formally, the principal's problem is given by

$$\max_{(\sigma, \Phi)} \Pi_P(\sigma, \Phi) \quad \text{s.t.} \quad (\text{IC-R}), (\text{IC-A}), \quad (\text{P})$$

where

$$\Pi_P(\sigma, \Phi) = \mathbb{E}_\sigma \left[\sum_{i=1}^n e_i \right] - \mathbb{E}_\sigma \left[\mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e,\theta}} \left[\sum_{i=1}^n t_i \right] \right] \right].$$

A contract Φ^* is *optimal* if there exists σ^* such that (σ^*, Φ^*) solves (P).

¹⁵ This formulation does not preclude the possibility that a reviewer randomizes over actions, because the randomization over probability measures can instead be written as a compound measure that is chosen with probability one. We impose the following regularity condition on contracts: for each measurable set $A \subseteq T$, $\mu^{e,\theta}(A)$ is a measurable function of (e, θ) . This ensures that expected payoffs are well-defined in contracts.

¹⁶ Formally, the constraints (IC-R) and (IC-A) characterize Perfect Bayesian equilibria of the following game. Given a delegation set D , the agents first simultaneously choose their efforts e . Nature then determines the reviewer's type θ . The reviewer finally observes e and θ and chooses from D . Constraint (IC-R) prescribes sequential rationality for the reviewer's (singleton) information sets, while (IC-A) prescribes sequential rationality (with weakly consistent beliefs) for the information sets in which the agents choose.

Finally, we introduce a specific class of contracts that will be referred to as *contests*. We say that a contract is a contest if it commits to a profile of prizes $y = (y_1, \dots, y_n) \in T$, some of which could be zero, and specifies how these prizes are allocated to the n agents as a function of their effort. More formally, let $P(y)$ denote the set of permutations of y .¹⁷ Then a contest C_y with prize profile y is a contract that satisfies $\mu^{e,\theta}(P(y)) = 1$ for all $(e, \theta) \in E \times \Theta$. Note that every contest is credible. Once the agents' efforts are sunk, any allocation of the prizes generates the same payoff for the principal and the same sum of utilities for the agents. Formally, $\forall (e, \theta), (e', \theta') \in E \times \Theta$,

$$\Pi_R(e, \mu^{e', \theta'}, \theta) = \sum_{i=1}^n e_i - \sum_{i=1}^n y_i + \theta \left(\sum_{i=1}^n u(y_i) - \sum_{i=1}^n c(e_i) \right)$$

is independent of (e', θ') . However, the set of credible contracts is substantially larger than the set of contests. For instance, it is possible to select from a much larger set of transfer profiles, not just permutations of given prizes, and still keep both the expected sum of transfers and the expected sum of the agents' utilities constant.

4.3 Optimal Contracts

4.3.1 The Optimality of Contests

To illustrate the key incentive problem in our model, suppose first that the preference parameter θ was known to the principal and the agents. The following example shows that, in this case, there may exist a credible contract which is not a contest but which implements the first-best effort levels and extracts the entire surplus.

Example. Consider our previous example for the special case of $n = 2$. Suppose the reviewer's type was common knowledge. First assume $\theta = 0$, so that there is also no misalignment of preferences between the principal and the reviewer. Consider a contract Φ^{FB} where, if both agents exert e^{FB} , each of them is paid t^{FB} . If one agent deviates, that agent is paid 0 while the non-deviating agent is paid $2t^{FB}$. In case both agents deviate, they are again each paid t^{FB} . It is easy to verify that this contract is credible, because the sum of transfers is constant across (t^{FB}, t^{FB}) , $(2t^{FB}, 0)$ and $(0, 2t^{FB})$, which makes the reviewer indifferent between these transfer profiles. It is also easy to verify that this contract implements (e^{FB}, e^{FB}) , because both agents receive a payoff of zero in equilibrium and a payoff of at most zero after any unilateral deviation. Thus the first-best is achievable. Observe that Φ^{FB} is not a contest, because the three transfer profiles are not permutations of each other. We will show below that the first-best is not achievable by a contest if the agents are risk-averse. Hence, Φ^{FB} performs strictly better than any contest in this example. This shows that non-verifiability of effort alone does not make contests optimal.¹⁸

¹⁷Profile t is a permutation of y if there exists a bijective mapping $s : I \rightarrow I$ such that $t_i = y_{s(i)} \forall i \in I$.

¹⁸This argument is related to MacLeod (2003), who considers an environment with a principal, a single agent, and non-verifiable performance signals. He shows that, if the principal can commit to burn money, he can credibly punish a shirking agent. In our example, the transfer to the non-deviating agent plays a role similar to money burning.

Now assume that $\theta > 0$ and adjust the contract Φ^{FB} as follows. The payment $2t^{FB}$ to a non-deviating agent is replaced by some t^{nd} , while everything else is kept unchanged. If t^{nd} is chosen such that the reviewer is indifferent between the transfer profiles (t^{FB}, t^{FB}) , $(t^{nd}, 0)$, and $(0, t^{nd})$, credibility is restored and the first-best can be implemented. For instance, with $\alpha = 1/2$, $\beta = 2$, and $\gamma = 1$ we have $e^{FB} \approx 0.63$ and $t^{FB} \approx 0.16$. For a reviewer of known type $\theta = 3$ we would then obtain $t^{nd} \approx 1.15$.¹⁹ This shows that the misalignment of preferences per se does also not make contests optimal. \square

The contracts described in the example no longer work if θ is the reviewer's private information. Just consider the optimal contract for type $\theta = 0$. Any reviewer with type $\theta' > 0$ will strictly prefer to allocate (t^{FB}, t^{FB}) , no matter what efforts the agents have exerted. This illustrates the leniency bias and the need for commitment discussed in the Introduction.

Our first main result shows that, despite the fact that the set of possible contracts is very large, optimal contracts with uncertainty about θ take a very simple form.

Theorem 4.1. *The set of optimal contracts contains a contest.*

We will establish Theorem 4.1 by proving a series of six lemmas. Since we have shown that the principal may be able to implement the first-best if she knew the reviewer's private type θ , it would seem reasonable to expect that the principal could benefit from screening these types. Lemmas 4.1 - 4.3 below show that it is not possible for the principal to benefit from screening. Lemmas 4.4 - 4.5 show that the principal can also not benefit from implementing mixed or asymmetric effort profiles. Lemma 4.6 shows that using contests is then without loss of generality. The proofs of all lemmas can be found in Appendix D.1.

Fix an arbitrary contract $\Phi = (\mu^{e,\theta})_{(e,\theta) \in E \times \Theta}$ and denote

$$S_t(e, \theta) = \mathbb{E}_{\mu^{e,\theta}} \left[\sum_{i=1}^n t_i \right], \quad S_u(e, \theta) = \mathbb{E}_{\mu^{e,\theta}} \left[\sum_{i=1}^n u(t_i) \right]$$

and

$$S(e, \theta) = -S_t(e, \theta) + \theta S_u(e, \theta).$$

We can then rewrite the credibility constraint (IC-R) as

$$-S_t(e, \theta) + \theta S_u(e, \theta) \geq -S_t(e', \theta') + \theta S_u(e', \theta') \quad \forall (e, \theta), (e', \theta') \in E \times \Theta.$$

Our first lemma provides a characterization of this multidimensional constraint.

Lemma 4.1. *A contract Φ is credible if and only if the conditions (i) - (iii) hold:*

$$(i) \quad \forall \theta \in \Theta, S(e, \theta) = S(e', \theta) \quad \forall e, e' \in E.$$

$$(ii) \quad \forall e \in E, S_u(e, \theta) \text{ is non-decreasing in } \theta.$$

¹⁹The indifference condition is $-2t^{FB} + \theta 2u(t^{FB}) = -t^{nd} + \theta u(t^{nd})$. Given our parameters, it has a second solution $t^{nd} \approx 3.72$, which would work as well. Note that the indifference condition is not guaranteed to have a solution for all parameter values, so our simple construction of a first-best contract does not work for all values of $\theta > 0$.

(iii) $\forall e \in E, S(e, \theta) = S(e, \underline{\theta}) + \int_{\underline{\theta}}^{\theta} S_u(e, s) ds \quad \forall \theta \in \Theta.$

Conditions (ii) and (iii) are familiar from the mechanism design literature. They have to hold separately for each fixed effort profile e . Condition (i) concerns the effort dimension and shows that the payoff of any reviewer has to be constant for any reported e .

Given the characterization provided by Lemma 4.1, the next lemma states an important implication of the credibility constraint. Not only does $S(e, \theta)$ have to be constant across different profiles e , also its constituent parts $S_t(e, \theta)$ and $S_u(e, \theta)$ cannot vary with e .

Lemma 4.2. *A contract Φ is credible only if there exists a pair of functions $x : \Theta \rightarrow \mathbb{R}_+$ and $\hat{x} : \Theta \rightarrow \mathbb{R}_+$ such that, $\forall e \in E$,*

$$S_t(e, \theta) = x(\theta), \quad S_u(e, \theta) = \hat{x}(\theta)$$

for almost all $\theta \in \Theta$.

We now show that there is no gain for the principal to screen the reviewer's private type by using a complex contract where $\mu^{e, \theta}$ varies with θ . Put differently, the principal can without loss of generality design the delegation set in a way such that all reviewers select the same actions.

Lemma 4.3. *For every contract Φ that implements a strategy profile σ , there exists a contract $\hat{\Phi}$ that also implements σ , yields the same expected payoff to the principal, and, $\forall \theta, \theta' \in \Theta$, satisfies $\hat{\mu}^{e, \theta} = \hat{\mu}^{e, \theta'} \quad \forall e \in E$.*

The proof of the lemma is constructive and shows how the contract $\hat{\Phi}$ without screening can be obtained from an arbitrary contract Φ . Given this result, we from now on focus without loss of generality on contracts where the agents' transfers depend on their efforts only, which we write as $\Phi = (\mu^e)_{e \in E}$. The next lemma shows that the principal does not benefit from implementing mixed strategies.

Lemma 4.4. *For every contract Φ that implements a strategy profile σ , there exists a contract $\hat{\Phi}$ that implements the pure-strategy profile $\bar{e} = (\bar{e}_1, \dots, \bar{e}_n)$, where $\bar{e}_i = \mathbb{E}_{\sigma_i}[e_i] \quad \forall i \in I$, and yields the same expected payoff to the principal.*

The intuition behind this result is simple: any randomness in transfers that is achieved by mixed strategies can equivalently be generated by the contract. On the other hand, since c is convex, the agents benefit from exerting the average effort \bar{e}_i instead of σ_i , while the principal is indifferent as to whether she obtains the efforts in expectation or deterministically.

The proofs of the above four lemmas do not rely on the symmetry of the agents' preferences. In fact, these intermediate results can be straightforwardly extended to the more general setting where each agent i 's utility function is given by $u_i(t_i) - c_i(e_i)$. The next lemma states that, if we consider the current symmetric setting, then it is without loss to restrict attention to the implementation of symmetric pure-strategy effort profiles.

Lemma 4.5. *For every contract Φ that implements a pure-strategy profile $\bar{e} = (\bar{e}_1, \dots, \bar{e}_n)$, there exists a contract $\hat{\Phi}$ that implements the symmetric pure-strategy profile $\hat{e} = (\hat{e}_1, \dots, \hat{e}_n)$, where $\hat{e}_1 = \dots = \hat{e}_n = \frac{1}{n} \sum_{i=1}^n \bar{e}_i$, and yields the same expected payoff to the principal.*

The next lemma completes the proof of Theorem 4.1 by demonstrating that the principal can achieve the same payoff with a contest as with any contract that implements a symmetric pure-strategy effort profile. Thus, the principal can obtain her maximal payoff with a contest.²⁰

Lemma 4.6. *For every contract Φ that implements a symmetric pure-strategy profile \hat{e} , there exists a contest C_y that also implements \hat{e} and yields the same expected payoff to the principal.*

To prove this lemma, we construct a contest which implements the effort profile \hat{e} . This contest features $n - 1$ identical prizes and one prize that is smaller. The small prize is used to punish agents who deviate in either direction from \hat{e} . In equilibrium, when the effort profile \hat{e} is realized, the n prizes are randomly allocated among the agents. As we will show in the next section, this prize structure is in fact a general feature of optimal contests, while the specific (non-monotonic) allocation rule is not required to achieve the optimum.

4.3.2 Optimal Contests

From the previous section, we know that the principal can restrict attention to contests when designing an optimal contract. In this section, we characterize general features of all optimal contests. When describing a contest C_y , in the following we always assume w.l.o.g. that the prize profile y is ordered such that $y_1 \geq y_2 \geq \dots \geq y_n$.

Theorem 4.2. *A contest is optimal if and only if the conditions (i) and (ii) hold:*

(i) *The prizes satisfy $y_n = 0$ and $\sum_{k=1}^n y_k = x^*$, with $x^* = \min\{\bar{x}, \bar{T}\}$ and \bar{x} given by*

$$u'\left(\frac{\bar{x}}{n-1}\right) = c' \left(c^{-1} \left(\frac{n-1}{n} u\left(\frac{\bar{x}}{n-1}\right) \right) \right).$$

If the agents are risk-averse, then the prize profile is unique and given by

$$y = (x^*/(n-1), \dots, x^*/(n-1), 0).$$

(ii) *The contest implements (e^*, \dots, e^*) , where e^* is given by*

$$e^* = c^{-1} \left(\frac{n-1}{n} u\left(\frac{x^*}{n-1}\right) \right).$$

Condition (i) in the theorem shows that the lowest prize will be zero in any optimal contest. This is not obvious, since the agents can be risk-averse and in equilibrium all agents face the risk of receiving the zero prize. The intuition is that in equilibrium an agent receives the zero prize with probability $1/n$, while a shirking agent would receive the zero prize with strictly larger probability (possibly one). Thus, any increase in y_n decreases the difference between the equilibrium and the deviation payoffs, and therefore decreases the amount of effort that can be demanded in equilibrium. When the agents are risk-averse, the optimal prize profile will feature $n - 1$ identical positive prizes in addition to the zero prize. In that sense, the commitment

²⁰To be exact, Theorem 4.1 follows only after it has been shown that problem (P) has a solution, so that an optimal contract exists. This will be shown in the proof of Theorem 4.2 in the next section.

to punishment is as small as possible. Condition (i) also characterizes the optimal total prize sum x^* , which is given by the point \bar{x} where marginal cost and benefit of inducing effort are equalized, or by the exogenous budget \bar{T} whenever it is sufficiently tight. Condition (ii) in the theorem shows that every optimal contest extracts the entire surplus from the agents, because it implements a symmetric pure-strategy effort profile such that each agent's equilibrium payoff is zero.

Having characterized the optimal contests, we turn to the question of efficiency loss.

Corollary 4.1. *If the agents are risk-neutral, the principal can achieve the first-best. If the agents are risk-averse, the principal cannot achieve the first-best.*

The efficiency loss is driven entirely by risk-aversion of the agents. The loss is a direct consequence of the commitment to punish, which is both inherent in the contest and the reason why the contest is optimal. Since the principal has to commit to a punishment, a punishment must be delivered even in equilibrium. Risk-averse agents have to be compensated for this, which increases the cost of inducing effort. Hence, the commitment problem prevents the principal from achieving the first-best. However, the loss will be small if the agents are only mildly risk-averse, as the following example illustrates.

Example. Consider again our example for $n = 2$. Applying the results from Theorem 4.2, it can be shown that $e^* = 2^{\frac{\alpha-1}{\beta-\alpha}} e^{FB}$ and $x^* = 2^{\frac{\beta-1}{\beta-\alpha}} t^{FB}$ holds in any optimal contest. The ratio of second-best to first-best profits of the principal is therefore

$$R = \frac{2e^* - x^*}{2e^{FB} - 2t^{FB}} = \frac{2^{\frac{\beta-1}{\beta-\alpha}}(e^{FB} - t^{FB})}{2(e^{FB} - t^{FB})} = 2^{\frac{\alpha-1}{\beta-\alpha}}.$$

This ratio is increasing in α , with $R \rightarrow 1$ in the limit as $\alpha \rightarrow 1$, so second-best profits approach first-best profits when the agents' risk-aversion vanishes. \square

The loss will also be small for any given risk-aversion if there are many agents. This follows immediately from Theorem 4.2, because the probability of not receiving any of the $n - 1$ identical prizes is $1/n$ in equilibrium and vanishes as the number of agents grows. We will illustrate this more formally in Section 4.4.3. In particular, these arguments imply that the absence of a contingent monetary transfer to the reviewer – the constitutive assumption of the delegation approach – comes with little loss if risk-aversion is small or if the number of agents is large.

4.3.3 Unique Implementation

We say a contract Φ *uniquely implements* some pure-strategy effort profile e if (i) it implements e , and (ii) it does not implement any other (possibly mixed) strategy profile $\sigma \neq e$. The next theorem states that the second-best effort profile e^* from Theorem 4.2 can be uniquely implemented by a contest that is similar to the familiar all-pay auctions. We refer to this contest as an *all-pay auction with censoring*.

An all-pay auction is one of the canonical contest types (see Konrad, 2009, Ch. 2.1). It is perfectly discriminating in the sense that the agent with the highest effort wins the highest prize with probability one, the agent with the second highest effort wins the second prize, and so on.

Ties are broken randomly. Our all-pay auction with censoring is the same as a standard all-pay auction (with $n - 1$ identical prizes) for all effort levels up to the censoring level. Efforts at or above the censoring level are not differentiated, i.e., an agent who exerts the effort exactly at the censoring level and an agent who exerts effort above the censoring level are treated the same and have the same chance of winning each of the prizes. The censoring level can also be thought of as a maximum admissible bid in an otherwise standard all-pay auction. The following result shows that censoring generates a unique equilibrium, which is in pure strategies.

Theorem 4.3. *The effort profile (e^*, \dots, e^*) is uniquely implemented by an all-pay auction with prize profile $y = (x^*/(n - 1), \dots, x^*/(n - 1), 0)$ and censoring level e^* .*

To see why all agents exerting e^* is an equilibrium, observe that upward deviations increase costs without increasing the probability of winning, while downward deviations guarantee the zero prize. The intuition for the result that no other pure-strategy equilibria exist is similar to that for all-pay auctions without censoring. For every positive effort profile $e \neq (e^*, \dots, e^*)$, either an upward deviation discretely increases the probability of winning, or a downward deviation decreases costs without changing the probability of winning. The crucial step in the proof is then to show that censoring destroys any potential mixed-strategy equilibrium.²¹

Theorem 4.3 shows that the optimum can be implemented by a contest with a simple allocation rule. The allocation rule is also weakly monotonic, in the sense that higher effort translates into weakly higher expected payments. Furthermore, the implementation is unique, so that the agents do not face the challenge of coordinating on a given equilibrium.

4.3.4 Implementation in Tullock Contests

The rent-seeking literature commonly studies contests that are imperfectly discriminating, which means that higher effort translates smoothly into a higher probability of winning (for example, see again Konrad, 2009). For the symmetric case with n agents but only one prize, such contests are characterized by a contest success function

$$p_i(e) = \frac{f(e_i)}{\sum_{j \in I} f(e_j)} \quad (4.1)$$

which determines the probability that agent i wins the prize as a function of the effort profile e , where f is continuous, strictly increasing and satisfies $f(0) = 0$. If all agents exert zero effort, each of them wins with equal probability. With more than one prize, as in our optimal contests, the contest success function can be applied in a nested fashion (see e.g. Clark and Riis, 1996): the first prize is allocated according to (4.1) among all n agents, the second prize is allocated according to (4.1) restricted to those $n - 1$ agents who have not received the first prize, and so on.

Tullock contests are a special case for $f(e_i) = e_i^r$, where $r \geq 0$ is a parameter measuring the randomness of the allocation rule. In particular, if $r = 0$ the winners are determined randomly irrespective of the exerted efforts. On the other hand, as $r \rightarrow \infty$ the contest approaches the

²¹The all-pay auction without censoring does not have an equilibrium in pure strategies. Therefore, the max-min optimal contest in Frankel (2014) would not be optimal in our setting with endogenous efforts, because it is unable to implement the optimal effort profile.

perfectly discriminating all-pay auction in which an agent exerting more effort wins a higher prize for sure.

The rent-seeking literature often treats the contest success function as a black box and refrains from explaining how technological and institutional circumstances generate its shape.²² Our next result shows that a specific contest success function arises as part of an optimally designed contract.

Theorem 4.4. *The effort profile (e^*, \dots, e^*) is implemented by a nested contest with prize profile $y = (x^*/(n-1), \dots, x^*/(n-1), 0)$ and the contest success function (4.1) for*

$$f(e_i) = c(e_i)^{r^*(n)} \text{ with } r^*(n) = \frac{n-1}{H_n-1},$$

where $H_n = \sum_{k=1}^n 1/k$ is the n -th harmonic number.

The optimal contest success function incorporates the agents' cost function and thus depends on the effort technology. It can be thought of as a generalization of the Tullock contest success function to settings with non-linear cost. Furthermore, it always reduces to the traditional Tullock shape $f(e_i) = e_i^r$ when the cost function is $c(e_i) = \gamma e_i^\beta$, as in our running example. The optimal randomness parameter $r^*(n)$ is strictly increasing in the number of agents, which means that the optimal contest becomes more discriminating as n grows. For instance, we have $r^*(2) = 2$, $r^*(3) = 2.4$ and $r^*(10) \approx 4.67$. It also holds that $r^*(n) \rightarrow \infty$ in the limit as $n \rightarrow \infty$, so the contest approaches the all-pay auction when the number of agents becomes large.

Theorem 4.4 shows that the optimum can be achieved by using an appropriately designed imperfectly discriminating contest. This contest is strictly monotonic, in the sense that higher effort always translates into strictly higher expected monetary payments.

4.4 Extensions

4.4.1 Imperfect Effort Observation

The assumption that the reviewer perfectly observes the individual effort of each agent can be seen as a strong one. In this subsection, we investigate two different observational constraints that may appear realistic. We will study a setting where only effort differences between the agents but not the levels are observable, and one where only noisy signals of the efforts are available. We will show that contests with stochastic allocation rules can help to overcome the problem of limited observation. We restrict attention to the case of two agents throughout this subsection.

We first assume that the reviewer does not observe the effort profile (e_1, e_2) but only the effort difference $\Delta e = e_1 - e_2$. We then define a *contest with additive noise* as a contest in which the optimal prize x^* is given to agent 1 if and only if $\Delta e + \tilde{\epsilon} \geq 0$, where $\tilde{\epsilon}$ is a random variable (see, for instance, Lazear and Rosen, 1981). Intuitively, agent 1 receives the prize whenever the effort difference Δe is larger than a randomly determined number. Such a contest can be conducted if only Δe is observable, but of course also if the entire profile e can be observed. Our

²²But for example see Jia et al. (2013) for foundations of various functional forms.

next result shows that a contest with additive noise implements the optimum for an appropriate choice of the distribution of $\tilde{\epsilon}$.

Proposition 4.1. *The effort profile (e^*, e^*) is implemented by a contest with additive noise for $\tilde{\epsilon} \sim \mathcal{U}[-c(e^*)/c'(e^*), c(e^*)/c'(e^*)]$.*

The randomness in the allocation rule ensures that unilateral deviations from (e^*, e^*) are not profitable, because the winning probability adapts appropriately. The distribution can neither be too noisy, which would create incentives to deviate to smaller effort levels, nor to concentrated, which would create incentives to deviate to larger effort levels. The uniform distribution described in the proposition is a particularly simple and convenient solution.

We next assume that the reviewer observes only noisy signals of the individual agents' efforts. The usual interpretation is that agent i exerts effort e_i but the final observable output is a random variable \tilde{e}_i that depends on e_i . In fact, the principal may care about output rather than effort, but since her payoffs are unaffected by any noise with zero mean, here we focus on randomness in the reviewer's observation of efforts. Since effort is non-negative we assume that noise is multiplicative, such that

$$\tilde{e}_i = e_i \tilde{\eta}_i$$

for a non-negative random variable $\tilde{\eta}_i$.²³ To ensure closed-form tractability, we assume that the effort cost function is given by $c(e_i) = \gamma e_i^\beta$ for some $\beta > 1$ and $\gamma > 0$. We furthermore assume that the pair $(\tilde{\eta}_1, \tilde{\eta}_2)$ follows a bivariate log-normal distribution,

$$(\tilde{\eta}_1, \tilde{\eta}_2) \sim \ln \mathcal{N} \left[\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right].$$

It is possible – but not necessary for our analysis – to impose parameter constraints that guarantee symmetry and/or that the expected value of $\tilde{e}_1 + \tilde{e}_2$ equals $e_1 + e_2$. We now define a *contest with multiplicative noise* as a contest in which the optimal prize x^* is given to agent 1 if and only if $\tilde{\eta} \tilde{e}_1 / \tilde{e}_2 \geq 1$, where $\tilde{\eta}$ is a non-negative random variable. Intuitively, agent 1 receives the prize whenever the observed effort ratio $\tilde{e}_1 / \tilde{e}_2$ is larger than a randomly determined number. As before, such a contest could also be conducted if there is actually no noise in the observation. Our next result shows that a contest with multiplicative noise can indeed implement the optimum when observation is not too noisy.

Proposition 4.2. *If $\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \leq 2/(\pi\beta^2)$, then the effort profile (e^*, e^*) is implemented by a contest with multiplicative noise for $\tilde{\eta} \sim \ln \mathcal{N}[\nu_\eta, \sigma_\eta^2]$ with*

$$\nu_\eta = \nu_2 - \nu_1 \quad \text{and} \quad \sigma_\eta^2 = \frac{2}{\pi\beta^2} - (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}).$$

As argued before, an appropriate level of randomness in the allocation is required to implement the optimum. Noisy observation of efforts already generates some baseline randomness. If this noise is too strong, then incentives to exert effort cannot be preserved. The condition

²³See Jia et al. (2013) for a survey of contests with multiplicative noise.

$\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \leq 2/(\pi\beta^2)$ in the proposition ensures that this is not the case. For instance, if the random variables $\tilde{\eta}_i$ are i.i.d. it simplifies to $\sigma_i^2 \leq 1/(\pi\beta^2)$. Positive correlation effectively reduces the observational noise and slackens the condition further. If it is satisfied, then the randomness due to noisy effort observation can be raised to the appropriate level by an additional random component in the contract. With $\tilde{\eta}$ as specified in the proposition, the compound random variable $(\tilde{\eta}_2/\tilde{\eta}_1)/\tilde{\eta}$ follows a log-normal distribution with location parameter $\nu = 0$ and scale parameter $\sigma^2 = 2/(\pi\beta^2)$, and we show in the proof that (e^*, e^*) is an equilibrium for the resulting stochastic allocation process.²⁴

4.4.2 Cheap Talk

In the main model we assumed that the principal delegates to the reviewer the decision on how to reward the agents. In this section, we consider a cheap talk model where the reviewer only reports the unknown state of the world to the principal, who then decides how to reward the agents. Similar to Kolotilin et al. (2013), we allow the principal to ex ante limit the set from which she can take her action ex post.

The timing is as follows. First, the principal commits to a set $D \subseteq \Delta T$ of possible actions. Next, the agents choose their efforts simultaneously. The reviewer then observes the efforts and reports back to the principal. After receiving the report, the principal chooses an action from D to reward or punish the agents.

Our previous analysis can be modified to capture this cheap talk setting. Given a credible contract as defined before, we reinterpret $\mu^{e,\theta}$ as the report of a type- θ reviewer who has observed e , rather than as the action that the reviewer can take himself. This report can be interchangeably interpreted as a direct communication of (e, θ) or as a recommendation to pay the agents according to $\mu^{e,\theta}$. The following additional constraint then ensures that the principal always has an incentive to follow the reviewer's recommendation:

$$\pi_P(e, \mu^{e,\theta}) \geq \pi_P(e, \mu^{e',\theta'}) \quad \forall (e, \theta), (e', \theta') \in E \times \Theta, \quad (\text{IC-P})$$

where $\pi_P(e, \mu^{e',\theta'}) = \mathbb{E}_{\mu^{e',\theta'}} [\pi_P(e, t)]$. We are interested in problem (P) with the additional cheap talk constraint (IC-P). The solution to this problem describes the optimum that the principal can achieve in the cheap talk setting.²⁵

Given this formulation of the problem, it is obvious that the cheap talk setting is weakly less permissive than the delegation setting. In the presence of a commitment problem, keeping the authority to make decisions may harm the principal. The following example illustrates possible consequences of the additional constraint (IC-P).

²⁴The formulation of the proposition allows for $\sigma_\eta^2 = 0$, by which we mean that $\tilde{\eta}$ is degenerate and takes the value e^{ν_η} with probability one.

²⁵The constraints (IC-R), (IC-A) and (IC-P) again characterize the Perfect Bayesian equilibria of an extensive form game. Consider the game described in footnote 16 and reinterpret the reviewer's choice from D as a recommendation to the principal. Then add a stage where the principal, after observing the recommendation but not the true state (e, θ) , makes the choice from D . Despite the complexity of the principal's information sets, many of which are off the equilibrium path, constraint (IC-P) prescribes sequential rationality (given any weakly consistent beliefs) for all these information sets. The reason is that the principal's best responses in these information sets are independent of her beliefs about (e, θ) .

Example. Reconsider the first-best contract Φ^{FB} for $n = 2$ and a reviewer of known type $\theta = 3$ derived for our parametric example in Section 4.3. This contract rewards the first-best efforts by the transfer profile (t^{FB}, t^{FB}) and punishes unilateral deviations by one of the profiles $(t^{nd}, 0)$ or $(0, t^{nd})$. Since $2t^{FB} \approx 0.32 < t^{nd} \approx 1.15$, the sum of transfers is not constant across these profiles. Hence the contract violates (IC-P). With this contract, the principal would exhibit a leniency bias. Intuitively, to induce the altruistic reviewer to report a deviator truthfully, the punishment has to be combined with a very large payment t^{nd} to the non-deviating agent. Ex post, the principal is not willing to carry out this costly punishment. \square

The next result shows that, with uncertainty about θ , the principal does not lose by keeping the authority to allocate rewards to the agents.

Proposition 4.3. *Any contest satisfies (IC-P).*

The result follows immediately from the observation that, $\forall(e, \theta), (e', \theta') \in E \times \Theta$,

$$\pi_P(e, \mu^{e', \theta'}) = \sum_{i=1}^n e_i - \mathbb{E}_{\mu^{e', \theta'}} \left[\sum_{i=1}^n t_i \right] = \sum_{i=1}^n e_i - \sum_{i=1}^n y_i$$

is independent of (e', θ') in any contest C_y . Since the principal commits to a transfer sum of $x = \sum_{i=1}^n y_i$ in a contest, once efforts have been exerted she can never increase her payoff by not implementing the reviewer's recommendation on how to allocate the prizes to the agents. Since by Theorem 4.1 the set of optimal contracts always contains a contest, the additional constraint (IC-P) does not restrict the set of achievable outcomes for the principal. Furthermore, the optimal contests derived above for the delegation setting remain optimal in the cheap talk setting.

4.4.3 Non-Separability and Asymmetry

The assumption that the agents have additively separable and symmetric utility functions serves as a natural starting point. It was used at several points in the analysis, but it matters mostly for our characterization of the credibility constraint (IC-R). Separability implies that the agents' sunk efforts do not influence the reviewer's optimal decision on how to distribute the prizes. Symmetry implies that the reviewer exhibits no favoritism. Therefore, a first question that arises is whether our contests are robust to reviewers who exhibit non-separable preferences or favoritism. A second question is whether contests are still optimal. We will address each of these questions in turn. In order to keep comparisons simple, we introduce non-separability and asymmetry directly in the reviewer's utility function, but leave the agents' utility functions unchanged.

Without separability, different distributions of the prizes will lead to different sums of the agents' utilities, which implies that the reviewer will no longer be indifferent between all possible allocations. For instance, non-separability could easily be captured by modifying the reviewer's payoff function to

$$\pi_R(e, t, \theta) = \pi_P(e, t) + \theta \sum_{i=1}^n h(\pi_i(e, t)),$$

for a strictly concave function $h : \mathbb{R} \rightarrow \mathbb{R}$. Whenever $\theta > 0$, which we will assume for the following discussion, this transformation implies a concern for equality of the entire utilities of the agents (“wide bracketing”) rather than just their utilities from monetary transfers (“narrow bracketing”). In particular, the reviewer will prefer giving larger prizes to agents who have exerted higher efforts. For a contest to remain credible, its allocation rule has to be perfectly discriminating. The all-pay auction discussed in Section 4.3.3 satisfies this property, except for efforts above the censoring level e^* . A reviewer who brackets widely may still want to discriminate between agents with efforts above the equilibrium level, thereby destroying the desirable equilibrium properties of the contract. This problem disappears if the censoring level is indeed a maximum bid, i.e., an upper bound on the effort that each agent can provide. In particular, the principal could benefit from setting this bound herself, for instance by imposing page limits on grant proposals or by enforcing maximal work hours. There is no cost associated to such measures in our setting, while they bring the benefit of ensuring robustness to non-separable preferences. The principal can also use tools from *information design* to achieve robustness. She could conceal deviations above e^* from the reviewer (e.g. by forbidding the manager to call the workers on the weekend) and conduct an otherwise standard all-pay auction. She could also structure the observation process in a way that adds the right amount of noise to make a perfectly discriminating contest optimal (see Proposition 4.2). Information design is also a response to favoritism, which can be captured by agent-specific functions h_i in the above expression. A blind reviewing process would make sure that the reviewer observes the chosen efforts but not the identity of the agent who chose each effort. Again, garbling the reviewer’s information in such a way comes at no loss with an optimal contest, but it helps to sustain credibility despite possible asymmetries in how the reviewer wants to treat the agents.

Rather than using these additional tools to achieve robustness of a contest, one may ask if the principal can exploit non-separable and/or asymmetric preferences of the reviewer by writing a contract which is not a contest. The answer to this second question will depend on the exact nature of the principal’s knowledge. With precise knowledge of preferences, it may indeed be the case that a standard contest is no longer optimal. For instance, we conjecture that asymmetries may make generalized contests optimal, in which different prizes are awarded for the same performance rank depending on the identity of the agent occupying that rank (as already proposed by Lazear and Rosen, 1981, p. 863). We leave this extension to future research. However, it is worth pointing out again that a robust contest will often achieve an outcome very close to the first-best, leaving little additional gain from designing a more complex mechanism. This is the case if the agents’ risk-aversion is moderate or the number of agents is sufficiently large, because then the risk imposed by an optimal contest on the agents in equilibrium is not very harmful. We illustrate this in the following example.

Example. Consider our running example and fix $\beta = 2$ and $\gamma = 1$. Figure 4.1 depicts the percentage of first-best payoffs that the principal can achieve with an optimal contest, as a function of the risk-aversion parameter α and for several values of n . Two observations are immediate. First, as $\alpha \rightarrow 1$ the share of the first-best payoffs that the principal can capture converges to one. Second, for any given level of risk-aversion, the principal obtains a larger share with a larger number of agents. This is intuitive, as the risk that an individual agent is

exposed to decreases in the number of agents. The example also shows that, even for a modest number of agents, the principal obtains a substantial share of the first-best payoffs by running an optimal contest. In particular, already for $n = 6$ the principal captures more than 90% of the first-best payoffs for any $\alpha \in (0, 1)$. \square

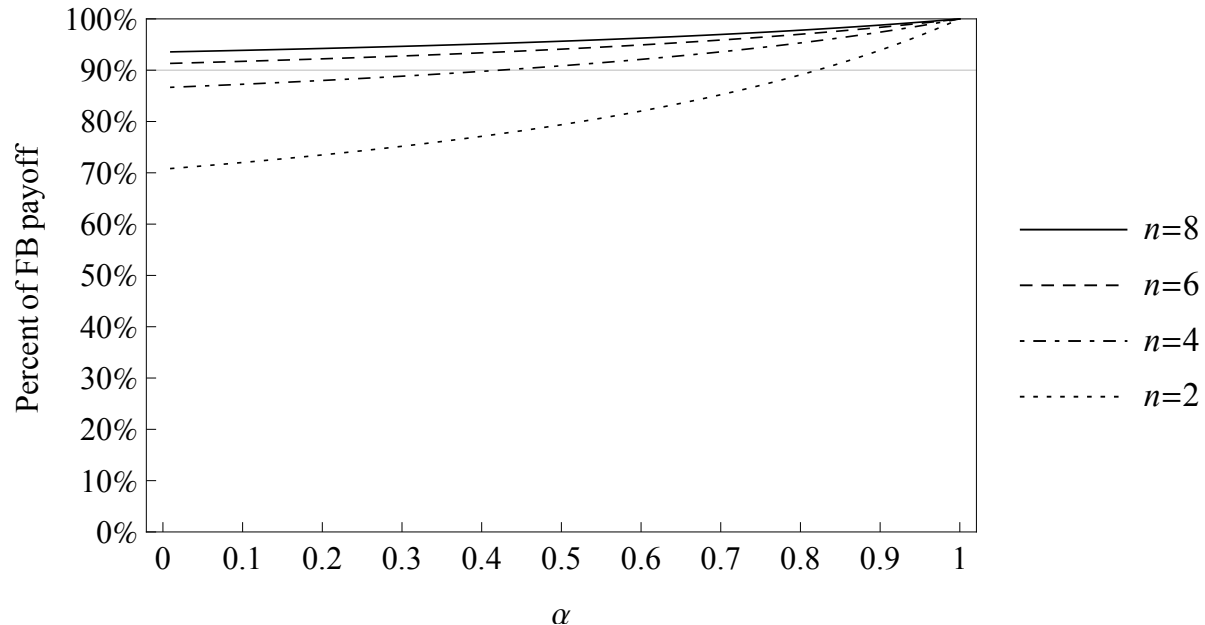


Figure 4.1: Share of first-best payoffs with an optimal contest.

4.5 Related Literature

Our contribution is related to three distinct groups of papers. First, the optimal mechanism in our paper is a contest, so we contribute to the literature examining conditions under which contests are optimal. Second, we work with a three-tiered hierarchical structure with a lenient reviewer. There are several papers featuring a similar structure. Third, the principal in our model delegates the decision on how to reward the agents to the reviewer. Our paper is therefore related to the literature on optimal delegation. We will discuss the connections and differences of our paper to each of these strands of literature in turn.

Optimality of contests. In their seminal paper, Lazear and Rosen (1981) show that contests can implement the socially optimal effort levels when agents are risk-neutral. They assume perfectly competitive labor markets in which the agents obtain all the surplus. Contests are then among the optimal mechanisms because they can induce first-best effort. At the same time, the set of optimal mechanisms also contains a piece-rate contract, among others. For the case of risk-averse agents, Lazear and Rosen (1981) compare the two specific mechanisms of piece-rate contracts and contests. They show that either of them sometimes dominates the other, but they do not establish results on global optimality.

A defining feature of contests is that the payoff of the agents depends on how well they perform relative to each other. This feature can make contests optimal in the presence of common

shocks. Both Green and Stokey (1983) and Nalebuff and Stiglitz (1983) show that contests can do better than individual contracts when agents are risk-averse and there is a random common shock to their outputs. If the relationship between effort and output is ambiguous and the agents are ambiguity-averse, Kellner (2015) shows that contests can be optimal because they filter out the common ambiguity.

In our paper, contests are optimal because they act as a commitment device. Contests provide a commitment for lenient reviewers to punish shirking agents.

Lenient reviewer. Several papers have looked at a three-tiered hierarchy with a lenient reviewer. Prendergast and Topel (1996) and Giebe and Gürtler (2012) consider a situation where the reviewer is facing a single agent and the principal can write contracts where the reviewer's pay is contingent on his behavior. In Prendergast and Topel (1996), both the reviewer and the principal receive a signal about the worker's effort. Their main result is that leniency need not be costly for the firm, because it can charge the reviewer for exercising leniency. In Giebe and Gürtler (2012), the principal offers a menu of contracts to screen lenient and non-lenient reviewers. They show that if the non-lenient type is common enough, the optimal solution can be to pay a flat wage to the reviewer, and rely on the non-lenient type for punishment of agents who shirk. The main difference between these papers and ours is that we consider multiple agents and do not allow contracts which condition payment to the reviewer on the reported evaluation.

Svensson (2003) applies a model with a lenient reviewer to the design of allocation mechanisms for foreign aid. The principal wants to use aid to incentivize countries to implement reforms. In his model, the principal determines the allocation mechanism, but the aid is then distributed by a country manager whose utility takes into account the well-being of the target countries. Svensson (2003) proposes a mechanism where each country manager is given a budget for several similar countries but has discretion in how to allocate the aid across countries. He shows that under certain conditions this mechanism can incentivize countries to reform. Like in our paper, Svensson (2003) considers multiple agents and does not allow for conditional monetary payments to the reviewer. The main difference is that we solve for the optimal delegation mechanism while Svensson (2003) compares two specific mechanisms.

Optimal delegation. Our paper is also related to the optimal delegation literature. In the usual delegation problem, the agent (the reviewer in our setting) is better informed about some exogenous state of the world. The principal delegates a unidimensional decision to the agent, but restricts the set of actions that the agent can choose. The question is how this set should be designed if the preferences of the principal and the agent are misaligned. The first to formulate the problem and show the existence of a solution was Holmström (1977, 1984), who focussed on interval delegation sets. Melumad and Shibano (1991) show that the optimal delegation set does not necessarily take the form of an interval. Alonso and Matouschek (2008) and Amador and Bagwell (2013) characterize the optimal delegation sets in progressively more general environments and find conditions under which the optimal delegation set is indeed an interval.

The canonical delegation model has been applied and extended in a number of ways.²⁶ In the multidimensional delegation model by Frankel (2014), which we have already discussed in the Introduction, optimal mechanisms exhibit what he calls the “aligned delegation” property, which means that all agents behave in the same way as the principal would behave. Our optimal contests also satisfy the aligned delegation property, i.e., the equilibrium behavior of reviewers is independent of their type. Krähmer and Kováč (2016) assume that the agent has a privately known type which encodes his ability to interpret the private information he receives later on. They also find that screening is not beneficial in a large range of cases. Tanner (2014) obtains a no-screening result in a standard delegation model with uncertain bias of the agent.

Most papers cited above focus on deterministic delegation mechanisms. Kováč and Mylovanov (2009) and Goltsman et al. (2009) allow for stochastic delegation mechanisms and derive conditions under which the optimal mechanism is deterministic. As in Frankel (2014), we allow for stochastic mechanisms, and our optimal mechanism is indeed non-deterministic, but in a special sense – the contest prizes are allocated randomly.

Finally, instead of delegating the decision to the agent, the principal could ask the agent to report the state of the world but take the action herself. This is the question addressed in the cheap talk literature in the tradition of Crawford and Sobel (1982). Several papers ask if the principal is better off delegating the decision or just asking for advice. Bester and Krähmer (2008) find that, if the agent needs to exert effort after selecting a project, delegation of the project selection is less likely to be optimal. Kolotilin et al. (2013) consider a model of cheap talk where the principal can ex ante commit not to take a certain action ex post. Again, they show that cheap talk with commitment can outperform delegation. On the other hand, Dessein (2002), Krishna and Morgan (2008), and Ivanov (2010) find that, in general, delegation is better than cheap talk. However, Fehr et al. (2013) and Bartling et al. (2014) provide experimental evidence showing that individuals value decision rights intrinsically, which implies that delegation may not take place even when it is beneficial. These issues do not arise in our model. We can implement in a cheap talk setting the same outcome as with the optimal delegation mechanism, provided the principal can commit to constraining her own actions in the same way as she can constrain the actions of the reviewer.

The main difference between our paper and the delegation literature is that the state of the world, on which the expert has private information, is exogenous in the delegation literature, while it is endogenous in our paper. In our model, the reviewer observes the efforts exerted by the agents. Since the incentives of the agents depend on the behavior of the reviewer, which in turn depends on the given delegation set, the state of the world is affected by the principal’s choice of the delegation set. To our knowledge, our paper is the first to consider this class of delegation problems.

²⁶See Armstrong and Vickers (2010) for an application to merger policy, Pei (2015a) for a model where delegation is used to conceal the principal’s private type, and Guo (2016) for a model of delegation of experimentation.

4.6 Conclusion

In this paper we have analyzed a three-tiered structure consisting of a principal, a reviewer, and n agents. The principal designs a reward scheme in order to incentivize the agents to exert effort. However, the principal herself does not observe the efforts, so she delegates the allocation of rewards to the reviewer. The reviewer has private information about the utility weights he puts on the payoffs of the principal and of the agents.

Our main result is that a very simple mechanism, a contest, is optimal. A contest is optimal because it acts as a commitment for the reviewer to punish shirking agents. We also characterize the set of all optimal contests and show that they have a flat reward structure with $n - 1$ equal positive prizes and one zero prize. Finally, we show that the optimum can be achieved with several common contest success functions, including modified all-pay auctions, nested Tullock contests, and contests with additive or multiplicative noise.

Given our results, other interesting questions can be examined in the framework of delegated performance evaluation. Here we will mention three immediate ones. First, while real-world contests indeed often feature only two prize levels (for instance, the size of research grants is often fixed, students sometimes receive only pass-fail grades, and tenure is either granted or declined), there are also contests with multiple prize levels. GE under Jack Welch separated their employees into three performance levels.²⁷ Grades are often given on a scale from A to D. An interesting question to ask would be why multiple prize levels are offered. We conjecture that they are a response to heterogeneity in agents' ability levels, so that agents of higher ability compete among themselves for higher prizes.²⁸ Second, in addition to incentivizing agents of heterogeneous abilities, principals will often be interested in screening the abilities of the agents in order to be able to assign more responsibilities to more capable agents. The purpose of a tenure or promotion contest is obviously not only to induce hard work, but also to select the right agents for a more advanced position. A question that could be examined in this framework is how screening and provision of incentives interact when both are delegated to potentially biased reviewers. Finally, the principal also hires the reviewer. At first blush, our results might seem to suggest that the principal would be better off by trying to recruit a selfish reviewer who will not take the well-being of the agents into account. However, that would be the case only if the principal was able to determine the reviewer's type with absolute certainty. If there is any remaining uncertainty, our results hold and imply that the principal's maximal payoff does not depend on the reviewer's type. This implies that the principal is free to select the reviewer based on other criteria. For instance, an altruistic mid-level manager may outperform a selfish one in uniting his team to face a common challenge.

Our results offer a novel explanation for the widespread use of contests. We also point to new applications where contest-like mechanisms could be profitably implemented. Settings where an intermediary allocates monetary rewards are widespread in the economy, and as the example of

²⁷See “‘Rank and Yank’ Retains Vocal Fans” (L. Kwok, *The Wall Street Journal*, January 31, 2012).

²⁸Moldovanu and Sela (2001) find, in a model with incomplete information and risk-neutral agents, that multiple prizes can be optimal when cost functions are convex. Olszewski and Siegel (2016) develop a novel approach to contest design, which can be used for very general classes of “large” contests. They characterize the distribution of prizes which maximizes the effort exerted by agents. Among other results, they find that multiple prizes of different levels are optimal when agents have convex costs or are risk-averse.

foreign aid illustrated, they can be found in unexpected places.

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Part III

Appendices

A Appendix: Chapter 1

A.1 Main Proofs and the Running Example

A.1.1 Proof of Proposition 1.1

I prove each of the three statements contained in Proposition 1.1 in turn.

Lemma A.1 (Existence). *An equilibrium in pure actions always exists.*

I provide a constructive proof of Lemma A.1 in three steps. Step 1 constructs the candidate equilibrium investment plan I^* . Step 2 proves that no firm can increase its expected profits by making additional investments. Step 3 proves that no firm can increase its expected profits by reducing investments. Finally, notice that any deviation from the investment plan I^* can be written as a collection of investments and divestments and by Steps 2 and 3, each such investment and divestment decreases expected profits and hence any such collection must decrease expected profits. Thus, no firm can profitably deviate from the investment plan I^* and then, by definition, I^* is an equilibrium.

Step 1. *Constructing the candidate equilibrium.*

Given a game, define m such that

$$\begin{aligned} m &= \max_{\{1, \dots, N\}} n \\ \text{s.t. } R(n, N) - r(n-1, N) - C(0) &> 0 \end{aligned}$$

As by assumption $R(1, N) - r(0, N) - C(0) > 0$, a solution to this maximization problem always exists.

Next, calculate each $\alpha_1, \alpha_2, \dots, \alpha_m$ such that the following condition holds:

$$\begin{aligned} R(1, N) - r(0, N) - C(\alpha_1) &= \\ R(2, N) - r(1, N) - C(\alpha_2) &= \\ R(3, N) - r(2, N) - C(\alpha_3) &= \\ &\vdots \\ R(m, N) - r(m-1, N) - C(\alpha_m) &= 0. \end{aligned}$$

By construction it holds $R(m, N) - r(m-1, N) - C(0) > 0$ and by Assumption 1.1 the reward of innovation are non-increasing, so the inequality holds for all $k < m$. As costs of innovation approach infinity as $j \rightarrow 1$, values $\alpha_1, \alpha_2, \dots, \alpha_m$ always exist by the Intermediate Value Theorem. Furthermore, as $C(j)$ is increasing and by applying Assumption 1.1 it is easy to see that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_m$.

Observe that $N \geq m$. For each $i \in \{1, \dots, m\}$, let $I_i^* = [0, \alpha_i)$. For each $i \in \{m+1, \dots, N\}$ let $I_i^* = \emptyset$. I will demonstrate that I^* is an equilibrium.

Step 2. Suppose that I^* is constructed as above. Then no firm can increase its expected profits by making additional investments.

Proof. First observe that as $\forall j \in (\alpha_1, 1)$, by construction $R(1, N) - r(0, N) - C(j) < 0$, no firm has an incentive to invest beyond the technology frontier. I will consider separately the firms which in I^* have some investment and those firms which do not.

First, fix a firm $i' \in \{1, \dots, m\}$ and take any feasible investment interval $L \subseteq [\alpha_{i'}, \alpha_1)$. It must be that $\min(L) \in [\alpha_k, \alpha_{k-1})^1$ for some $k \leq i'$ and $k \geq 2$ and $\sup(L) \in (\alpha_{k'}, \alpha_{k'-1}]$ for some $k' \leq i'$ and $k' \geq 2$, with $k' \leq k$.

First consider the case where $k' = k$. Then $L \subseteq [\alpha_k, \alpha_{k-1})$. Observe that $R(k, N) - r(k-1, N) - C(\alpha_k) = 0$ and $n(j, I^*) = k-1$ for all $j \in [\alpha_k, \alpha_{k-1})$ by construction. As $C(\cdot)$ is assumed to be strictly increasing, then $R(k, N) - r(k-1, N) - C(j) < 0$ for all $j \in (\min(L), \sup(L))$. Hence,

$$-\int_L C(j)dj + \int_L R(k, N)dj - \int_L r(k-1, N)dj < 0$$

and the firm i' has no incentive to invest in the interval L .

Next consider the case where $k' < k$. Then we can write $L = [\min(L), \alpha_{k-1}) \cup [\alpha_{k-1}, \alpha_{k-2}) \cup \dots \cup [\alpha_{k'}, \sup(L))$. Denote these subintervals as $L^{k-1}, L^{k-2}, \dots, L^{k'-1}$. Observe that by construction, the following statements hold:

$$\begin{aligned} R(k, N) - r(k-1, N) - C(\alpha_k) &= 0 \text{ and } n(j, I^*) = k-1 && \text{for all } j \in L^{k-1} \\ R(k-1, N) - r(k-2, N) - C(\alpha_{k-1}) &= 0 \text{ and } n(j, I^*) = k-2 && \text{for all } j \in L^{k-2} \\ &\vdots && \\ R(k', N) - r(k'-1, N) - C(\alpha_{k'}) &= 0 \text{ and } n(j, I^*) = k'-1 && \text{for all } j \in L^{k'-1} \end{aligned}$$

As $C(\cdot)$ is assumed to be strictly increasing, the following statements hold:

$$\begin{aligned} R(k, N) - r(k-1, N) - C(j) &< 0 && \text{for all } j \in L^{k-1} \\ R(k-1, N) - r(k-2, N) - C(j) &< 0 && \text{for all } j \in L^{k-2} \\ &\vdots && \\ R(k', N) - r(k'-1, N) - C(j) &< 0 && \text{for all } j \in L^{k'-1} \end{aligned}$$

But then it holds

$$\begin{aligned} \int_L R(n(j, I^*) + 1, N) - r(n(j, I^*), N) - C(j)dj &= \\ &= \left(\int_{L^{k-1}} R(k, N) - r(k-1, N) - C(j)dj \right) + \left(\int_{L^{k-2}} R(k-1, N) - r(k-2, N) - C(j)dj \right) + \\ &\quad \dots + \left(\int_{L^{k'-1}} R(k', N) - r(k'-1, N) - C(j)dj \right) < 0 \end{aligned}$$

¹If $\alpha_k = \alpha_{k-1}$, let $L = \{\alpha_k\}$ and $\min(L) = \alpha_k$.

and the firm i' has no incentive to invest in the interval L .

Next, fix a firm $i' \in \{m+1, \dots, N\}$ and take any feasible investment interval $L \subseteq [0, \alpha_1)$. Observe that we can write L as a union of two sets, $L = L' \cup L''$ where $L' \subseteq [0, \alpha_m)$ and $L'' \subseteq [\alpha_m, \alpha_1)$. By the same argument as above, it holds that any investment in the set L'' cannot be profitable. Consider now an investment in the set L' . By construction, m is the maximum number of firms that can profitably invest in the project $j = 0$. As $C(j)$ is strictly increasing it is also the maximum number of firms that can invest in any project. By construction, there are m firms investing in all projects in $[0, \alpha_m)$ and as a result the firm i' cannot profitably invest in the set L' . Thus, the investment in the set L cannot be profitable. \square

Step 3. Suppose that I^* is constructed as above. Then no firm can increase its expected profits by decreasing investments.

Proof. First observe that all firms $i > m$ have zero investments by construction and hence cannot decrease their investments. Fix a firm $i' \in \{1, \dots, m\}$ and take any feasible investment interval $L \subseteq [0, \alpha_{i'})$. Consider a disinvestment from the set L . It must be that $\min(L) \in [\alpha_k, \alpha_{k-1})$ for some $k-1 \geq i'$ with $k \leq m+1$ and $\alpha_{m+1} = 0$ and $\sup(L) \in (\alpha_{k'}, \alpha_{k'-1}]$ for some $k'-1 \geq i'$ and $k' \leq k$.

Consider the case where $k' = k$. Then $L \subseteq [\alpha_k, \alpha_{k-1})$. Observe that $R(k-1, N) - r(k-2, N) - C(j) > 0$ and $n(j, I^*) = k-1$ for all $j \in (\alpha_k, \alpha_{k-1})$ by construction. Hence,

$$\int_L R(k-1, N) - r(k-2, N) - C(j) dj > 0.$$

and the firm i' has no incentive to divest from the interval L .

Next consider the case where $k' < k$. Then we can write $L = [\min(L), \alpha_{k-1}) \cup [\alpha_{k-1}, \alpha_{k-2}) \cup \dots \cup [\alpha_{k'}, \sup(L))$. Denote these subintervals as $L^{k-1}, L^{k-2}, \dots, L^{k'-1}$. Observe that by construction, the following statements hold:

$$\begin{aligned} R(k-1, N) - r(k-2, N) - C(j) &> 0 \text{ and } n(j, I^*) = k-1 && \text{for all } j \in L^{k-1} \\ R(k-2, N) - r(k-3, N) - C(j) &> 0 \text{ and } n(j, I^*) = k-2 && \text{for all } j \in L^{k-2} \\ &\vdots \\ R(k'-1, N) - r(k'-2, N) - C(j) &> 0 \text{ and } n(j, I^*) = k'-1 && \text{for all } j \in L^{k'-1} \end{aligned}$$

But then it holds

$$\begin{aligned} \int_{L''} R(n(j, I^*), N) - r(n(j, I^*) - 1, N) - C(j) dj &= \\ = \int_{L^{k-1}} R(k-1, N) - r(k-2, N) - C(j) dj &+ \int_{L^{k-2}} R(k-2, N) - r(k-3, N) - C(j) dj + \\ + \dots + \int_{L^{k'-1}} R(k'-1, N) - r(k'-2, N) - C(j) dj &> 0 \end{aligned}$$

and the firm i' has no incentive to divest from the interval L .

Thus no firm can increase its expected profits by divesting from any feasible interval L . \square

Lemma A.2. *If I^* is an equilibrium and $0 < n(j, I^*) < N$ for some $j \in [0, 1)$, then infinitely many equilibria exist.*

Proof. Let I^* be an equilibrium and fix some $j \in [0, 1)$ such that $0 < n(j, I^*) < N$. Then there exist firms i and i' such that $j \in I_i$ and $j \notin I_{i'}$. Then there must exist some $\epsilon > 0$ such that $[j, j + \epsilon) \cap I_i = [j, j + \epsilon)$ and $[j, j + \epsilon) \cap I_{i'} = \emptyset$.

Consider an investment plan \hat{I} such that $\hat{I}_{i''} = I_{i''}^*$, for all $i'' \neq i, i'$. For i and i' let $\hat{I}_i = I_i^* \setminus [j, j + \epsilon)$ and $\hat{I}_{i'} = I_{i'}^* \cup [j, j + \epsilon)$. In words, only transfer the ownership of investment in projects $[j, j + \epsilon)$ from firm i to firm i' and leave everything else unchanged. I will demonstrate that \hat{I} is also an equilibrium and hence, because there is an infinite number of ways to choose ϵ , there exists an infinity of equilibria.

Suppose that \hat{I} is not an equilibrium. Then, there exists a firm that can profitably change its investment plan. This means that there exists an interval $L \subset [0, 1)$ and a firm i^l such that firm can increase its expected profits by either investing in the interval L or divesting from the interval L . Consider first those firms $i'' \neq i, i'$. By construction $n(j, I^*) = n(j, \hat{I})$ for all $j \in [0, 1)$. From Equation 1.1, it is clear that strategic effects only influence the expected profit through $n(j, I)$. Thus, if a firm can profitably deviate from \hat{I} it can also profitably deviate from I^* .

Next, consider firms i and i' . As their investment plans are unchanged in the set $[0, j) \cup [j + \epsilon, 1)$ by an argument identical to the one above, if they could profitably deviate in this set from \hat{I} , they could also profitably deviate from I^* . Now consider the set $[j, j + \epsilon)$. Firm i' can deviate in this set only by not investing. Suppose that there exists an interval $L' \subseteq [j, j + \epsilon)$, such that not investing in this set increases the expected profits of firm i' . Then it must be the case that

$$\int_{L'} R(n(j, \hat{I}), N) - r(n(j, \hat{I}) - 1, N) - C(j) dj < 0.$$

But in this case, firm i could profitably deviate from I^* by not investing in the interval L' . Next, firm i can deviate in the set $[j, j + \epsilon)$ only by investing. Suppose that there exists an interval $L' \subseteq [j, j + \epsilon)$, such that investing in this set increases the expected profits of firm i . Then it must be the case that

$$\int_{L'} R(n(j, \hat{I}) + 1, N) - r(n(j, \hat{I}), N) - C(j) dj > 0.$$

But in this case, firm i' could profitably deviate from I^* by investing in the interval L' .

Thus, in each case, a profitable deviation from \hat{I} implies a profitable deviation from I^* which contradicts the initial assumption that I^* is an equilibrium. \square

Lemma A.3. *If there are multiple equilibria they all result in the same market portfolio of investment in innovation. That is, if I_1^* and I_2^* are equilibrium investment plans, then $n(j, I_1^*) = n(j, I_2^*)$ for all $j \in [0, 1)$. If I_1^* is an equilibrium then any investment plan I_3^* such that $n(j, I_1^*) = n(j, I_3^*)$ for all $j \in [0, 1)$ is also an equilibrium.*

I prove this Lemma in two steps, each proving one part of the Lemma.

Step 1. *If there are multiple equilibria they all result in the same market portfolio of investment*

in innovation. That is, if I_1^* and I_2^* are equilibrium investment plans, then $n(j, I_1^*) = n(j, I_2^*)$ for all $j \in [0, 1)$.

Proof. Suppose not. Then, there exists a point $j \in [0, 1)$ such that $n(j, I_1^*) \neq n(j, I_2^*)$. Suppose, without loss of generality, that $n(j, I_1^*) > n(j, I_2^*)$. Fix a firm i and a point $\epsilon > 0$ such that it holds $[j, j+\epsilon) \cap I_{1,i}^* = [j, j+\epsilon)$ and $[j, j+\epsilon) \cap I_{2,i}^* = \emptyset$ and $n(l, I_1^*) = \text{const}$, $n(l, I_2^*) = \text{const}$, $\forall l \in [j, j+\epsilon)$. Such a firm and a point always exist.

(1) Suppose $R(n(j, I_1^*), N) - r(n(j, I_1^*) - 1, N) \geq C(j + \epsilon)$. As $C(\cdot)$ is increasing it holds $R(n(j, I_1^*), N) - r(n(j, I_1^*) - 1, N) > C(l)$ for all $l \in [j, j + \epsilon)$. By Assumption 1.1 it holds $R(n(j, I_2^*) + 1, N) - r(n(j, I_2^*), N) > C(l)$ for all $l \in [j, j + \epsilon)$. Then it holds

$$\int_j^{j+\epsilon} R(n(l, I_2^*) + 1, N) - r(n(l, I_2^*), N) - C(l) dl > 0.$$

Then I_2^* cannot be an equilibrium as firm i could increase its expected profits by investing in the interval $[j, j + \epsilon)$.

(2) Suppose $R(n(j, I_1^*), N) - r(n(j, I_1^*) - 1, N) < C(j + \epsilon)$. Then there exists an $\epsilon' > 0$ such that $R(n(j, I_1^*), N) - r(n(j, I_1^*) - 1, N) < C(l)$ for all $l \in [j + \epsilon - \epsilon', j + \epsilon)$. Then it holds

$$\int_{j+\epsilon-\epsilon'}^{j+\epsilon} R(n(l, I_1^*), N) - r(n(l, I_1^*) - 1, N) - C(l) dl < 0.$$

Then I_1^* cannot be an equilibrium as firm i could increase its expected profits by not investing in the interval $[j + \epsilon - \epsilon', j + \epsilon)$. \square

Step 2. If I_1^* is an equilibrium then any investment plan I_3^* such that $n(j, I_1^*) = n(j, I_3^*)$ for all $j \in [0, 1)$ is also an equilibrium.

Proof. Suppose not. Then in the investment plan I_3^* exists a firm i and an interval L such that firm i would be better off by either investing in the interval L or by divesting from interval L .

(1) Suppose that the firm i can profitably invest in the interval L . Then there exists $L' \subseteq L$ such that $R(n(j, I_3^*) + 1, N) - r(n(j, I_3^*), N) > C(l)$ for all $l \in L'$. But then there exists a firm i'' and a set $L'' \subseteq L'$ such that $L'' \cap I_{1,i''}^* = \emptyset$. Then I_1^* cannot be an equilibrium as the firm i'' could profitably deviate by investing in the interval L'' .

(2) Suppose that the firm i can profitably divest from the interval L . Then there exists $L' \subseteq L$ such that $R(n(j, I_3^*), N) - r(n(j, I_3^* - 1), N) < C(l)$ for all $l \in L'$. But then there exists a firm i'' and a set $L'' \subseteq L'$ such that $L'' \cap I_{1,i''}^* = L''$. Then I_1^* cannot be an equilibrium as the firm i'' could profitably deviate by divesting from the interval L'' . \square

A.1.2 Proof of Proposition 1.2

Proof. Observe that by Assumption 1.2, for all $n \geq 1$ we have $r(n, N) = 0$. Let I^* be the equilibrium constructed in the proof of Lemma A.1. If $n^*(j)$ as constructed in Proposition 1.2 is equal to $n(j, I^*)$ for all $j \in [0, 1)$, then by statement 3 in Proposition 1.1 it characterizes the equilibrium market portfolio of research projects.

I here show that $n^*(j) = n(j, I^*)$. First, as noted in the proof of Lemma A.1, observe that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_m$. First suppose that $j \in [\alpha_1, 1)$. Then it must be that $j \geq \alpha_k$ for all

$k \in \{1, \dots, m\}$. Hence $n^*(j) = 0$. By construction, $n(l, I^*) = 0$ for all $l \in [\alpha_1, 1)$. Next, suppose that $j \in [0, \alpha_1)$. Then

$$\begin{aligned} n^*(j) &= \max_{1, \dots, m} k \\ \text{s.t. } j &< \alpha_k \end{aligned}$$

Let $\hat{k} = n^*(j)$. It holds that $j < \alpha_{\hat{k}} \leq \alpha_{\hat{k}-1} \leq \dots \leq \alpha_1$. By construction, each firm i such that $i \in \{1, \dots, \hat{k}\}$ invests in j . Hence, $n(j, I^*) = \hat{k}$. \square

A.1.3 Proof of Proposition 1.3

I will prove each statement in the proposition in turn. Let the number of firms in the pre-innovation market increase from N to N' . Denote the maximum number of firms investing in two cases as m and m' and the k -firm frontiers as α_k and α'_k .

Lemma A.4. *In equilibrium, the variety of projects developed and the probability of developing an innovation weakly increases, that is $\alpha_1 \leq \alpha'_1$.*

Proof. By Proposition 1.2 the variety of projects developed in the two equilibria is equal to the sets $[0, \alpha_1)$ and $[0, \alpha'_1)$ and the probability of successfully developing an innovation is equal to α_1 and α'_1 . Thus we need to show that $\alpha_1 \leq \alpha'_1$. By Proposition 1.2 we have $R(1) - r(0, N) = C(\alpha_1)$ and $R(1) - r(0, N') = C(\alpha'_1)$. By assumption 1.3 we have $r(n-1, N') \leq r(n-1, N)$ hence $C(\alpha'_1) \geq C(\alpha_1)$. As $C(j)$ is assumed to be increasing this implies $\alpha'_1 \geq \alpha_1$. \square

Lemma A.5. *The maximum number of firms investing also increases, that is $m \leq m'$.*

Proof. By Proposition 1.2 we have $m = \max_{\{1, \dots, N\}} n$ such that $R(n) - r(n-1, N) - C(0) > 0$. Observe that $m \in \{1, \dots, N\} \subseteq \{1, \dots, N'\}$. If $n = 1$, by assumption 1.3 we have $R(n) - r(n-1, N') - C(0) \geq R(n) - r(n-1, N) - C(0)$. If $n > 1$, by assumption 1.2 we have $R(n) - r(n-1, N') - C(0) = R(n) - r(n-1, N) - C(0)$. Hence m is chosen from a subset from which m' is chosen and it satisfies a stricter condition. Thus m' cannot be lower than m . \square

Lemma A.6. *Apart from the increase in variety of projects developed and in the maximum number of firms investing, the equilibrium portfolio remains the same. That is, $n(j, I) = n(j, I')$ for all $j \in [0, 1) \setminus \{[0, \alpha'_{m'}] \cup [\alpha_1, \alpha'_1]\}$.*

Proof. Because $\alpha'_1 \geq \alpha_1$, we have $n(j, I) = n(j, I') = 0$ for all $j \in [\alpha'_1, 1)$. As $R(k)$ does not depend on N or N' , by Proposition 1.2 it follows that $\alpha_k = \alpha'_k$ for all $2 \leq k \leq m$. Again by Proposition 1.2 it follows that $n(j, I) = n(j, I')$ for all $j \in (\alpha'_{m'}, \alpha_1)$. \square

A.1.4 Proof of Proposition 1.4

Proof. Suppose the intensity of competition increases from (R, r) to (R', r') . Denote the respective equilibrium investment plans as I and I' . Then the following holds by direct application of Proposition 1.2:

$$m' \begin{cases} = m & \text{if } R'(m, N) - C(0) > 0 \\ < m & \text{otherwise} \end{cases}$$

$$\begin{aligned}\alpha'_k &< \alpha_k \quad \forall k \in \{2, \dots, m'\} \\ \alpha'_1 &> \alpha_1\end{aligned}$$

and

$$\begin{aligned}n(j, I') &\leq n(j, I) && \text{for all } j \in [0, \alpha_1) \\ n(j, I') &> n(j, I) && \text{for all } j \in [\alpha_1, \alpha'_1) \\ n(j, I') &= n(j, I) = 0 && \text{for all } j \in [\alpha'_1, 1).\end{aligned}$$

Because $\alpha'_1 > \alpha_1$ the variety of research projects undertaken and the probability of discovering an innovation increase. Because $\alpha'_k < \alpha_k \quad \forall k \in \{2, \dots, m'\}$ there are some projects which are developed by fewer firms than with less intense competition. Hence the amount of duplication of research decreases. □

A.1.5 Proof of Proposition 1.5

Proof. The portfolio given in Proposition 1.5 can always be constructed. I show that it is optimal. Suppose not. Then, there exists a project $j \in [0, 1)$ such that investing either more or less than $n^o(j)$ marginally increases the expected welfare. There are two cases: (1) there exists a possibility to profitably increase investment in some project and (2) there exists a possibility to profitably decrease investment in some project.

(1) Suppose that there exists a possibility to profitably increase investment in some project j . Then there exists some n such that $n^o(j) < n \leq N$ and

$$W(n) - nC(j) > W(n^o(j)) - n^o(j)C(j).$$

Then we can write

$$\left[\sum_{k=n^o(j)+1}^n W(k) - W(k-1) - C(j) \right] + W(n^o(j)) - n^o(j)C(j) > W(n^o(j)) - n^o(j)C(j)$$

$$\sum_{k=n^o(j)+1}^n W(k) - W(k-1) - C(j) > 0.$$

Suppose $n^o(j) = m^o$. Then, $\forall k > n^o(j)$ it holds:

$$\left(W(k) - W(k-1) - C(0) \right) + \left(C(0) - C(j) \right) \leq 0$$

the first bracketed expression is by construction not positive whereas the second is not positive because the function $C(\cdot)$ is increasing. A sum of non-positive elements cannot be positive. A contradiction.

Suppose now that $n^o(j) < m^o$. By construction it holds

$$W(n^o(j) + 1) - W(n^o(j)) - C(\alpha_{n^o(j)+1}^o) = 0$$

and for every $k > n^0(j)$ by assumption holds

$$W(k) - W(k-1) - C(\alpha_{n^0(j)+1}^o) \leq 0$$

By construction $j > \alpha_{n^0(j)+1}^o$, so that $C(j) > C(\alpha_{n^0(j)+1}^o)$. Plugging it into the expression above, it follows

$$W(k) - W(k-1) - C(j) \leq 0 \quad \forall k > n^0(j).$$

Again, a sum of non-positive elements cannot be positive. A contradiction.

(2) Suppose that there exists a possibility to profitably decrease investment in some project j . Then there exists some n such that $0 \leq n < n^0(j)$ and

$$W(n) - nC(j) > W(n^0(j)) - n^0(j)C(j).$$

Then we can write

$$\begin{aligned} W(n) - nC(j) &> W(n) - nC(j) + \left[\sum_{k=n+1}^{n^0(j)} W(k) - W(k-1) - C(j) \right] \\ 0 &> \sum_{k=n+1}^{n^0(j)} W(k) - W(k-1) - C(j). \end{aligned}$$

By construction $W(n^0(j)) - W(n^0(j)-1) - C(j) > 0$ and by assumption it holds for any $k < n^0(j)$ that

$$W(k) - W(k-1) - C(j) > 0$$

A sum of positive elements has to be positive. A contradiction. □

A.1.6 Proof of Corollary 1.1

Using the notation of Propositions 1.2 and 1.4 the variety of R&D projects in the market portfolio is $[0, \alpha_1]$ and the variety of R&D projects in the optimal portfolio is $[0, \alpha_1^o]$. Thus the market will underinvest in the variety of R&D projects if and only if $\alpha_1 < \alpha_1^o$. As $C(\cdot)$ is increasing this is equivalent to $C(\alpha_1) < C(\alpha_1^o)$. By Propositions 1.2 and 1.4 it then holds $R(1) - r(0, N) < W(1) - W(0)$. Decomposing $W(1)$ into $CS(1) + R(1)$ and $W(0)$ into $CS(0) + Nr(0, N)$ yields the desired result. Overinvestment and optimal investment cases are proven analogously.

A.1.7 Proof of Proposition 1.6

As assumptions 1.1, 1.2 and 1.4 hold, by Corollary 1.1 this market will underinvest in the variety of R&D projects if and only if $CS(1) - W(0) + r(0, N) > 0$. Denote with q_1 the quantity supplied by a monopolist with the innovation and with q_0 the quantity supplied by a single firm if no innovation is developed. As the innovation is drastic $P(q_1) < P(Nq_0)$ or equivalently $q_1 > Nq_0$. We can write the consumer surplus as the difference between total utility and the total expense paid by consumers, so it holds $CS(1) = \int_0^{q_1} P(s)ds - P(q_1)q_1$. Welfare is total utility less the

total cost of production, so it holds $W(0) = \int_0^{Nq_0} P(s)ds - N\bar{c}(q_0)$. Then this market will underinvest if and only if:

$$\int_0^{q_1} P(s)ds - P(q_1)q_1 - \left[\int_0^{Nq_0} P(s)ds - N\bar{c}(q_0) \right] + r(0, N) > 0.$$

Subtracting the integrals and rearranging terms gives:

$$\int_{Nq_0}^{q_1} P(s)ds - P(q_1)q_1 + N\bar{c}(q_0) + r(0, N) > 0.$$

By assumption $P'(\cdot) < 0$ so that $\int_{Nq_0}^{q_1} P(s)ds \geq (q_1 - Nq_0)P(q_1)$. The inequality above will hold whenever the following inequality holds:

$$(q_1 - Nq_0)P(q_1) - P(q_1)q_1 + N\bar{c}(q_0) + r(0, N) > 0.$$

Rearranging gives:

$$N\bar{c}(q_0) - Nq_0P(q_1) + r(0, N) > 0.$$

By assumption $\bar{c}'(\cdot) \geq 0$ so that $\bar{c}(q_0) = \int_0^{q_0} \bar{c}'(s)ds \geq (q_0 - 0)\bar{c}'(0)$. The inequality above will hold whenever the following inequality holds:

$$Nq_0(\bar{c}'(0) - P(q_1)) + r(0, N) > 0.$$

As $r(0, N) \geq 0$ by rationality of firms and $\bar{c}'(0) > P(q_1)$ by definition of a drastic process innovation, the above inequality always holds.

A.1.8 Proof of Corollary 1.2

Consider first the case where $\alpha_k^o < \alpha_k$. As $C(\cdot)$ is increasing then it holds $C(\alpha_k^o) < C(\alpha_k)$. As assumptions 1.1, 1.2 and 1.4 hold, then Propositions 1.2 and 1.5 hold. Applying them yields $W(k) - W(k-1) < R(k)$ and decomposing the expression for $W(\cdot)$ yields $kR(k) + CS(k) - (k-1)R(k-1) - CS(k-1) < R(k)$. Rearranging gives:

$$\delta(k) = \left[(k-1)(R(k) - R(k-1)) \right] + \left[CS(k) - CS(k-1) \right] < 0.$$

Hence $\alpha_k^o < \alpha_k$ if and only if $\delta(k) < 0$. The other cases follow analogously.

A.1.9 Proof of Proposition 1.7

Observe that the proof of Proposition 1.1 does not require Assumption 1.2. Proof of Proposition 1.7 exactly mirrors the proof of Proposition 1.2, except without setting $r(n, N) = 0$ for all $n \geq 1$. In essence, Proposition 1.2 is a special case of Proposition 1.7.

A.1.10 Example: Process innovation in a Cournot market

As an illustrative example, consider a simple Cournot model with homogeneous products, linear costs and linear demand. Suppose that there are three firms facing inverse demand of the form

$$P(q_1, q_2, q_3) = 1 - (q_1 + q_2 + q_3)$$

where q_i is the quantity supplied by the firm i . Denote with \bar{c} the marginal cost of production with the old technology and with \underline{c} the marginal cost of production with the new technology, where $\underline{c} \leq \bar{c} \leq 1$. That is, firms have the possibility to develop a process innovation which reduces their production cost from \bar{c} to \underline{c} . The innovation is drastic if

$$\bar{c} \geq \frac{1 + \underline{c}}{2}, \quad (\text{A.1})$$

where the right hand side of the inequality is the price which would be obtained if there was a monopolist with marginal cost \underline{c} in the market. Suppose that the costs of research are given by

$$C(j) = b\sqrt{\frac{j}{1-j}}, \quad j \in [0, 1),$$

where $b > 0$ is a slope parameter. Observe that this choice of cost function implies $C(0) = 0$ so that $m = N$, that is at least some of the innovation projects are developed by all the firms in the market.

Using standard methods, the profits in Cournot markets with n firms and marginal costs c are given by $\Pi(n, c) = (1 - c)^2 / (n + 1)^2$. From this equation it is possible to derive the ex post payoffs:

$$\begin{aligned} r(0, 3) &= \frac{(1 - \bar{c})^2}{16}, & R(1) &= \frac{(1 - \underline{c})^2}{4}, \\ R(2) &= \frac{(1 - \underline{c})^2}{9}, & R(3) &= \frac{(1 - \underline{c})^2}{16}. \end{aligned}$$

In order to be able to apply Proposition 1.2, we have to check if Assumptions 1.1 and 1.2 hold. Assumption 1.2 holds whenever Equation (A.1) is satisfied. In addition, Assumption 1.1 holds whenever $\underline{c} \leq \bar{c} \leq 1$, which is assumed.² Hence, Proposition 1.2 can be used to characterize the equilibrium R&D portfolio.

Applying Proposition 1.2 yields the following k -firm frontiers for $k \in \{1, 2, 3\}$:

$$\begin{aligned} \alpha_3 &= \frac{R(3)^2}{b^2 + R(3)^2}, \\ \alpha_2 &= \frac{R(2)^2}{b^2 + R(2)^2}, \end{aligned}$$

²If Assumption 1.2 is satisfied then the sufficient condition for Assumption 1.1 to hold is $\frac{(1 - \underline{c})^2}{4} - \frac{(1 - \bar{c})^2}{16} \geq \frac{(1 - \underline{c})^2}{9}$. If $\bar{c} = 1$, the inequality is satisfied. If $\bar{c} < 1$, the expression simplifies to $\left(\frac{1 - \underline{c}}{1 - \bar{c}}\right)^2 \geq \frac{9}{20}$, which is always satisfied because the left-hand expression is always greater than 1.

$$\alpha_1 = \frac{(R(1) - r(0, 3))^2}{b^2 + (R(1) - r(0, 3))^2}.$$

All projects in the interval $[0, \alpha_3)$ are developed by all three firms whereas the projects in the interval $[\alpha_3, \alpha_2)$ are developed by two firms. Projects in the interval $[\alpha_2, \alpha_1)$ are developed by just one firm whereas the projects in the interval $[\alpha_1, 1)$ are not developed at all. Thus, if the successful project is from the interval $[0, \alpha_1)$, the market will successfully develop the innovation and all firms which invested in the successful project will compete with the production costs \underline{c} . However, if the successful project is from the interval $[\alpha_1, 1)$ the market will not develop the innovation and all firms will compete with the production costs \bar{c} .

Figure 1.1 (in Section 1.4) illustrates the equilibrium market portfolio in the case where $b = 0.05$, $\bar{c} = 3/4$ and $\underline{c} = 1/2$.

Merger of two firms

Suppose now that two of the three firms merge, leaving everything else unchanged. That is, suppose that the merger affects only the number of firms which are active in the market. Denote with $\{r', R'\}$ payoffs after the merger and with $\{r, R\}$ payoffs without the merger. Clearly, $\{r, R\}$ are the same as before. The new payoff functions are given by:

$$\begin{aligned} r'(0, 2) &= \frac{(1 - \bar{c})^2}{9}, \\ R'(1) &= \frac{(1 - \underline{c})^2}{4}, \\ R'(2) &= \frac{(1 - \underline{c})^2}{9}. \end{aligned}$$

It is immediately clear that $r'(0, 2) > r(0, 3)$ whereas $R'(1) = R(1)$ and $R'(2) = R(2)$. The intuition behind this is that the merger increases profits in the market when all firms are active, because there are fewer competitors, hence $r'(0, 2) > r(0, 3)$. However, due to the drastic nature of innovation, post-innovation profits only depend on the number of firms which successfully innovated, hence $R'(1) = R(1)$ and $R'(2) = R(2)$. As after the merger there are only two firms in the market, the maximum number of firms investing in any project is at most 2.

Applying Proposition 1.2 yields $m' = 2 < m = 3$, $\alpha'_2 = \alpha_2$ but $\alpha'_1 < \alpha_1$. This is in line with results derived in Proposition 1.3. Figure 1.2 (in Section 1.5) graphically illustrates the change in the market portfolio of research projects after the merger.

From Cournot to Bertrand competition

Consider again the scenario with three firms and suppose that the type of competition changes from Cournot to Bertrand. This change can be interpreted as an increase in the intensity of competition among the firms. How will the market portfolio of research projects change? Applying Proposition 1.4, the variety of research projects developed will increase whereas the duplication of research projects will decrease.

From above we know that with three firms engaged in a Cournot competition, the market portfolio will be characterized by the maximum number of firms investing m and the firm-

frontiers α_3 , α_2 , and α_1 . The equilibrium values of the market under Bertrand competition are denoted with a prime. When there are multiple symmetric firms competing in a homogeneous goods Bertrand market, in equilibrium, firms set prices equal to marginal cost of production and earn zero profits. Hence, the payoff functions will be $r'(0, 3) = 0$ and $R'(2) = R'(3) = 0$. The monopolist earns the same profits in both cases, that is $R'(1) = R(1) = (1 - \underline{c})^2/4$.

As Assumptions 1.1 and 1.2 clearly hold, Proposition 1.2 can be applied. It immediately follows that $m' = 1$, hence $n(j) < n'(j)$ for all $j < \alpha_2$. This drastic change in the amount of duplication is due to the fact that firms make no profits if there is a competitor, so firms choose to do no duplication at all. Simple calculations show that $\alpha'_1 > \alpha_1$. Figure 1.3 (in section 1.5) illustrates the change in the market portfolio of research projects due to the change of competition from Cournot to Bertrand.

Market and optimal portfolios in a Cournot model

Consider again the Cournot example from the appendix A.1.10. Social welfare generated in this product market by firms supplying total quantity Q is given by:

$$W_Q = \int_0^Q P(s)ds - Qc = \int_0^Q (1 - s)ds - Qc = Q \left(1 - \frac{Q}{2} - c \right), \quad (\text{A.2})$$

where c is the constant marginal cost of production. Using standard results, the total quantity supplied in a Cournot market with n firms is given by

$$Q(n, c) = \frac{n(1 - c)}{n + 1}.$$

Assumptions 1.1 and 1.2 hold and simple calculations show that Assumption 1.4 holds as well. Hence Propositions 1.5 and 1.6 can be applied. Proposition 1.6 immediately informs us that there will be underinvestment in the variety of research projects. Figure 1.4 illustrates the difference between the optimal and the market portfolio in this market.

A.2 Further Extensions

In this Appendix I consider four extensions of the basic model. First two deal with the possibility that a merger could generate efficiencies which could overturn the result that a merger decreases the variety of approaches to innovation. In Section A.2.2 I consider a mixed strategy equilibrium and show that the equilibrium structure and determinants of comparative static results found for pure strategy equilibria are robust. Finally, in Section A.2.3 I consider the case when the research budgets are limited or when financing of research is costly. Proofs are presented sequentially in the end of the Appendix.

A.2.1 Efficiency defense

General cost reduction

Consider the original setting, but suppose that if two firms merge, they become more efficient at developing innovations. That is, suppose that for the merged firm the fixed cost of developing

any given approach j is given by $\tilde{C}(j; \epsilon) : [0, 1) \rightarrow \mathbb{R}^+$ such that $\tilde{C}(j; \epsilon) \leq C(j)$ for all j . Like C , assume that \tilde{C} is continuous, differentiable, strictly increasing and that $\lim_{j \rightarrow 1} \tilde{C}(j; \epsilon) = \infty$. Finally, suppose that in this setting ϵ captures the size of the efficiency gains resulting from the merger, such that $\partial \tilde{C}(j; \epsilon) / \partial \epsilon < 0$. Simple functional forms that satisfy these assumptions (for the appropriate domain of ϵ) are: (i) additive $\tilde{C}(j; \epsilon) = C(j) - \epsilon$ and (ii) multiplicative $\tilde{C}(j; \epsilon) = (1 - \epsilon)C(j)$. Let the number of symmetric firms with innovation cost functions $C(j)$ in the pre-merger market be N . Suppose that after the merger, the merged firm has the cost function $\tilde{C}(j; \epsilon)$ and that the remaining $N - 2$ firms are active with cost functions $C(j)$.

In this setting, if the efficiency gain from the merger is sufficiently large, there will be no loss of diversity in the approaches to innovation as a consequence of the merger.

Proposition A.1 (Merger with general cost reductions).

Suppose that Assumptions 1.1, 1.2, and 1.3 hold and that the merger results in efficiency gains as above. Then:

1. *A PSE in the post-merger market always exists.*
2. *In any PSE in the post-merger market the set of developed approaches is $[0, \tilde{\alpha}_1)$, where $\tilde{\alpha}_1$ is given by $\tilde{C}(\tilde{\alpha}_1; \epsilon) = R(1) - r(0, N - 1)$.*
3. *If the technology frontier in the market without a merger is given by α_1 , then the merger does not reduce the variety of approaches to innovation if and only if*

$$C(\alpha_1) - \tilde{C}(\alpha_1; \epsilon) \geq r(0, N - 1) - r(0, N). \quad (\text{A.3})$$

From Proposition 1.3 we know that a merger, via the Arrow effect, reduces the incentives to invest. This is captured by the right-hand side of the inequality (A.3). However, Proposition A.1 states that if the efficiency caused by the merger is large enough, which is given by the left-hand side of the inequality (A.3), it can outweigh the decrease in the incentive to invest. In this case, the merger does not lead to a decrease in the variety of approaches to innovation. In the case of additive efficiency gains, that is if $\tilde{C}(j; \epsilon) = C(j) - \epsilon$, the inequality (A.3) would simplify to $\epsilon \geq r(0, N - 1) - r(0, N)$. In the case of multiplicative efficiency gains, that is if $\tilde{C}(j; \epsilon) = (1 - \epsilon)C(j)$, the inequality (A.3) simplifies to $\epsilon C(\alpha_1) \geq r(0, N - 1) - r(0, N)$. It is clear that there always exists ϵ large enough such that these inequalities are satisfied, and that such a merger would not lead to a decrease in the variety of approaches to innovation.

Approach-specific synergies

In the previous section, I considered a situation in which a merger between any two firms leads to the same efficiency gains. Now, suppose that each firm has some specific knowledge and that if the two firms merged, they could combine this specific knowledge in a way that would enable the merged entity to conduct research over some specific interval of approaches more efficiently. In this setting, it will not only be the size of the efficiency gains that will be required for a successful efficiency defense, but also that the efficiency gains occurs over approaches that would not have been developed in the post-merger market absent the efficiency gains.

For concreteness, consider this simple extension of the model. Suppose that each firm $i \in \{1, \dots, N\}$ is located on the unit line in an equidistant manner. That is, the location of the firm is given by $i/(N+1)$. Firm's location represents its specific knowledge. On its own, this knowledge is worthless. However, suppose that firm i merged with some firm $l \in \{1, \dots, N\}$, $i \neq l$. Then the merged entity would receive efficiency gains over an interval midway between the location of the firms i and l . That is, the merged entity is more efficient over an interval

$$\left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta \right)$$

for some $0 < \delta \leq 1/(N+1)$.³ For simplicity, suppose that on the above interval the cost of developing an approach is zero for the merged entity. That is, a firm which has not merged has the innovation cost function $C(j)$ and the merged firm (where the merging firms are i and l) has the innovation cost function

$$\tilde{C}_{i,l}(j) = \begin{cases} 0 & \text{if } j \in \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta \right) \\ C(j) & \text{otherwise} \end{cases}.$$

In this setting, a merger will not lead to a decrease in the variety of approaches if the efficiency gain covers large enough interval (δ is large enough) and if the efficiency gain occurs over projects which would not be developed absent the efficiency gain. The latter depends on which firms actually merge. Thus, for the same size of the efficiency gain from the merger, some mergers will lead to a decrease in the variety of approaches whereas others will not.

Proposition A.2 (Merger with approach-specific synergies).

Suppose that Assumptions 1.1, 1.2, and 1.3 hold and that the merger results in efficiency gains as above. Then, if firms i and l merge:

1. *A PSE in the post-merger market always exists.*
2. *In any PSE in the post-merger market the set of developed approaches is $[0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta \right)$, where $\tilde{\alpha}_1$ is given by $C(\tilde{\alpha}_1) = R(1) - r(0, N-1)$.*
3. *If the technology frontier in the market without a merger is given by α_1 , then the merger does not reduce the variety of approaches to innovation if and only if*

$$\int_{x \in [0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta \right)} dx \geq \alpha_1. \quad (\text{A.4})$$

As the innovation cost function for the merged firm is not strictly increasing, the set of developed approaches need not be convex any more. However, the intuition is clear — the efficiency gain must be both large enough and must materialize over the projects which would not have been developed otherwise for the efficiency defense to be successful.

³The upper bound is a simplification that ensures that efficiency gains are always in the unit interval. It would be straightforward to remove it, at the cost of more cumbersome notation.

A.2.2 Mixed strategies

Consider the original setting, but suppose that firms are using mixed strategies. As a simplifying assumption, I will consider only the following pure strategy space

$$\mathcal{I}^m = \{0\} \cup \{[0, j) : j \in (0, 1)\}$$

and I will look only at symmetric mixed strategy equilibria (SMSE). Because now the pure strategy of a firm is restricted to choosing an interval $[0, j)$, it can be identified with the upper bound of the interval j . Denote with $f_i(j)$ the density that the firm i chooses the interval $[0, j)$ and with $F_i(j)$ the related cumulative distribution function.

Proposition A.3 (Characterization of SMSE). *Suppose $N = 2$ and the Assumptions 1.1 and 1.2 hold. Then the unique SMSE is characterized by the cumulative distribution function:*

$$F(j) = \begin{cases} 0 & \text{if } \frac{C(j) - R(2)}{R(1) - r(0, 2) - R(2)} < 0 \\ \frac{C(j) - R(2)}{R(1) - r(0, 2) - R(2)} & \text{if } \frac{C(j) - R(2)}{R(1) - r(0, 2) - R(2)} \in [0, 1] \\ 1 & \text{if } \frac{C(j) - R(2)}{R(1) - r(0, 2) - R(2)} > 1 \end{cases}$$

for $j \in [0, 1)$.

Suppose that $R(2) - C(0) > 0$. In pure actions, by Proposition 1.2 it holds: $m = 2$, $C(\alpha_1) = R(1) - r(0, N)$ and $C(\alpha_2) = R(2)$. Thus, both firms will invest in the interval $[0, \alpha_1)$, only one firm will invest in the interval $[\alpha_1, \alpha_2)$ and no firm will invest in $[\alpha_2, 0)$. Now consider SMSE. By Proposition A.3, for $j \in [0, \alpha_1)$ it holds $F(j) = 0$, thus both firms invest in this interval with probability 1. For $j \in (\alpha_1, \alpha_2)$ it holds $0 < F(j) < 1$, thus firms invest with some probability less than one. If $j \in [\alpha_2, 0)$, then $F(j) = 1$, so that firms do not invest in this interval. Similar results hold if $R(2) - C(0) \leq 0$. Thus, the basic structure of the model is the same in both pure and mixed strategy equilibria. In particular the k -firm frontiers are the same. Furthermore, comparative statics results regarding variety of projects undertaken remain qualitatively the same, as anything that affects the one-firm frontier has qualitatively the same effect both in pure action and in mixed strategy equilibria. Figure A.1 illustrates the difference between the (expected) equilibrium market portfolios for the Cournot duopoly example from the appendix A.1.10. The mixed strategy equilibrium is “smoother” than the pure strategy equilibrium. The reason for this is that the integer problem is not present in the mixed strategy setting. In pure strategy equilibrium, some projects have higher expected profits than others (i.e. project $\alpha_2 + \epsilon$ is more profitable than $\alpha_2 - \epsilon$ for some small positive ϵ). In mixed strategy equilibria, all projects in the interval where the mixing occurs have the same expected profits.

A.2.3 Limited budget and costly financing

This section considers the case where firms face an exogenous constraint on their research budgets. This constraint can take the form of a budget constraint, or it can (equivalently) take the form of costly financing for research. The main result is that a binding budget constraint

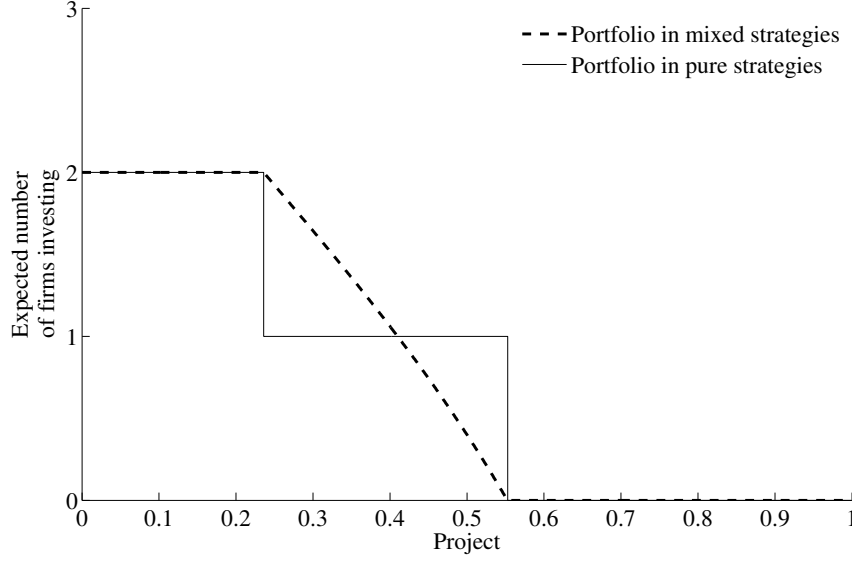


Figure A.1: Symmetric mixed strategy equilibrium.

or a costly source of financing imposes a positive opportunity cost on investments in research projects, but that the main mechanics of the model remain unchanged.

First, suppose that there are two firms in a market and that each firm has a budget B and suppose that the budget is binding, in the sense that firms would want to invest more in research if they had more resources.⁴ Then the following result is obtained:

Proposition A.4 (Equilibrium in a game with limited budget).

Suppose that Assumptions 1.1 and 1.2 hold, and that there are two firms with a budget B . Then, a PSE always exists, the induced PSE market portfolio is unique and any investment plan which induces a portfolio identical to the market PSE portfolio is itself a PSE. Furthermore, there exists a unique $\beta > 0$ such that:

1. *the maximum number of firms investing in any project m^b is given by*

$$m^b = \max_{\{1,2\}} n \quad \text{s.t.} \quad R(n) - r(n-1, N) - C(0) > \beta.$$

2. *Firm frontiers are determined by*

$$\begin{aligned} R(1) - r(0, N) - C(\alpha_1^b) &= \\ R(m^b) - r(m^b - 1, N) - C(\alpha_m^b) &= \beta. \end{aligned}$$

3. *Let $\alpha_{m+1}^b = 0$ and $\alpha_0^b = 1$. The total expenditure is*

$$m \int_0^{\alpha_m^b} C(j) dj + (m-1) \int_{\alpha_m^b}^{\alpha_{m-1}^b} C(j) dj = 2B.$$

⁴Formally, if $m = 2$ then $2B < 2 \int_0^{\alpha_2^b} C(j) dj + \int_{\alpha_2^b}^{\alpha_1^b} C(j) dj$ and if $m = 1$ then $2B < \int_0^{\alpha_1^b} C(j) dj$.

Then the PSE portfolio $n^b(j)$ is given by

$$n^b(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}^b, \alpha_k^b).$$

As can be seen from conditions 1. and 2., the basic form of the market equilibrium portfolio will remain unchanged. The only difference is that the budget constraint will impose positive opportunity cost β on the choice of research projects, as opposed to the unconstrained equilibrium where the opportunity costs was 0. In the scenario where firms can borrow unlimited funds at some positive price, the equilibrium characterized above still holds, but now β is exogenously given and as a function of the cost of financing.

A.2.4 Proof of Proposition A.1

I prove each of the three statements contained in Proposition A.1 in turn. The proof is analogous to the proof of Proposition 1.1.

Lemma A.7 (Existence). *A PSE in the post-merger market always exists.*

I provide a constructive proof of Lemma A.7 in three steps. Step 1 constructs the candidate equilibrium investment plan \tilde{I} . Step 2 proves that no firm can increase its expected profits by making additional investments. Step 3 proves that no firm can increase its expected profits by reducing investments. Finally, notice that any deviation from the investment plan \tilde{I} can be written as a collection of investments and divestments and by Steps 2 and 3, each such investment and divestment decreases expected profits and hence any such collection must decrease expected profits. Thus, no firm can profitably deviate from the investment plan \tilde{I} and then, by definition, \tilde{I} is an equilibrium.

Step 1. *Constructing the candidate equilibrium.*

Given a game, define m such that

$$\begin{aligned} m &= \max_{\{1, \dots, N-1\}} n \\ \text{s.t. } & R(n) - r(n-1, N-1) - C(0) > 0 \end{aligned}$$

As by assumption $R(1) - r(0, N-1) - C(0) > 0$, a solution to this maximization problem always exists.

Next, calculate each $\tilde{\alpha}_1, \alpha_2, \dots, \alpha_m$ such that the following condition holds:

$$\begin{aligned} R(1) - r(0, N-1) - \tilde{C}(\tilde{\alpha}_1; \epsilon) &= \\ R(2) - C(\alpha_2) &= \\ R(3) - C(\alpha_3) &= \\ &\vdots \\ R(m) - C(\alpha_m) &= 0. \end{aligned}$$

By construction it holds $R(m) - r(m-1, N) - C(0) > 0$ and by Assumption 1.1 the reward of innovation are non-increasing, so the inequality holds for all $k < m$. As $\tilde{C}(j; \epsilon) \leq C(j)$, the inequality also holds for the merged firm. As costs of innovation approach infinity as $j \rightarrow 1$, values $\tilde{\alpha}_1, \alpha_2, \dots, \alpha_m$ always exist by the Intermediate Value Theorem. Furthermore, as $C(j)$ is increasing, $\tilde{C}(j; \epsilon) \leq C(j)$, and by applying Assumption 1.1 it is easy to see that $\tilde{\alpha}_1 \geq \alpha_2 \geq \dots \geq \alpha_m$.

Observe that $N-1 \geq m$. Label the merged firm with subscript $i = 1$ and all other firms with $i \in \{2, \dots, N-1\}$. For the merged firm, let $\tilde{I}_1 = [0, \tilde{\alpha}_1)$. For each $i \in \{2, \dots, m\}$, let $\tilde{I}_i = [0, \alpha_i)$. For each $i \in \{m+1, \dots, N-1\}$ let $\tilde{I}_i = \emptyset$. I will demonstrate that \tilde{I} is an equilibrium.

Step 2. *Suppose that \tilde{I} is constructed as above. Then no firm can increase its expected profits by making additional investments.*

Proof. First observe that for all firms $i \in \{2, \dots, N-1\}$ the argument is identical as in the proof of Proposition 1.1, as the investment decision of the firm only depends on their investment costs and the number of firms investing in any given project. Thus we only need to show that the merged firm cannot increase profits by making additional investments. This holds by construction. The merged firm already invests in the entire interval $[0, \tilde{\alpha}_1)$. For any $j > \tilde{\alpha}_1$ it holds $R(1) - r(0, N-1) - \tilde{C}(j; \epsilon) < 0$ as $\tilde{C}(j; \epsilon)$ is strictly increasing in j . Thus, no additional profitable investments exist for the merged firm. \square

Step 3. *Suppose that \tilde{I} is constructed as above. Then no firm can increase its expected profits by decreasing investments.*

Proof. Similar to the argument in the previous step, it is sufficient to show that the merged firm cannot increase profits by decreasing investments. First, observe that for $j \in [\alpha_2, \tilde{\alpha}_1)$ the investment is profitable as it holds $R(1) - r(0, N-1) - \tilde{C}(j; \epsilon) > 0$ for all j in the interval. For all j in $[0, \alpha_2)$ it holds $R(n(j, \tilde{I})) - C(j) > 0$ (otherwise non-merged firms would have an incentive to divest) and as $\tilde{C}(j; \epsilon) \leq C(j)$, it also holds $R(n(j, \tilde{I})) - \tilde{C}(j; \epsilon) > 0$ for all $j \in [0, \alpha_2)$. Hence, the merged firm cannot increase profits by divesting. \square

Lemma A.8. *In any PSE in the post-merger market the set of developed approaches is $[0, \tilde{\alpha}_1)$, where $\tilde{\alpha}_1$ is given by $\tilde{C}(\tilde{\alpha}_1; \epsilon) = R(1) - r(0, N-1)$.*

Proof. Suppose not. Then, either there exist an interval $l \subseteq [0, \tilde{\alpha}_1)$ where no firm invests, or there exists an interval $l' \subseteq [\tilde{\alpha}_1, 1)$ where at least one firm invests, or both. First suppose that an interval l exists. As $\tilde{C}(j; \epsilon)$ is strictly increasing, then for all $j \in l$ it holds $\tilde{C}(j; \epsilon) < R(1) - r(0, N-1)$. Hence the merged firm can profitably invest in the subset of l . Next, suppose an interval l' exists. Observe that for any $j > \tilde{\alpha}_1$ it holds $C(j) \geq \tilde{C}(j; \epsilon) > R(1) - r(0, N-1)$. By Assumption 1.1 it then also holds $C(j) \geq \tilde{C}(j; \epsilon) > R(k)$ for all $k \geq 2$. Thus, no firm can profitably invest any subset of l' . \square

Lemma A.9. *If the technology frontier in the market without a merger is given by α_1 , then the merger does not reduce the variety of approaches to innovation if and only if*

$$C(\alpha_1) - \tilde{C}(\alpha_1; \epsilon) \geq r(0, N-1) - r(0, N).$$

Proof. The merger does not reduce variety if and only if $\tilde{\alpha}_1 \geq \alpha_1$. As $\tilde{C}(j; \epsilon)$ is strictly increasing, this will hold if and only if $\tilde{C}(\alpha_1; \epsilon) \leq R(1) - r(0, N - 1)$. By Proposition 1.2, we know that $C(\alpha_1) = R(1) - r(0, N)$. Subtracting the above inequality, the claim follows. \square

A.2.5 Proof of Proposition A.2

I prove each of the three statements contained in Proposition A.2 in turn. The proof is analogous to the proof of Proposition 1.1.

Lemma A.10 (Existence). *A PSE in the post-merger market always exists.*

I provide a constructive proof of Lemma A.10 in three steps, analogous to the proof of Lemma A.7.

Step 1. *Constructing the candidate equilibrium.*

Given a game, define m such that

$$m = \max_{\{1, \dots, N-1\}} n$$

$$\text{s.t. } R(n) - r(n - 1, N - 1) - C(0) > 0$$

As by assumption $R(1) - r(0, N - 1) - C(0) > 0$, a solution to this maximization problem always exists.

Next, calculate each $\tilde{\alpha}_1, \alpha_2, \dots, \alpha_m$ such that the following condition holds:

$$\begin{aligned} R(1) - r(0, N - 1) - C(\tilde{\alpha}_1) &= \\ R(2) - C(\alpha_2) &= \\ R(3) - C(\alpha_3) &= \\ &\vdots \\ R(m) - C(\alpha_m) &= 0. \end{aligned}$$

By construction it holds $R(m) - r(m - 1, N) - C(0) > 0$ and by Assumption 1.1 the reward of innovation are non-increasing, so the inequality holds for all $k < m$. As costs of innovation approach infinity as $j \rightarrow 1$, values $\tilde{\alpha}_1, \alpha_2, \dots, \alpha_m$ always exist by the Intermediate Value Theorem. Furthermore, by Assumption 1.1, it is easy to see that $\tilde{\alpha}_1 \geq \alpha_2 \geq \dots \geq \alpha_m$.

Observe that $N - 1 \geq m$. Label the merged firm with subscript $i = 1$ and all other firms with $i \in \{2, \dots, N - 1\}$. For the merged firm, let $\tilde{I}_1 = [0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta \right)$. For each $i \in \{2, \dots, m\}$, let $\tilde{I}_i = [0, \alpha_i)$. For each $i \in \{m + 1, \dots, N - 1\}$ let $\tilde{I}_i = \emptyset$. I will demonstrate that \tilde{I} is an equilibrium.

Step 2. *Suppose that \tilde{I} is constructed as above. Then no firm can increase its expected profits by making additional investments.*

Proof. For firms $i \in \{2, \dots, N - 1\}$ the argument is identical as in the proof of Proposition A.1. Thus we only need to show that the merged firm cannot increase profits by making additional

investments. The merged firm already invests in the entire set $[0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)$. For any $j > \tilde{\alpha}_1$ and $j \notin \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)$ it holds $R(1) - r(0, N-1) - \tilde{C}(j) < 0$ as $C(j)$ is strictly increasing. Thus, no additional profitable investments exist for the merged firm. \square

Step 3. Suppose that \tilde{I} is constructed as above. Then no firm can increase its expected profits by decreasing investments.

Proof. Similar to the argument in the previous step, it is sufficient to show that the merged firm cannot increase profits by decreasing investments. First, observe that for $j \in [\alpha_2, \tilde{\alpha}_1)$ the investment is profitable as it holds $R(1) - r(0, N-1) - C(j) > 0$ for all j in the interval. For all j in $[0, \alpha_2)$ it holds $R(n(j, \tilde{I})) - C(j) > 0$ (otherwise non-merged firms would have an incentive to divest) and as $\tilde{C}(j) \leq C(j)$, it also holds $R(n(j, \tilde{I})) - \tilde{C}(j) > 0$ for all $j \in [0, \alpha_2)$. For all $j \in \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)$ the investment is costless. Hence, the merged firm cannot increase profits by divesting. \square

Lemma A.11. In any PSE in the post-merger market the set of developed approaches is $[0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)$, where $\tilde{\alpha}_1$ is given by $C(\tilde{\alpha}_1) = R(1) - r(0, N-1)$.

Proof. Suppose not. Then, either there exists an interval $l \subseteq [0, \tilde{\alpha}_1)$ or an interval $l' \subseteq \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)$ where no firm invests, or there exists an interval $l'' \subseteq [0, 1) \setminus \left([0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)\right)$ where at least one firm invests. First suppose that an interval l exists. Because by construction it holds $C(j) < R(1) - r(0, N-1)$ for all $j < \tilde{\alpha}_1$, any firm can profitably invest in the set l . Next, suppose l' exists. The merged firm can invest in the set l' without any cost, hence it can increase its expected profit by investing. Finally, suppose an interval l'' exists. Observe that for any $j > \tilde{\alpha}_1$ and $j \notin \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)$ it holds $\tilde{C}(j) > R(1) - r(0, N-1)$. By Assumption 1.1 it then also holds $C(j) \geq \tilde{C}(j) > R(k)$ for all $k \geq 2$. Thus, no firm can profitably invest any subset of l'' . \square

Lemma A.12. If the technology frontier in the market without a merger is given by α_1 , then the merger does not reduce the variety of approaches to innovation if and only if

$$\int_{x \in [0, \tilde{\alpha}_1) \cup \left[\frac{i+l}{2(N+1)} - \delta, \frac{i+l}{2(N+1)} + \delta\right)} dx \geq \alpha_1.$$

Proof. Without the merger, the set of developed approaches by Proposition 1.2 is $[0, \alpha_1)$. The result follows by Claim 2 of the Proposition. \square

A.2.6 Proof of Proposition A.3

Suppose, without loss of generality, that the firm 2 invests according to some probability density function f_2 , with the cumulative density function F_2 . Consider any pure action x_1 of firm 1. The profit of the firm 1 can be expressed as:

$$\begin{aligned} \pi_1(x_1 | F_2) = & - \int_0^{x_1} C(j) dj + \int_0^{x_1} \left[\int_0^{x_2} R(2) dj + \int_{x_2}^{x_1} R(1) dj + \int_{x_1}^1 r(0, 2) dj \right] f_2(x_2) dx_2 + \\ & + \int_{x_1}^1 \left[\int_0^{x_1} R(2) dj + \int_{x_2}^1 r(0, 2) dj \right] f_2(x_2) dx_2. \end{aligned}$$

Deriving:

$$\begin{aligned} \frac{d\pi_1(x_1|F_2)}{dx_1} = & -C(x_1) + \left[\int_0^{x_1} R(2)dj + \int_{x_1}^{x_1} R(1)dj + \int_{x_1}^1 r(0,2)dj \right] f_2(x_1) - \\ & - \left[\int_0^{x_1} R(2)dj + \int_{x_1}^1 r(0,2)dj \right] f_2(x_1) + \\ & + \int_0^{x_1} [R(1) - r(0,2)] f_2(x_2)dx_2 + \int_{x_1}^1 R(2)f_2(x_2)dx_2 \end{aligned}$$

and simplifying:

$$\frac{d\pi_1(x_1|F_2)}{dx_1} = -C(x_1) + [R(1) - r(0,2)] F_2(x_1) + R(2) (1 - F_2(x_1)).$$

Next, use the assumption that the equilibrium is symmetric, that is $F_1 = F_2 = F$. In equilibrium it has to hold $d\pi_1(x_1|F)/dx_1 = 0$ for all x_1 in the support of f . This condition is uniquely satisfied by

$$\tilde{F}(j) = \frac{C(j) - R(2)}{R(1) - r(0,2) - R(2)}$$

for j in the support of f .

Observe that \tilde{F} is strictly increasing and, for all j such that $\tilde{F}(j) < 0$ it follows that $d\pi_1(x_1, F)/dx_1 > 0$, and for all j such that $\tilde{F}(j) > 1$ it follows that $d\pi_1(x_1, F_2)/dx_1 < 0$. Hence, the unique symmetric equilibrium is given by the profile (F, F) where

$$F(j) = \begin{cases} 0 & \text{if } \frac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} < 0 \\ \frac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} & \text{if } \frac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} \in [0, 1] \\ 1 & \text{if } \frac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} > 1 \end{cases}$$

for $j \in [0, 1)$.

A.2.7 Proof of Proposition A.4

I prove this statement in three steps. First I show that some β satisfying all conditions in the proposition always exists and is unique. Next, I construct an investment plan inducing the same portfolio as the one in Proposition A.4. Finally I show that the constructed investment plan is an equilibrium and that any investment plan inducing the same portfolio is an equilibrium as well.

Lemma A.13. *β always exists and is unique.*

Proof. Define functions $\psi^1(\beta) : [0, \beta^1] \rightarrow \mathbb{R}^+$, $\psi^2(\beta) : [0, \beta^2] \rightarrow \mathbb{R}^+$ such that

$$\begin{aligned} \psi^1(\beta) &= \int_0^{C^{-1}(R(1)-r(0,2)-\beta)} C(j)dj \\ \psi^2(\beta) &= \int_0^{C^{-1}(R(2)-\beta)} C(j)dj + \int_0^{C^{-1}(R(1)-r(0,2)-\beta)} C(j)dj \end{aligned}$$

with $\beta^1 = R(1) - r(0, 2) - C(0)$ and $\beta^2 = R(2) - C(0)$. As $C(\cdot)$ is continuous, strictly increasing and defined on an interval, its inverse is continuous and strictly increasing as well. Hence both $\psi^1(\beta)$ and $\psi^2(\beta)$ are continuous and strictly decreasing. Furthermore, by Assumption 1.1 it holds $\beta^1 \geq \beta^2$.

Either (i) $\psi^1(\beta^2) \geq 2B$ or (ii) $\psi^1(\beta^2) < 2B$. If (i) is true, $\psi^1(\beta^2) \geq 2B$ and $\psi^1(\beta^1) = 0 < 2B$. By the Intermediate Value Theorem there exists some $\beta^* \in [\beta^2, \beta^1]$ such that $\psi^1(\beta^*) = 2B$ and furthermore β^* is unique because $\psi^1(\beta)$ is strictly decreasing. Observe that $\beta^* \in [R(2) - C(0), \beta^1]$, hence $R(1) - r(0, 2) - C(0) > \beta^*$ and $R(2) - C(0) \leq \beta^*$. Thus, by the condition 1. of Proposition A.4 we have $m^b = 1$. By the condition 2. the firm frontier is $\alpha_1^b = C^{-1}(R(1) - r(0, 2) - \beta^*)$. Finally, the condition 3. holds because $\int_0^{\alpha_1^b} C(j) dj = 2B$ by construction. Hence, β^* uniquely satisfies all three conditions of the Proposition A.4.

If (ii) is true, then $\psi^2(\beta^2) < 2B$ and $\psi^2(0) > 2B$, by the assumption of the binding budget constraint. By the Intermediate Value Theorem there exists some $\beta^* \in (0, \beta^2)$ such that $\psi^2(\beta^*) = 2B$ and furthermore β^* is unique because $\psi^2(\beta)$ is strictly decreasing. Observe that $\beta^* \in (0, \beta^2)$, hence $R(2) - C(0) > \beta^*$. Thus, by the condition 1. of Proposition A.4 we have $m^b = 2$. By the condition 2. the firm frontiers are $\alpha_1^b = C^{-1}(R(1) - r(0, 2) - \beta^*)$ and $\alpha_2^b = C^{-1}(R(2) - \beta^*)$. Finally, the condition 3. holds because $\int_0^{\alpha_2^b} C(j) dj + \int_0^{\alpha_1^b} C(j) dj = 2B$ by construction. Hence, β^* uniquely satisfies all three conditions of the Proposition A.4. \square

Lemma A.14. *An equilibrium inducing portfolio equivalent to the one characterized in Proposition A.4 can always be constructed.*

Proof. Either $m = 2$ or $m = 1$. If $m = 1$, then it holds $\int_0^{\alpha_1^b} C(j) dj = 2B$. Then there exists a point x such that $0 < x < \alpha_1^b$ and $\int_0^x C(j) dj = B$ and $\int_x^{\alpha_1^b} C(j) dj = B$. Let one firm invest in the interval $[0, x)$ and the other firm in the interval $[x, \alpha_1^b]$. This investment plan generates a portfolio equivalent to the one characterized.

If $m = 2$, then it holds $2 \int_0^{\alpha_2^b} C(j) dj + \int_{\alpha_2^b}^{\alpha_1^b} C(j) dj = 2B$. Then there exists a point x such that $\alpha_2^b \leq x \leq \alpha_1^b$ and $\int_0^x C(j) dj = B$ and $\int_0^{\alpha_2^b} C(j) dj + \int_x^{\alpha_1^b} C(j) dj = B$. Let one firm invest in the interval $[0, x)$ and the other firm in the set $[0, \alpha_2^b) \cup [x, \alpha_1^b]$. This investment plan generates a portfolio equivalent to the one characterized. \square

Lemma A.15. *The investment plan constructed in Lemma A.14 is an equilibrium and any investment plan inducing the same portfolio is an equilibrium as well.*

Proof. The proof is analogous to the proof of Proposition 1.1, with the opportunity cost equal to β as opposed to 0. \square

B Appendix: Chapter 2

B.1 Basics

In the following, we introduce some notation that we use throughout the Appendix. We also formulate the restrictions implied by subgame perfection.

B.1.1 Notation

We consistently use subscripts B for buyers, $i = 1, 2$ for suppliers and T for "total" (buyers plus suppliers). Superscripts such as fpt for fixed-price tournament, bt for bonus tournament or a for auction refer to the contest \mathcal{P} under consideration. We will drop these superscripts whenever there is no danger of confusion.

1. $p_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b, \Psi]^2}$ is a *price strategy function*.¹
2. $\pi_i(p_i, p_j | q_i, q_j)$ is the realized revenue that supplier i earns with prices p_1 and p_2 , conditional on qualities q_1 and q_2 , assuming that the buyer chooses the i sequentially rationally, i.e., the i that maximizes $q_i - p_i$ in contest \mathcal{P} .²
3. $\hat{\Pi}_i(v_i, v_j, p_i, p_j)$ is the expectation over $\pi_i(p_i, p_j | q_i, q_j)$ when suppliers choose $v_1, v_2, p_1()$ and $p_2()$, where the expectation is taken over all pairs of quality realizations for given (v_1, v_2) .
4. $\Pi_i^{\mathcal{P}}(v_i, v_j) = \hat{\Pi}_i(v_i, v_j, p_i, p_j)$, where $p_i()$ and $p_j()$ are the subgame equilibria for the contest \mathcal{P} as in Lemma 2.2, is the (*expected*) *revenue* of supplier i .
5. $S_i^{\mathcal{P}}(v_i, v_j) = \Pi_i^{\mathcal{P}}(v_i, v_j) + t - C$ is the (*expected*) *surplus* of supplier i .
6. $S_B^{\mathcal{P}}(v_i, v_j) = E_{\sigma}[\max\{q(v_1, \sigma), q(v_2, \sigma)\}] - \Pi_1^{\mathcal{P}}(v_i, v_j) - \Pi_2^{\mathcal{P}}(v_i, v_j) - 2t$ is the (*expected*) *surplus* of the buyer.

B.1.2 Subgame-Perfect Equilibrium

A subgame-perfect equilibrium of the innovation contest given by \mathcal{P} consists of supplier strategies $s_i = (v_i, p_i) \in [0, 1] \times \mathcal{P}^{[\Psi-b, \Psi]^2}$ and buyer strategies $\nu \in \{v_1, v_2\}^{(\mathcal{P} \times [\Psi-b, \Psi])^2}$ such that:

(DC1) ν_1 and ν_2 are sequentially rational.

(DC2) $\pi_i(p_i(q_i, q_j), p_j(q_j, q_i) | q_i, q_j) \geq \pi_i(p'_i, p_j(q_j, q_i) | q_i, q_j)$ for all $p'_i \in \mathcal{P}, (q_i, q_j) \in [\Psi - b, \Psi]^2$ (sequential rationality of supplier i)

¹For sets X and Y , Y^X is the set of all mappings from X to Y .

²When $q_1 - p_1 = q_2 - p_2$, we appeal to tie-breaking rule (T1) below.

(DC3) $\hat{\Pi}_i(v_i, v_j, p_i(q_i, q_j), p_j(q_j, q_i)) \geq \hat{\Pi}_i(v'_i, v_j, \tilde{p}_i(q_i, q_j), p_j(q_j, q_i))$ for all $v'_i \in [0, 1]$ and all $\tilde{p}_i(q_i, q_j) \in \mathcal{P}^{[\Psi-b, \Psi] \times [\Psi-b, \Psi]}$ (best-response condition for supplier i).

B.2 Proofs of Auxiliary Results (Section 2.3.1)

B.2.1 Proof of Lemma 2.1

Suppose, without loss of generality, that $v_1 \leq v_2$. The total surplus is

$$S_T(v_1, v_2) - 2C = \int_0^1 \max\{q(v_1, \sigma), q(v_2, \sigma)\} dF(\sigma) - 2C =$$

$$\Psi - b \left(\begin{array}{c} \int_0^{v_1} (v_1 - \sigma) dF(\sigma) + \int_{v_1}^{(v_1+v_2)/2} (\sigma - v_1) dF(\sigma) + \\ \int_{(v_1+v_2)/2}^{v_2} (v_2 - \sigma) dF(\sigma) + \int_{v_2}^1 (\sigma - v_2) dF(\sigma) \end{array} \right) - 2C.$$

This is a continuous function with a compact domain, hence it attains the maximum. Note that

$$\frac{\partial S_T(v_1, v_2)}{\partial v_1} = b(-2F(v_1) + F((v_1 + v_2)/2)) \quad (\text{B.1})$$

$$\frac{\partial S_T(v_1, v_2)}{\partial v_2} = b(1 - 2F(v_2) + F((v_1 + v_2)/2)). \quad (\text{B.2})$$

(B.1) and (B.2) imply that there are no boundary optima. To see this, first note that $\frac{\partial S_T(0, v_2)}{\partial v_1} > 0 \forall v_2 > 0$ and $\frac{\partial S_T(v_1, 1)}{\partial v_2} < 0 \forall v_1 < 1$. Moreover $(v_1, v_2) = (0, 0)$ and $(1, 1)$ are both dominated by $(1/2, 1/2)$. Thus, the optimum must satisfy

$$-2F(v_1) + F((v_1 + v_2)/2) = 0 \quad (\text{B.3})$$

$$1 - 2F(v_2) + F((v_1 + v_2)/2) = 0. \quad (\text{B.4})$$

Together these conditions imply $F(v_2^*) = 1/2 + F(v_1^*)$.

For $v_1 \in [0, 1/2]$, let $g(v_1) = F^{-1}\left(F(v_1) + \frac{1}{2}\right)$. F^{-1} is well-defined because of (A1)(iii). Inserting $v_2 = g(v_1)$ in (B.3) and (B.4), the first-order conditions hold for $(v_1, v_2) = (v_1, g(v_1))$ if

$$v_1 = F^{-1}\left(\frac{F((v_1 + g(v_1))/2)}{2}\right). \quad (\text{B.5})$$

(B.5) has at least one solution $v_1^* \in (0, 1/2)$. This holds because both sides of (B.5) are strictly increasing, and the r.h.s. is positive for $v_1 = 0$ and strictly less than $1/2$ for $v_1 = 1/2$. Now consider $(v_1^*, v_2^*) = (v_1^*, g(v_1^*))$ such that $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Thus $F(v_2^*) = F(v_1^*) + 1/2$. Moreover, symmetry implies $v_1^* + v_2^* = 1$ and thus the r.h.s. of (B.5) is $F^{-1}\left(\frac{1}{4}\right)$, so that the first-order condition holds for (v_1^*, v_2^*) .

Finally, consider the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 S_T}{\partial v_1^2} & \frac{\partial^2 S_T}{\partial v_1 \partial v_2} \\ \frac{\partial^2 S_T}{\partial v_1 \partial v_2} & \frac{\partial^2 S_T}{\partial v_2^2} \end{bmatrix} = \begin{bmatrix} -2f(v_1) + \frac{1}{2}f((v_1 + v_2)/2) & \frac{1}{2}f((v_1 + v_2)/2) \\ \frac{1}{2}f((v_1 + v_2)/2) & -2f(v_2) + \frac{1}{2}f((v_1 + v_2)/2) \end{bmatrix}.$$

First, H is negative definite at (v_1^*, v_2^*) if and only if $f(1/2) < 2f(v_1^*)$. To see this, note that $f(v_1^*) = f(v_2^*)$ and $f((v_1^* + v_2^*)/2) = f(1/2)$. Hence,

$$-2f(v_1^*) + \frac{1}{2}f((v_1^* + v_2^*)/2) = -2f(v_1^*) + \frac{1}{2}f(1/2) < 0 \Leftrightarrow f(1/2) < 4f(v_1^*).$$

In addition,

$$|H| = 4f(v_1^*)f(v_2^*) - (f(v_1^*) + f(v_2^*))f((v_1^* + v_2^*)/2) = 4f(v_1^*)^2 - 2f(v_1^*)f(1/2).$$

This condition holds if and only if $f(1/2) < 2f(v_1^*)$, which holds by (A1)(iv).

Second, H is negative definite $\forall (v_1, v_2)$ if $f(1/2) < 2f(0)$. To see this, note that $f(v)$ is minimized at $v = 0$ and maximized at $v = 1/2$. Hence, $f(1/2) < 2f(0) < 4f(0)$ implies

$$-2f(v_i) + \frac{1}{2}f\left(\frac{v_1 + v_2}{2}\right) \leq -2f(0) + \frac{1}{2}f\left(\frac{1}{2}\right) < 0 \quad \forall i \in \{1, 2\}.$$

and

$$|H| = f(v_1) \left(2f(v_2) - f\left(\frac{v_1 + v_2}{2}\right) \right) + f(v_2) \left(2f(v_1) - f\left(\frac{v_1 + v_2}{2}\right) \right) > 0.$$

Therefore, $f(1/2) < 2f(0)$, which holds by (A1)(iv), is a sufficient condition for (v_1^*, v_2^*) to be the unique global optimum.

B.2.2 Proof of Lemma 2.2

Step 1: Pricing subgame for $q_1 = q_2$.

Consider the equilibrium for the subgame defined by (v_1, v_2, σ) and the resulting quality vector (q_1, q_2) . If $q_1 = q_2$, the standard Bertrand logic implies that $(\bar{p}(\sigma), \bar{p}(\sigma)) = (\underline{P}, \underline{P})$ is the unique equilibrium.

Step 2: Pricing subgame for $q_i > q_j$

Clearly, if $q_i > q_j$, the suggested strategy profile is a subgame equilibrium. To see that i must bid $\bar{p}(\sigma)$ in equilibrium, first suppose $p_i > \bar{p}(\sigma)$. If $p_i > p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier j wins. By setting $p_i = \bar{p}(\sigma) \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i can ensure that he wins, which is a profitable deviation by (T2). If $p_i > \bar{p}(\sigma)$ and $p_i \leq p_j + q(v_i, \sigma) - q(v_j, \sigma)$, supplier i wins. By setting $p_j = \underline{P}$, supplier j can profitably deviate. If $p_i < \bar{p}(\sigma)$, supplier i can deviate upwards to $\bar{p}(\sigma)$. He then still wins by (T1), and revenues are higher.

B.2.3 Proof of Lemma 2.3

(i) The result is trivial for $v_1 = v_2$. For $v_1 < v_2$, we show that supplier 1 can profitably deviate to some $v'_1 > v_1$ if $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$. This immediately follows from the following two steps:

Step 1: If $v_1 < v_2$ and $\Delta q(v_1, v_2) + \underline{P} \notin \mathcal{P}$, there exists a deviation to $v'_1 \in (v_1, v_2]$ such that the set of states in which supplier 1 wins after the deviation is a strict superset of the set of states in which the supplier wins before the deviation.

Before the deviation, by Lemma 2.2, if $\sigma \in [0, v_1]$, supplier 1 wins and $\bar{p}(\sigma) < \Delta q(v_1, v_2) + \underline{P}$. By continuity, $\exists v'_1 \in (v_1, v_2]$ such that $\bar{p}(\sigma) < \Delta q(v'_1, v_2) + \underline{P} < \Delta q(v_1, v_2) + \underline{P}$. By deviating to v'_1 , supplier 1 wins whenever $\sigma < (v'_1 + v_2)/2$ rather than when $\sigma < (v_1 + v_2)/2$. Step 1 thus follows.

Step 2: After this deviation, the buyer pays a weakly higher price than before.

For $\sigma \in [0, v_1]$, the price is unaffected. For $\sigma \in (v_1, (v'_1 + v_2)/2]$, the price is at least as high as before the deviation. Thus, v'_1 is a profitable deviation by (T2).

(ii) follows directly from Lemmas 2.2 and 2.3 (i).

B.3 Proofs of Main Optimality Results (Section 2.3.2)

B.3.1 Proof of Proposition 2.1

Let $A = \Delta q(v_1, v_2)$ for some (v_1, v_2) . We will show that, in the bonus tournament with $P = \{A, 0\}$ and sufficiently high subsidies, the strategy profiles $(v_1, v_2, p_1(\cdot), p_2(\cdot))$ such that $p_i(q_i, q_j) = A$ if $q_i - q_j \geq A$ and 0 otherwise, form an equilibrium.

Sequential rationality of $p_i(\cdot)$ follows from Lemma 2.2. We now show that $(v_1, p_1(\cdot))$ is a best response of supplier 1 to $(v_2, p_2(\cdot))$; the argument for supplier 2 is analogous. For $A = 0$, only $(v_1, v_2) = (1/2, 1/2)$ satisfies the above conditions. Thus, the statement for $A = 0$ will follow from Proposition 2.2(ii). If $v_1 < v_2$, $\Delta q(v_1, v_2) > 0$, and the probability that supplier 1 wins with a positive prize is $F(v_1)$. Deviating to $v'_1 < v_1$ is not profitable, because the winning probability falls to $F(\hat{v}_1)$, with $\hat{v}_1 < v_1$ implicitly defined by $q(v'_1, \hat{v}_1) - q(v_2, \hat{v}_1) = \Delta q(v_1, v_2)$, and the prize does not rise. It is not profitable to deviate to $v''_1 \in (v_1, \tilde{v})$, where $\tilde{v} = \min(2v_2 - v_1, 1) \geq 1/2$: For such deviations, $\Delta q(v''_1, v_2) < \Delta q(\tilde{v}, v_2) \leq \Delta q(v_1, v_2) \forall \sigma$, so that the probability of winning a positive prize is 0. Finally, if $\tilde{v} < 1$, deviating to $v'''_1 \in [\tilde{v}, 1]$ is not profitable, because $\tilde{v} \geq 1/2 + v_2 - v_1$ implies $1 - \tilde{v} \leq 1/2 - (v_2 - v_1) \leq v_2 - (v_2 - v_1) = v_1$ and therefore, by symmetry of the state distribution, $1 - F(v'''_1) \leq 1 - F(\tilde{v}) \leq F(v_1)$. By analogous arguments, there are no profitable deviations for supplier 2.

By Lemma 2.1, the social optimal satisfies $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$. Clearly, it must be that $0 < v_1^* \leq 1/2 \leq v_2^* < 1$, and the social optimum can be implemented.

B.3.2 Proof of Theorem 2.1

The buyer optimally chooses $(v_1, v_2, p_1, p_2, \mathcal{P}, t) \in [0, 1]^2 \times \mathcal{P}^{[\Psi-b, \Psi]^2} \times \mathcal{I}(\mathbb{R}^+) \times [0, +\infty)$ so as to maximize

$$S_T(v_1, v_2) - \hat{\Pi}_1(v_1, v_2, p_1, p_2) - \hat{\Pi}_2(v_1, v_2, p_1, p_2) - 2t$$

such that, for all $i \in \{1, 2\}$ and $j \neq i$, (DC1)-(DC3) hold and

$$\hat{\Pi}_i(v_i, v_j, p_i, p_j) + t - C \geq 0 \text{ for all } i, j \in \{1, 2\} \text{ and } i \neq j. \quad (\text{B.6})$$

(i) The statement follows from two lemmas. Lemma B.1 shows that allocations maximizing buyer surplus satisfy the conditions of Proposition 2.1 and can thus be implemented by a bonus tournament. Lemma B.2 shows that implementation requires lower expected transfer than any alternative; hence buyer surplus is maximal.

Lemma B.1. *If (v_1^B, v_2^B, p_1, p_2) is an equilibrium of a contest that maximizes buyer surplus, then $0 < v_1^B \leq \frac{1}{2} \leq v_2^B < 1$.*

We prove this lemma in two steps.

Step 1: *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium where w.l.o.g. $v_1^B \leq v_2^B$, then $v_1^B \leq 1/2 \leq v_2^B$.*

Proof: We will show that $v_1 \leq 1/2 \leq v_2$ must hold in any contest equilibrium. Suppose, to the contrary, that $v_1 \leq v_2 < 1/2$. The case that $1/2 < v_1 \leq v_2$ follows analogously. Let p_1, p_2 be the associated pricing strategies. Then, the expected revenue of supplier 1 is $\Pi_1(v_1, v_2) = \int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma)$. Consider the deviation $v'_1 = 2v_2 - v_1 < 1$ with the same pricing function. Supplier 1 now wins whenever $\sigma > (v_2 + v'_1)/2$. We can write the expected revenue as $\Pi_1(v'_1, v_2) = \int_{\frac{v'_1+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma) + \int_{2v_2}^1 p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma)$.

Clearly, $(v_1 + v_2)/2 = 2v_2 - \frac{v'_1+v_2}{2}$. Moreover, there exists a bijective mapping $[0, (v_1 + v_2)/2] \rightarrow [(v'_1 + v_2)/2, 2v_2]$; $\sigma' \mapsto \sigma''$ such that $q(v_1, \sigma') - q(v_2, \sigma') = q(v'_1, \sigma'') - q(v_2, \sigma'')$ and $f(\sigma') \leq f(\sigma'')$. Thus $\int_0^{\frac{v_1+v_2}{2}} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma) \leq \int_{\frac{v'_1+v_2}{2}}^{2v_2} p_1(q_1(\sigma), q_2(\sigma)) dF(\sigma)$. As a result, $\Pi_1(v_1, v_2) \leq \Pi_1(v'_1, v_2)$ and v'_1 leads to strictly higher probability of winning, hence v'_1 is a profitable deviation.³ Thus, $v_1 \leq 1/2 \leq v_2$ must hold in any equilibrium; in particular, therefore $v_1^B \leq 1/2 \leq v_2^B$.

Step 2: *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium maximizing buyer surplus, then $0 < v_i^B < 1$ for $i \in \{1, 2\}$.*

Proof: By Step 1, we know that $v_1 \leq 1/2 \leq v_2$. Suppose $v_1^B = 0$ and $v_2^B = 1$. We will distinguish two cases, $C = 0$ and $C > 0$. First suppose $C = 0$. By single-peakedness (A1), $v_1 = v_2 = 1/2$ results in weakly higher total surplus than (v_1^B, v_2^B) . As the allocation $(v_1, v_2) = (1/2, 1/2)$ can be implemented with an FPT and $A = 2C$ by Proposition 2.2(ii), the buyer would be strictly better off than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$ where the suppliers earn positive surplus. Finally, observe that $v_1^B = 0$ and $v_2^B = 1$ cannot be implemented so that the suppliers earn zero surplus, as the suppliers could increase their probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. Next suppose $C > 0$. There exists some small ε such that $S_T(v_1^B = 0, v_2^B = 1) < S_T(\varepsilon, 1 - \varepsilon)$ and $F(\varepsilon)\Delta q(\varepsilon, 1 - \varepsilon) < C$. But then a bonus tournament with subsidy $t' = C - F(\varepsilon)\Delta q(\varepsilon, 1 - \varepsilon)$, and $\mathcal{P} = \{\Delta q(\varepsilon, 1 - \varepsilon), 0\}$ implements $(\varepsilon, 1 - \varepsilon)$, achieves higher total surplus, and the supplier surplus not higher than in any contest implementing $v_1^B = 0$ and $v_2^B = 1$. Hence, the buyer surplus is higher, which is a contradiction.

³Given the tie-breaking rule T2, this is even true for $p = 0$.

Next suppose $v_1 = 0$ and $v_2 < 1$ (the case that $v_1 > 0$ and $v_2 = 1$ follows analogously). By Lemma 2.2, the revenue is $\Pi_1(0, v_2) = \int_0^{\frac{v_2}{2}} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma)$ for supplier 1 and $\Pi_2(v_2, 0) = \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma)) dF(\sigma) + \int_{v_2}^1 \bar{p}(q_2(\sigma), q_1(\sigma)) dF(\sigma)$ for supplier 2. Moreover, $\Pi_1(0, v_2) > 0$, because otherwise supplier 1 could increase his probability of winning by deviating to the interior, which by (T2) would be a profitable deviation. By single-peakedness (A1) it holds $\int_0^{\frac{v_2}{2}} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma) \leq \int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma)) dF(\sigma)$. Suppose that this equilibrium is implemented with transfers t such that $t + \Pi_1(0, v_2) \geq C$. This implies $t + \Pi_2(v_2, 0) > C$. Further, using (B.1), $dS_T(v_1^B, v_2^B) / dv_1^B|_{v_1^B=0} = bF(v_2/2) > 0$, so that there exists some $\bar{\varepsilon} > 0$ such that $S_T(\varepsilon, v_2^B) > S_T(0, v_2^B)$ for every $\varepsilon \in (0, \bar{\varepsilon})$. Fix ε such that $F(\varepsilon)\Delta q(\varepsilon, v_2) \leq \Pi_1(0, v_2)$ and $F(\varepsilon) < 1 - F(v_2)$. Let $t' = t + \Pi_1(0, v_2) - F(\varepsilon)\Delta q(\varepsilon, v_2)$. Now consider a bonus tournament with subsidy t' and $\mathcal{P} = \{\Delta q(\varepsilon, v_2), 0\}$. By Proposition 2.1, this bonus tournament will implement (ε, v_2) if the participation constraint is met. This condition holds for both suppliers, because $t' + (1 - F(v_2))\Delta q(\varepsilon, v_2) > t' + F(\varepsilon)\Delta q(\varepsilon, v_2) \geq C$. Compared to the original situation with $v_1 = 0$ and $v_2 < 1$, the rent of supplier 1 is unchanged, but the rent of supplier 2 decreases since $\int_{\frac{v_2}{2}}^{v_2} \bar{p}(q_2(\sigma), q_1(\sigma)) dF(\sigma) + t > t'$ and $\int_{v_2}^1 \bar{p}(q_2(\sigma), q_1(\sigma)) dF(\sigma) > (1 - F(v_2))\Delta q(\varepsilon, v_2)$. Since the total surplus increases and the suppliers' surplus decreases, the buyer's surplus must increase. Therefore, the bonus tournament that implements (ε, v_2) increases the buyer surplus, which is a contradiction.

Lemma B.2. *If $(v_1^B, v_2^B, p_1^B, p_2^B)$ is an equilibrium of a contest maximizing buyer surplus, then it can be implemented by a contest with $\mathcal{P} = \{A, 0\}$.*

Proof: From Proposition 2.1 and Lemma B.1, we know that the bonus tournament with $A = \Delta q(v_1^B, v_2^B)$ implements (v_1^B, v_2^B) . It remains to be shown that the buyer cannot implement (v_1^B, v_2^B) with lower expected total transfers with any other contest. First, suppose that $v_1^B + v_2^B = 1$. By Lemmas 2.2 and 2.3, in any contest that implements (v_1^B, v_2^B) the price paid by the buyer is exactly $\Delta q(v_1^B, v_2^B) + \underline{P}$ if $\sigma \in [0, v_1^B] \cup [v_2^B, 1]$ and it is at least 0 if $\sigma \in (v_1^B, v_2^B)$. Thus, if $\Delta q(v_1^B, v_2^B)F(v_1^B) > C$, a bonus tournament implements (v_1^B, v_2^B) with the lowest possible expected total transfers. If $\Delta q(v_1^B, v_2^B)F(v_1^B) \leq C$, a bonus tournament with an appropriate t implements (v_1^B, v_2^B) with zero expected supplier surplus. Next, consider an arbitrary contest implementing (v_1^B, v_2^B) with $v_1^B + v_2^B < 1$ with subsidy t (the case $v_1^B + v_2^B > 1$ is analogous).

The surplus of supplier 1 is then $S_1 = \Delta q(v_1^B, v_2^B)F(v_1^B) + \int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma) + t - C$, and for supplier 2 it is $S_2 = \Delta q(v_1^B, v_2^B)(1 - F(v_2^B)) + \int_{\frac{v_1^B+v_2^B}{2}}^{v_2^B} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma) + t - C$. By similar arguments as in Lemma B.1, $\Delta q(v_1^B, v_2^B)F(v_1^B) < \Delta q(v_1^B, v_2^B)(1 - F(v_2^B))$ and $\int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma) \leq \int_{\frac{v_1^B+v_2^B}{2}}^{v_2^B} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma)$. Now consider a bonus

tournament with $\mathcal{P} = \{\Delta q(v_1^B, v_2^B), 0\}$ and $t' = \int_{v_1^B}^{\frac{v_1^B+v_2^B}{2}} \bar{p}(q_1(\sigma), q_2(\sigma)) dF(\sigma) + t$. The surplus of supplier 1 now becomes $S'_1 = S_1$ by construction. On the other hand, $S'_2 \leq S_2$, but $S'_2 > S'_1$. Thus, the proposed bonus tournament implements (v_1^B, v_2^B) with lowest possible net supplier surplus, which implies that the buyer surplus is maximized.

(ii) Suppose $C \geq F(v_1^*)\Delta q(v_1^*, v_2^*)$. From Proposition 2.1 we know that for the proposed

$\mathcal{P} = \{A, 0\}$, (v_1^*, v_2^*) emerges in equilibrium; and the result also gives the pricing strategies p_1 and p_2 . For $t = C - F(v_1^*) \Delta q(v_1^*, v_2^*)$, the buyer surplus in the proposed equilibrium is

$$\begin{aligned} & S_T(v_1^*, v_2^*) - \Pi_1(v_1^*, v_2^*) - \Pi_2(v_1^*, v_2^*) + 2t \\ &= S_T(v_1^*, v_2^*) - 2F(v_1^*) \Delta q(v_1^*, v_2^*) + 2(F(v_1^*) \Delta q(v_1^*, v_2^*) - C) = S_T(v_1^*, v_2^*) - 2C \end{aligned} \quad (\text{B.7})$$

This is the highest surplus that the buyer can achieve without violating the suppliers' participation constraints.

(iii) Suppose $C < F(v_1^*) \Delta q(v_1^*, v_2^*)$. The proof for this case relies on the fact that implementation with minimal revenues uses bonus tournaments. It shows that (v_1^B, v_2^B) must satisfy $v_1^B + v_2^B = 1$. Among all the bonus tournaments implementing (v_1, v_2) with $v_1 \leq v_2$ and $v_1 + v_2 = 1$, the buyer has highest surplus (ignoring participation constraint) at $(1/2, 1/2)$. Using these facts, the proof shows that the buyer always chooses the minimal value of the subsidy t , and she just implements enough diversity so that the suppliers (who benefit from some diversity) break even on expectation.

Step 1: *The outcome of an optimal contest can be implemented by $\mathcal{P} = \{A, 0\}$ for some $A \geq 0$.*

This follows from Part (i).

Step 2: *In an optimal contest $v_1^B + v_2^B = 1$.*

Consider any (v_1, v_2) such that $v_1 + v_2 < 1$. We show that $(v_1, v_2) \neq (v_1^B, v_2^B)$; the case $v_1 + v_2 > 1$ follows analogously. By Step 1, the optimal outcome can be implemented by some $\mathcal{P} = \{A, 0\}$ and $t \geq 0$. The equilibrium values of p_i in this contest are zero whenever $\sigma \in (v_1, v_2)$. Hence, the participation constraint for supplier 1 implies that $F(v_1)A + t \geq C$; thus $v_1 + v_2 < 1$ implies $(1 - F(v_2))A + t > C$. Now suppose the buyer implements $(v_1 + \varepsilon, v_2 + \varepsilon)$, where ε is sufficiently small. We know that $(v_1 + \varepsilon, v_2 + \varepsilon)$ can also be implemented with $\mathcal{P} = \{A, 0\}$. Thus, we can write the buyer surplus as

$$S_B(\varepsilon) = S_T(v_1 + \varepsilon, v_2 + \varepsilon) - F(v_1 + \varepsilon)A - (1 - F(v_2 + \varepsilon))A - 2t$$

for $\varepsilon \geq 0$. Thus

$$\frac{dS_B(\varepsilon)}{d\varepsilon} = dS_T(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon - Af(v_1 + \varepsilon) + Af(v_2 + \varepsilon).$$

Since $v_1 + v_2 < 1$, single-peakedness and symmetry (A1) imply $f(v_1 + \varepsilon) < f(v_2 + \varepsilon)$. Thus $dS_B(\varepsilon)/d\varepsilon > dS_T(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon$. We will show that $dS_T(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon > 0$; because $F(v_1 + \varepsilon)A + t > C$ and (for sufficiently small ε) $(1 - F(v_2))A + t \geq C$, the buyer will thus be better off implementing $(v_1 + \varepsilon, v_2 + \varepsilon)$ than (v_1, v_2) . Maximizing total surplus is equivalent to minimizing the expected distance

$$\begin{aligned} D(v_1 + \varepsilon, v_2 + \varepsilon) &= \int_0^{v_1 + \varepsilon} (v_1 + \varepsilon - \sigma) f(\sigma) d\sigma + \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2} + \varepsilon} (\sigma - v_1 - \varepsilon) f(\sigma) d\sigma \\ &\quad + \int_{\frac{v_1 + v_2}{2} + \varepsilon}^{v_2 + \varepsilon} (v_2 + \varepsilon - \sigma) f(\sigma) d\sigma + \int_{v_2 + \varepsilon}^1 (\sigma - v_2 - \varepsilon) f(\sigma) d\sigma. \end{aligned}$$

From this we obtain

$$\begin{aligned} \frac{dD(v_1 + \varepsilon, v_2 + \varepsilon)}{d\varepsilon} &= \int_0^{v_1 + \varepsilon} f(\sigma) d\sigma - \int_{v_1 + \varepsilon}^{\frac{v_1 + v_2}{2} + \varepsilon} f(\sigma) d\sigma + \int_{\frac{v_1 + v_2}{2} + \varepsilon}^{v_2 + \varepsilon} f(\sigma) d\sigma - \int_{v_2 + \varepsilon}^1 f(\sigma) d\sigma \\ &= 2F(v_1 + \varepsilon) + 2(F(v_2 + \varepsilon)) - 2F\left(\frac{v_1 + v_2}{2} + \varepsilon\right) - 1. \end{aligned}$$

We will show that this expression is negative for $v_1 + v_2 < 1$ and sufficiently small ε . To see this, fix any v_2 such that $1/2 \leq v_2 < 1$. Note that $h(v_1, v_2) := \frac{dD(v_1 + \varepsilon, v_2 + \varepsilon)}{d\varepsilon} \Big|_{\varepsilon=0} = 0$ for $v_1 = 1 - v_2$. Furthermore

$$\frac{\partial h}{\partial v_1} = 2f(v_1) - f\left(\frac{v_1 + v_2}{2}\right) > 0,$$

where the last inequality follows by (A1)(iv). Thus, $v_1 + v_2 < 1$ implies $2F(v_1) + 2(F(v_2)) - 2F((v_1 + v_2)/2) - 1 < 0$ and thus $dD(v_1 + \varepsilon, v_2 + \varepsilon)/d\varepsilon < 0$ for small enough ε . This in turn implies that $S_T(v_1 + \varepsilon, v_2 + \varepsilon)$ increases in ε so that buyer surplus also increases in ε .

Step 3: For $v_1 \in [0, 1/2]$ and for fixed t , buyer surplus $S_B^{bt}(v_1, 1 - v_1)$ increases in v_1 .

For any $\sigma \in [0, v_1]$, buyer surplus equals $q_2(1 - v_1, \sigma)$, which increases in v_1 . For any $\sigma \in (v_1, 1/2]$, buyer surplus equals $q_1(v_1, \sigma)$; for a small marginal change, the expected payoff from states on $(v_1, 1/2]$ thus also increases in v_1 . By similar arguments, buyer surplus increases for any $\sigma \in (1/2, 1 - v_1]$ and any $\sigma \in (1 - v_1, 1]$.

Step 4: Suppose $C < F(v_1^*) \Delta q(v_1^*, v_2^*)$. Then, there exists $v_1 \in (v_1^*, 1/2]$ such that: $C = F(v_1) \Delta q(v_1, 1 - v_1)$.

The result follows from $F(1/2) \Delta q(1/2, 1/2) = 0$ because $F(v_1) \Delta q(v_1, 1 - v_1)$ is a continuous function.

Step 5: Fix $\tilde{v}_1 = \max_{[0, 1/2]} v_1$ s.t. $C = F(v_1) \Delta q(v_1, 1 - v_1)$. Then $v_1^B \geq \tilde{v}_1$.

Suppose not. According to Step 1, (v_1^B, v_2^B) can be implemented in a bonus tournament with a subsidy $t \geq 0$. By Step 4, \tilde{v}_1 exists. Suppose $v_1^B < \tilde{v}_1$. By Step 3, moving from (v_1^B, v_2^B) to $(\tilde{v}_1, 1 - \tilde{v}_1)$ the buyer could increase her payoff while the participation constraint would remain satisfied.

Step 6: For $v_2 = 1 - v_1$ and $v_1 > v_1^*$, $S_T(v_1, 1 - v_1)$ decreases in v_1 .

Using the same argument as in Step 2, the derivative of the expected distance is

$$\frac{dD(v_1, 1 - v_1)}{dv_1} = \int_0^{v_1} f(\sigma) d\sigma - \int_{v_1}^{\frac{1}{2}} f(\sigma) d\sigma - \int_{\frac{1}{2}}^{1 - v_1} f(\sigma) d\sigma + \int_{1 - v_1}^1 f(\sigma) d\sigma = 4F(v_1) - 1$$

This function is monotonic and positive for all $v_1 > v_1^*$. Hence the total expected distance increases in v_1 , and the total expected surplus decreases.

Step 7: Fix \tilde{v}_1 as in Step 5. Then $v_1^B \leq \tilde{v}_1$.

Suppose not. By Step 1, (v_1^B, v_2^B) can be implemented in a bonus tournament with a subsidy $t \geq 0$. Suppose $v_1^B > \tilde{v}_1$. By Step 6, moving from (v_1^B, v_2^B) to $(\tilde{v}_1, 1 - \tilde{v}_1)$ increases total surplus. Since $(\tilde{v}_1, 1 - \tilde{v}_1)$ can be implemented with $t' = 0$ because $C = F(\tilde{v}_1) \Delta q(\tilde{v}_1, 1 - \tilde{v}_1)$, buyer surplus increases.

Step 8: The optimal (v_1^B, v_2^B) for the buyer (i) satisfies $v_1^B = \tilde{v}_1$ and $v_2^B = 1 - v_1^B$. It can be implemented with (ii) a bonus tournament such that $t = 0$.

The first part of the statement follows by combining Steps 5 and 7. The second part follows

from (i) and the fact that the suggested contest implements $(\tilde{v}_1, 1 - \tilde{v}_1)$ with minimal subsidies required to satisfy the participation constraint.

B.3.3 Proof of Corollary 2.1

Proof. As $F(v_1^*) \Delta q(v_i^*, v_j^*) < C$ by assumption, both prizes are positive. For firm 1, the expected profit of following the candidate equilibrium is $\Pi_1(v_1^*, v_2^*) = F(v_1^*) A + (1/2 - F(v_1^*)) a - C$. Inserting the values of A and a and $F(v_1^*) = 1/4$, $\Pi_1(v_1^*, v_2^*) = 0$. By symmetry, both suppliers break even on expectation. Thus, the suggested allocation maximizes total surplus, with full rent appropriation by the buyer. It thus suffices to show that $\{A, a\}$ implements (v_1^*, v_2^*) . Consider supplier 1. First, any deviation $v_1 = v_2^* + \varepsilon$ is dominated by $v_1' = v_2^* - \varepsilon$. Next, a deviation to $v_1' < v_1^*$ cannot increase expected supplier profit, as the probability of winning decreases and the price charged in any state of the world does not increase. Thus, the only remaining case is a deviation to $v_1' \in (v_1^*, v_2^*]$. The expected gross profit can be written as $\Pi_1(v_1', v_2^*) = aF((v_1' + v_2^*)/2)$. This is clearly increasing in v_1' and the profit of supplier 1 is at most $\Pi_1(v_1', v_2^*) = aF(v_2^*)$. The expected profit of following the candidate equilibrium is $\Pi_1(v_1^*, v_2^*) = F(v_1^*) A + (1/2 - F(v_1^*)) a$. Thus there is no profitable deviation to values just below v_2^* if and only if $F(v_1^*) A + (1/2 - F(v_1^*)) a \geq aF(v_2^*)$. Inserting the values of A and a and $F(v_1^*) = 1/4$ and $F(v_2^*) = 3/4$ shows that (v_1^*, v_2^*) is an equilibrium. \square

B.4 Proofs on Auctions and Tournaments (Section 2.4)

B.4.1 Proof of Proposition 2.2

(i) By Lemma 2.2, the unique equilibrium of the pricing subgame induced by q_1 and q_2 is $p_i = \max\{q_i - q_j, 0\}$ for $i, j \in \{1, 2\}; j \neq i$. Suppose that an auction does not implement the social optimum (v_1^*, v_2^*) . Then, for some i , there exists $\bar{v}_i \neq v_i^*$ such that $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$. Let $\Theta_i(v_i, v_j) = \{\sigma \in [0, 1] \mid q(v_i, \sigma) \geq q(v_j, \sigma)\}$ and $\Theta_{-i}(v_i, v_j) = [0, 1] \setminus \Theta_i(v_i, v_j)$. Thus $\Pi_i(\bar{v}_i, v_j^*) > \Pi_i(v_i^*, v_j^*)$ if and only if

$$\int_{\Theta_i(\bar{v}_i, v_j^*)} (q(\bar{v}_i, \sigma) - q(v_j^*, \sigma)) dF(\sigma) > \int_{\Theta_i(v_i^*, v_j^*)} (q(v_i^*, \sigma) - q(v_j^*, \sigma)) dF(\sigma),$$

or equivalently

$$\begin{aligned} & \int_{\Theta_i(\bar{v}_i, v_j^*)} (q(\bar{v}_i, \sigma) - q(v_j^*, \sigma)) dF(\sigma) + \int_0^1 q(v_j^*, \sigma) dF(\sigma) > \\ & \int_{\Theta_i(v_i^*, v_j^*)} (q(v_i^*, \sigma) - q(v_j^*, \sigma)) dF(\sigma) + \int_0^1 q(v_j^*, \sigma) dF(\sigma) \end{aligned}$$

Splitting $[0, 1]$ into $\Theta_i(\bar{v}_i, v_j^*)$ and $\Theta_{-i}(\bar{v}_i, v_j^*)$ in the first line and into $\Theta_i(v_i^*, v_j^*)$ and $\Theta_{-i}(v_i^*, v_j^*)$ in the second line and simplifying, this is equivalent with

$$\int_{\Theta_i(\bar{v}_i, v_j^*)} q(\bar{v}_i, \sigma) dF(\sigma) + \int_{\Theta_{-i}(\bar{v}_i, v_j^*)} q(v_j^*, \sigma) dF(\sigma) >$$

$$\int_{\Theta_i(v_i^*, v_j^*)} q(v_i^*, \sigma) dF(\sigma) + \int_{\Theta_{-i}(v_i^*, v_j^*)} q(v_j^*, \sigma) dF(\sigma).$$

and thus

$$\int_0^1 \max\{q(\bar{v}_i, \sigma), q(v_j^*, \sigma)\} dF(\sigma) > \int_0^1 \max\{q(v_i^*, \sigma), q(v_j^*, \sigma)\} dF(\sigma),$$

contradicting optimality of (v_1^*, v_2^*) .

(ii) This follows from the more general statement in Corollary 2.5(ii) below.

(iii) Using Proposition 2.2(ii), any FPT such that the supplier breaks even has a unique equilibrium with $(v_1, v_2) = (1/2, 1/2)$. For $A = 2C$ and $t = 0$, the participation constraint of the suppliers binds. Hence, buyer surplus is maximized in the class of FPTs. It is

$$\begin{aligned} S_B^{fpt} &= \int_0^{1/2} \left(\Psi - b \left(\frac{1}{2} - \sigma \right) \right) f(\sigma) d\sigma + \int_{1/2}^1 \left(\Psi - b \left(\sigma - \frac{1}{2} \right) \right) f(\sigma) d\sigma - 2C \\ &= \Psi + \int_0^{1/2} b\sigma f(\sigma) d\sigma - \int_{1/2}^1 b\sigma f(\sigma) d\sigma - 2C \end{aligned}$$

The surplus of supplier 1 (supplier 2 follows by symmetry) is

$$\begin{aligned} S_1^a &= F(v_1^*) \Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{1/2} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma \\ &= \frac{b(v_2^* - v_1^*)}{4} + \int_{v_1^*}^{1/2} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma. \end{aligned}$$

Thus whenever $C < b(v_2^* - v_1^*)/4$, the participation constraint of the suppliers is satisfied even with $t = 0$. By Lemma 2.2, in an auction the winning supplier bids exactly the quality difference. This implies that the value the buyer receives, in any state of the world, is equal to the quality of the losing supplier. Then, the buyer surplus in an auction with $t = 0$ is

$$\begin{aligned} S_B^a &= \int_0^{1/2} (\Psi - b(v_2^* - \sigma)) f(\sigma) d\sigma + \int_{1/2}^1 (\Psi - b(\sigma - v_1^*)) f(\sigma) d\sigma \\ &= \Psi + \int_0^{1/2} b\sigma f(\sigma) d\sigma - \int_{1/2}^1 b\sigma f(\sigma) d\sigma - \frac{bv_2^*}{2} + \frac{bv_1^*}{2} \end{aligned}$$

The buyer prefers FPT to the auction if $S_B^{fpt} - S_B^a > 0$, which holds whenever $\frac{bv_2^*}{2} - \frac{bv_1^*}{2} - 2C > 0$ or equivalently $\frac{b(v_2^* - v_1^*)}{4} > C$.

When $\frac{b(v_2^* - v_1^*)}{4} < C$, the participation constraints require positive subsidies. In this case, the buyer implements the social optimum by using an auction with $t = C - \Pi_1^a$ with zero supplier surplus. Obviously this outperforms the inefficient FPT.

B.4.2 Proof of Corollary 2.3

Denote the minimum allowable price with \underline{P} . If $v_1 \neq v_2$ in equilibrium, by Proposition 2.2(ii), the contest is not an FPT. Suppose that $v_1 < v_2$. By Lemmas 2.2 and 2.3, the buyer pays $q_i - q_j + \underline{P}$ to the supplier with $q_i \geq q_j$ in equilibrium. Thus, for any σ , the buyer surplus is

$\min\{q_1, q_2\} - \underline{P}$. Hence, the surplus of a buyer who induces $v_1 < v_2$ with \underline{P} is

$$\begin{aligned} S_B(v_1, v_2; \underline{P}) &= \int_0^1 \min\{q_i(v_i, \sigma), q_j(v_j, \sigma)\} dF(\sigma) - \underline{P} \\ &= \int_0^{\frac{v_1+v_2}{2}} q_2(v_2, \sigma) dF(\sigma) + \int_{\frac{v_1+v_2}{2}}^1 q_1(v_1, \sigma) dF(\sigma) - \underline{P} \end{aligned}$$

Thus

$$\frac{dS_B}{dv_1} = \int_{\frac{v_1+v_2}{2}}^1 \frac{\partial q_1}{\partial v_1} dF(\sigma) > 0; \quad \frac{dS_B}{dv_2} = \int_0^{\frac{v_1+v_2}{2}} \frac{\partial q_2}{\partial v_2} dF(\sigma) < 0.$$

Thus, the buyer surplus is maximal for $v_1 = v_2$ and $\underline{P} = 0$. Given $v_1 = v_2$, the buyer surplus is maximal for $v_1 = v_2 = 1/2$, the unique equilibrium of an FPT with A arbitrarily close to 0. Given (T2), it is an equilibrium for $A = 0$.

B.5 Extensions: $n > 3$ (Section 2.5.1)

B.5.1 Proof of Lemma 2.4

(i) Arguing as for two suppliers, $v_i^* \neq v_j^*$ for all $i \neq j \in \{1, \dots, n\}$. Thus

$$S_T(\mathbf{v}) = \int_0^{\frac{v_1+v_2}{2}} q_1(v_1, \sigma) d\sigma + \sum_{k=2}^{n-1} \int_{\frac{v_{k-1}+v_k}{2}}^{\frac{v_k+v_{k+1}}{2}} q_k(v_k, \sigma) d\sigma + \int_{\frac{v_{n-1}+v_n}{2}}^1 q_n(v_n, \sigma) d\sigma$$

The maximum of this function exists and it obviously does not involve corner solutions. Hence, it is given by the first order conditions

$$\frac{\partial S_T(\mathbf{v})}{\partial v_1} = -bv_1 + b\frac{v_2 - v_1}{2} = 0 \quad (\text{B.8})$$

$$\frac{\partial S_T(\mathbf{v})}{\partial v_k} = -b\frac{v_k - v_{k-1}}{2} + b\frac{v_{k+1} - v_k}{2} = 0 \quad (\text{B.9})$$

$$\text{for } k \in \{2, \dots, n-1\}$$

$$\frac{\partial S_T(\mathbf{v})}{\partial v_n} = -b\frac{v_n - v_{n-1}}{2} + b(1 - v_n) = 0 \quad (\text{B.10})$$

(B.9) can be rearranged to give $v_k - v_{k-1} = v_{k+1} - v_k \equiv \Delta^v$ for $k = 2, \dots, n-1$. (B.8) and (B.10) give $v_1 = 1 - v_n = \Delta^v/2$. Inserting these equations into $v_1 + (v_2 - v_1) + \dots + (v_n - v_{n-1}) + (1 - v_n) = 1$ gives $\Delta^v = \frac{1}{n}$. Thus, $v_1 = \frac{1}{2n}$ and $v_k = \frac{1}{2n} + \frac{k-1}{n} = \frac{2k-1}{2n}$ for $k \in \{2, \dots, n\}$.

(ii) The proof of the result on auctions is analogous to the proof of Proposition 2.2(i) above. Consider the bonus tournament. If suppliers $1, \dots, n$ choose $v_1^*, v_2^*, \dots, v_n^*$, then suppliers $2, \dots, n-1$ receive no revenues, but they break even because of the subsidy. There are no feasible deviations for which they can earn a positive price. Consider supplier 1 (supplier n is analogous): His surplus is $\frac{1}{2n} \left(\frac{b}{n}\right) + C - C = \frac{b}{2n^2}$. Deviating to $v_1 < v_1^*$ would reduce the probability of winning the prize, with no compensating benefits. Deviating to $v_n > v_n^*$ would mean that supplier 1 would only win the low prize 0. This is clearly not profitable.

(iii) Let $\mathbf{v} = [v_1, \dots, v_n]$ be the vector of approaches, ordered so that $v_1 \leq \dots \leq v_n$. In Step 1-5, we show that diversity is less than socially optimal in the FPT. In Step 6, we consider the effect

of increasing n .

Step 1: In any equilibrium of the FPT, $v_1 = v_2$ and $v_{n-1} = v_n$. This implies that there are at most $n - 2$ active approaches.

Suppose $v_1 < v_2$. Then the revenue of supplier 1 is $A \frac{v_1+v_2}{2}$. For $v'_1 = v_1 + \varepsilon$, $\varepsilon > 0$, such that $v'_1 < v_2$, the revenue is $A \frac{v'_1+v_2}{2} > A \frac{v_1+v_2}{2}$. A similar argument holds for $v_{n-1} < v_n$.

We prove the second claim (that there is an inefficiently low amount of diversity) in several steps. For any supplier i , let $P_{\sigma < v_i}^i$ ($P_{\sigma > v_i}^i$) be the probability that supplier i wins and, in addition, $\sigma < v_i$ ($\sigma > v_i$). Let $P^i = P_{\sigma < v_i}^i + P_{\sigma > v_i}^i$ be the total probability that supplier i wins.

Step 2: If for suppliers i and j there exist $k \neq i$ and $l \neq j$ such that $v_i = v_k$ and $v_j = v_l$, then $P_{\sigma < v_i}^i = P_{\sigma > v_i}^i = P_{\sigma < v_j}^j = P_{\sigma > v_j}^j$ in any equilibrium.

Suppose first that $P_{\sigma < v_i}^i \neq P_{\sigma > v_i}^i$ for some supplier i using the same approach as another one. Suppose that $P_{\sigma < v_i}^i > P_{\sigma > v_i}^i$ (the opposite case is analogous). Then, a deviation to $v_i - \varepsilon$ for some sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P_{\sigma < v_i}^i > P_{\sigma < v_i}^i + P_{\sigma > v_i}^i$,⁴ which is a profitable deviation. Next, suppose that $P_{\sigma > v_i}^i < P_{\sigma < v_j}^j$ (the opposite case is analogous). Then, a deviation of supplier i to $v_j - \varepsilon$ for sufficiently small $\varepsilon > 0$ leads to a winning probability of $2P_{\sigma < v_j}^j > P_{\sigma < v_i}^i + P_{\sigma > v_i}^i$,⁵ which is a profitable deviation.

Step 3: In any equilibrium of an FPT with n suppliers, $\bar{P} := P^1 = P^2 = P^{n-1} = P^n \geq \frac{1}{2(n-2)}$.

By Step 2, all extreme approaches are duplicate. The three equalities thus follow from Step 1. Suppose that the inequality does not hold. Then $P^1 + P^2 + P^{n-1} + P^n < \frac{2}{n-2}$ which in turn implies that $\sum_{j=3}^{n-2} P^j \geq \frac{n-4}{n-2}$. But then there exist at least one $k \in \{3, \dots, n-2\}$ such that $P^k \geq \frac{1}{n-2}$. By deviating to v_k , each supplier $1, 2, n-1$ or n would win with a probability of at least $\frac{1}{2(n-2)}$, which would be a profitable deviation.

Step 4: Any equilibrium of an FPT with n suppliers satisfies $\max_i v_i^T - \min_i v_i^T \leq \frac{n-3}{n-2}$.

Suppose not. As $\frac{2(n-2)-1}{2(n-2)} - \frac{1}{2(n-2)} = \frac{n-3}{n-2}$, there exists an equilibrium of an FPT such that either $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$ or $\min_i v_i^T < \frac{1}{2(n-2)}$ or both. If $\max_i v_i^T > \frac{2(n-2)-1}{2(n-2)}$, then Steps 1 and 2 imply $P^n < \frac{1}{2(n-2)}$, which is impossible by Step 3. If $\min_i v_i^T < \frac{1}{2(n-2)}$, then $P^1 < \frac{1}{2(n-2)}$ by Steps 1 and 2, which is again impossible by Step 3.

Step 5: The diversity in an FPT is lower than socially optimal.

By (i), the socially optimal diversity is $\frac{n-1}{n}$. By Step 4, the diversity in an FPT is at most $\frac{n-3}{n-2} < \frac{n-1}{n}$.

Step 6: The difference between the FPT and the social optimum converges to zero as the number of suppliers increases.

By Step 3, we know that each supplier $1, 2, n-1, n$ wins with probability \bar{P} . Then in any equilibrium of an FPT, there exists a supplier j such that $P^j \leq \frac{1-4\bar{P}}{n-4}$. A deviation to $v_1 - \varepsilon$ would result in a probability of winning approximately \bar{P} . Then, a necessary condition for an equilibrium is that $\bar{P} \leq \frac{1-4\bar{P}}{n-4}$, which implies that $\bar{P} \leq 1/n$ and consequently $v_1 \leq 1/n$ and $v_n \geq (n-1)/n$. Then, $\max_i v_i^T - \min_i v_i^T \geq \frac{n-2}{n}$ in any equilibrium of an FPT. By (i), the socially optimal diversity is $(n-1)/n$, so the difference between the socially optimal diversity and diversity in any equilibrium of an FPT is at most $\frac{n-1}{n} - \frac{n-2}{n} = 1/n$. Thus, the difference converges to zero as n increases.

⁴The winning probability is approximately $2P_{\sigma < v_i}^i$ if $v_i = \min\{v_1, \dots, v_n\}$.

⁵The winning probability is approximately $2P_{\sigma < v_j}^j$ if $v_j = \min\{v_1, \dots, v_n\}$.

B.5.2 Sufficient Conditions for FPT equilibria

We now provide sufficient conditions for equilibria in the FPT. These conditions hold in the equilibria described in Figure 2.3.

Lemma B.3. *An outcome with k active approaches (r_1, \dots, r_k) can be supported in an equilibrium if the following conditions both hold:*

- (a) $k \in \{\underline{k}, \dots, \bar{k}\}$, where $\bar{k} = n - 2$ and $\underline{k} = n/2$ if n is even and $\underline{k} = (n + 1)/2$ if n is odd;
- (b) $(r_1, \dots, r_k) = (1/2k, 3/2k, 5/2k, \dots, (2k - 1)/2k)$.

Two suppliers choose the extreme approaches r_1 and r_k ; each of the intermediate approaches r_2, \dots, r_{k-1} is chosen by one or two suppliers.

Proof. Step 1: Suppose n is even and $k = n/2$. Then any choice of r_1, \dots, r_k as stated in part (b) of the lemma can be supported as an equilibrium.

In the suggested equilibria, the active approaches are equidistant. Also, $r_1 = 1/n$ and $r_{n/2} = 1 - 1/n$. For any $1 < m < n/2$, $r_m - r_{m-1} = 2/n$, any of the active approaches offers the highest quality with probability $1/k = 2/n$. Now suppose each approach r_1, \dots, r_k is chosen by exactly two suppliers. Then each supplier has a revenue of $\Pi_i = A/n$. Deviating to any other active approach leads to payoff of $2A/3n$; hence it is not profitable. A deviation to $[0, r_1)$ or $(r_{n/2}, 1]$ results in a winning probability strictly lower than $1/n$, so this is not a profitable deviation either. Finally, consider a deviation to $v \in (r_{m-1}, r_m)$, $m \in \{2, \dots, n/2\}$. The deviating supplier wins if and only if σ is in the set $[\frac{v+r_{m-1}}{2}, \frac{v+r_m}{2}]$, so that the winning probability is $1/n$ and this is also not a profitable deviation.

Step 2: Now suppose n is even or odd and $k > n/2$. Then any choice of r_1, \dots, r_k as stated in part (b) of the lemma is an equilibrium.

Arguing as in Step 1, any of the active approaches offers the highest quality with probability $1/k$. Suppose two suppliers choose r_1 and r_k , respectively. Moreover, suppose that each of the approaches r_2, \dots, r_{k-1} is chosen by one or two suppliers. Thus, if there are two suppliers using an approach, each of them wins with probability $1/2k$, and if there is only one supplier using this approach, he wins with probability $1/k$. Consider a supplier who wins with probability $1/2k$. By the same argument as in Step 1, if he deviates to $[0, r_1)$ or $(r_k, 1]$, he wins with probability strictly lower than $1/2k$. Deviating to any approach in some interval (r_l, r_{l+1}) ; $l \in \{1, \dots, k - 1\}$, he wins with probability of at most $1/2k$; hence such a deviation is not profitable either. If he deviates to any active approach, he wins with a probability of at most $1/2k$. Thus, such suppliers do not have profitable deviations. Finally consider a deviation by a supplier who is the only one to choose some r_m , where $1 < m < k$. Any deviation to $[0, r_{m-1}]$ or $[r_{m+1}, 1]$ leads to strictly lower revenues, by the same argument as above. For any approach $v \in (r_{m-1}, r_{m+1})$, he wins whenever $\sigma \in [\frac{v+r_{m-1}}{2}, \frac{v+r_{m+1}}{2}]$, so that the winning probability is $\frac{v+r_{m+1}}{2} - \frac{v+r_{m-1}}{2} = \frac{r_{m+1}-r_{m-1}}{2} = 1/k$. Hence, this is not a profitable deviation either. \square

B.5.3 Proof of Proposition 2.3

(i) Arguing as in Lemma 2.3, the bonus prize of b/n is necessary for implementation of the social optimum with a bonus tournament. Thus, the total expected transfer from the buyer to the suppliers is $b/n^2 + nC$. In an auction, the conditional transfers to suppliers 1 and n differ from

those for the remaining suppliers. The revenue of supplier 1 is

$$\Pi_1 = \frac{b}{2n^2} + \int_{1/2n}^{2/2n} \left(\Psi - b \left(\sigma - \frac{1}{2n} \right) - \left(\Psi - b \left(\frac{3}{2n} - \sigma \right) \right) \right) d\sigma = \frac{3b}{4n^2}$$

For supplier 2 it is

$$\Pi_2 = 2 \int_{2/2n}^{3/2n} \left(\Psi - b \left(\frac{3}{2n} - \sigma \right) - \left(\Psi - b \left(\sigma - \frac{1}{2n} \right) \right) \right) d\sigma = \frac{2b}{4n^2}$$

By symmetry, $\Pi_1 = \Pi_n$ and $\Pi_2 = \Pi_j$ for all $j \in \{2, \dots, n-1\}$. As $\Pi_1 > \Pi_2$, the participation constraint of suppliers $j \in \{2, \dots, n-1\}$ will be binding. Suppose first $C > \Pi_2$. Then, the buyer optimally sets $t = C - \Pi_2$ in the auction. The total transfers of the buyer to the suppliers are thus $\Sigma \Pi_i + nt = 2(\Pi_1 - \Pi_2) + nC = \frac{b}{2n^2} + nC$. In this case, the total transfers of the buyer are strictly greater in the bonus tournament than in the auction. Since both contests implement the social optimum, the buyer is better off in an auction.

Next, suppose $C \leq \Pi_2$. Then, the buyer optimally sets $t = 0$. The total transfers of the buyer to the suppliers are therefore $\Sigma \Pi_i = 2 \frac{3b}{4n^2} + (n-2) \frac{2b}{4n^2} = \frac{b}{2n^2} (1+n)$. The buyer prefers the bonus tournament iff $\frac{b}{n^2} + nC < \frac{b}{2n^2} (1+n)$ or, equivalently, $C < \frac{b}{2} \left(\frac{n-1}{n^3} \right)$.

(ii) According to the proof of Lemma 2.4(iii), an FPT can implement at most $n-2$ different approaches. By Lemma B.3, an FPT implementing $n-2$ approaches exists. The FPT implementing maximum diversity (hence maximizing total surplus) thus implements $k = n-2$ with $A = 0$ and $t = C$. The participation constraint of all suppliers binds, so this is the best outcome for the buyer. In the FPT, the buyer has expected costs from suboptimal quality of $\frac{b}{4k}$. Moreover, she pays subsidies nC . In the bonus tournament, the buyer has expected costs from suboptimal quality of $\frac{b}{4n}$, pays revenues $\frac{b}{n^2}$ and subsidies nC ; together $\frac{b}{n^2} + \frac{b}{4n} + nC = \frac{b}{4} \frac{n+4}{n^2} + nC$. Thus, the buyer is better off in the bonus tournament if $\frac{b}{4} \frac{n+4}{n^2} \leq \frac{b}{4k}$ or $k \leq \frac{n^2}{n+4}$. The maximum value of k in any tournament equilibrium is $n-2$. For $k = n-2$ and $n > 4$, this condition holds strictly; for $k = n-2$ and $n = 4$ it holds with equality.

B.5.4 Proof of Corollary 2.4

According to Lemma 2.4(i), the social optimum is given by the choices $v_k^* = (2k-1)/2n$ ($k \in \{1, \dots, n\}$). The average quality in the social optimum is thus $\Psi - b/4n$. Therefore the total surplus is $\Psi - b/4n - nC$. The maximum of this expression in \mathbb{R}^+ is $n = \sqrt{b}/2\sqrt{C}$. By concavity of the objective function, the optimal choice of $n \in \mathbb{N}$ is thus given by $n_-(C)$ or $n_+(C)$. According to Lemma 2.4(ii), the social optimum for any given number of suppliers can be implemented with an auction.

B.6 Other Extensions (Section 2.5.2)

B.6.1 Proof of Corollary 2.5

Proof. (i) The proof of the result on auctions is the same as the proof of Proposition 2.2(i) above. By the generalized Proposition 2.1, the social optimum can be implemented with a bonus tournament if $0 < v_1^* \leq 1/2 \leq v_2^* < 1$. Thus, we only need to show that the social optimum

always satisfies these conditions. Therefore, first consider any $v_1 = 0$ ($v_2 = 1$ is analogous). Clearly, $\frac{\partial S_T(v_1, v_2)}{\partial v_1} \Big|_{v_1=0} > 0$. Hence, in the social optimum $v_1^* > 0$. Next, consider (v_1, v_2) such that $v_1 \leq v_2 < 1/2$ (the case $1/2 < v_1 \leq v_2$ is analogous). Supplier 2 offers higher quality than supplier 1 in the interval $[\frac{v_1+v_2}{2}, 1]$. We can write the total surplus from this interval as

$$\begin{aligned} S_T(v_1, v_2) \Big|_{\sigma \geq \frac{v_1+v_2}{2}} = & \Psi \left(1 - F \left(\frac{v_1 + v_2}{2} \right) \right) - \int_{\frac{v_1+v_2}{2}}^{v_2} \delta(|v_2 - \sigma|) f(\sigma) d\sigma - \int_{v_2}^{1/2} \delta(|v_2 - \sigma|) f(\sigma) d\sigma \\ & - \int_{1/2}^{1-v_2} \delta(|v_2 - \sigma|) f(\sigma) d\sigma - \int_{1-v_2}^{1-\frac{v_1+v_2}{2}} \delta(|v_2 - \sigma|) f(\sigma) d\sigma - \int_{1-\frac{v_1+v_2}{2}}^1 \delta(|v_2 - \sigma|) f(\sigma) d\sigma \end{aligned}$$

Consider a deviation to $v'_2 = 1 - v_2$. Symmetry of $f(\sigma)$ implies that $\int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}} \delta(|v_2 - \sigma|) f(\sigma) d\sigma = \int_{\frac{v_1+v_2}{2}}^{1-\frac{v_1+v_2}{2}} \delta(|v'_2 - \sigma|) f(\sigma) d\sigma$. As the highest available quality determines the total surplus, it follows $S_T(v_1, v_2) \Big|_{\frac{v_1+v_2}{2} \leq \sigma \leq 1-\frac{v_1+v_2}{2}} \leq S_T(v_1, v'_2) \Big|_{\frac{v_1+v_2}{2} \leq \sigma \leq 1-\frac{v_1+v_2}{2}}$. Observe that $\int_{1-\frac{v_1+v_2}{2}}^1 \delta(|v_2 - \sigma|) f(\sigma) d\sigma < \int_{1-\frac{v_1+v_2}{2}}^1 \delta(|v'_2 - \sigma|) f(\sigma) d\sigma$, because δ is increasing. Thus $S_T(v_1, v_2) \Big|_{\sigma \geq \frac{v_1+v_2}{2}} < S_T(v_1, v'_2) \Big|_{\sigma \geq \frac{v_1+v_2}{2}}$. For $\sigma < \frac{v_1+v_2}{2}$, the highest quality always comes from v_1 . Thus we have $S_T(v_1, v_2) \Big|_{\sigma < \frac{v_1+v_2}{2}} = S_T(v_1, v'_2) \Big|_{\sigma < \frac{v_1+v_2}{2}}$ and obtain $S_T(v_1, v_2) < S_T(v_1, v'_2)$. Thus, there can be no social optimum with $v_1^* \leq v_2^* < 1/2$.

(ii) *The unique equilibrium in an FPT is such that $v_1 = v_2$ and $F(v_i) = 1/2$ for $i = 1, 2$.*

First, we show that the suggested (v_1, v_2) emerges as an equilibrium. Denote the prize with A . Let v_j be such that $F(v_j) = 1/2$. Since f is everywhere positive, such a v_j is unique. Now if supplier $i \in \{1, 2\}$ plays $v_i = v_j$, his revenue is $\Pi_i(v_i, v_j) = A/2$. For any $v_i < v_j$ the revenue is $\Pi_i(v_i, v_j) = AF((v_i + v_j)/2) < A/2$. Similarly, for any $v_i > v_j$ the revenue is $\Pi_i(v_i, v_j) = A(1 - F((v_i + v_j)/2)) < A/2$. Thus, $v_i = v_j$ is an equilibrium. Second, $v'_i = v'_j$ is an equilibrium only if $F(v'_j) = 1/2$. Suppose not. Then, a supplier i can profitably deviate to v_i such that $F(v_i) = 1/2$, since his revenue will be $\Pi_i(v_i, v_j) > A/2$. Third, $v_i \neq v_j$ is never an equilibrium. Suppose it was. Let $v_1 < v_2$. Then, the revenue of supplier 1 is $\Pi_1(v_1|v_2) = AF((v_1 + v_2)/2)$, while deviating to $(v_1 + v_2)/2$ leads to a revenue of $AF((v_1 + 3v_2)/4) > AF((v_1 + v_2)/2)$. \square

B.6.2 Proof of Proposition 2.4

(i) By Corollary 2.5(ii), the FPT uniquely implements $v_1 = v_2 = 1/2$ and $F(v_i) = 1/2$ for $i = 1, 2$. Then there exists $\varepsilon > 0$, such that $F(1/2 - \varepsilon) \Delta q(1/2 - \varepsilon, 1/2 + \varepsilon) = (1 - F(1/2 - \varepsilon)) \Delta q(1/2 - \varepsilon, 1/2 + \varepsilon) < C$. Then, by the generalized version of Proposition 2.1, a bonus tournament with prices $\mathcal{P} = \{\Delta q(1/2 - \varepsilon, 1/2 + \varepsilon), 0\}$ and transfers $t = C - F(1/2 - \varepsilon) \Delta q(1/2 - \varepsilon, 1/2 + \varepsilon)$ implements $(v_1, v_2) = (1/2 - \varepsilon, 1/2 + \varepsilon)$. This yields strictly greater total surplus, with weakly lower supplier surplus than any FPT. Hence, buyer surplus is strictly greater in such a bonus tournament than in any FPT.

(iia) By Corollary 2.5(i), both the auction and the bonus tournament implement the social optimum with $t = 0$. When $\sigma \in [0, v_1^*] \cup \{(v_1^* + v_2^*)/2\} \cup [v_2^*, 1]$, the price paid is equal in both the auction and the bonus tournament. Everywhere else the price paid is in the bonus tournament is zero, while in the auction it is strictly positive. Hence, the buyer is strictly better off in the

bonus tournament than in the auction.

(iib) If $v_1^* + v_2^* = 1$, then $F(v_1^*) \Delta q(v_1^*, v_2^*) = (1 - F(v_2^*)) \Delta q(v_1^*, v_2^*)$. Moreover, we obtain $\int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma = \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma) d\sigma$. By (iia), we can focus on the case that $F(v_1^*) \Delta q(v_1^*, v_2^*) < C$. If, in addition, $C < F(v_1^*) \Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma$, the auction implements the social optimum with positive supplier surplus. The bonus tournament implements the social optimum (with appropriate choice of t) in such a way that the suppliers make zero surplus. Hence, the buyer is strictly better off in a bonus tournament. If $C \geq F(v_1^*) \Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma$, both contests implement the social optimum with zero supplier surplus and the buyer is indifferent.

(iic) Suppose that neither (a) nor (b) hold and suppose (w.l.o.g.) that $v_1^* + v_2^* < 1$. We can write the revenues for each buyer as

$$\begin{aligned}\Pi_1^a &= F(v_1^*) \Delta q(v_1^*, v_2^*) + \int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma \\ \Pi_2^a &= (1 - F(v_2^*)) \Delta q(v_1^*, v_2^*) + \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma) d\sigma\end{aligned}$$

From $v_1^* + v_2^* < 1$, it follows that $F(v_1^*) \Delta q(v_1^*, v_2^*) < (1 - F(v_2^*)) \Delta q(v_1^*, v_2^*)$. Furthermore, symmetry and single-peakedness of $f(\sigma)$ implies that

$$\int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma < \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma) d\sigma.$$

Let $t' \geq 0$ be the lowest subsidy needed to satisfy participation constraints in an auction; t' guarantees that supplier 1 breaks even. Then, the bonus tournament with $\mathcal{P} = \{\Delta q(v_1^*, v_2^*), 0\}$ and $t = t' + \int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma$ implements the social optimum. Again, the participation constraint of supplier 1 binds, while supplier 2 obtains positive surplus. However, the surplus of supplier 2 is lower in the bonus tournament than in the auction since

$$\int_{v_1^*}^{\frac{v_1^* + v_2^*}{2}} (q(v_1^*, \sigma) - q(v_2^*, \sigma)) f(\sigma) d\sigma < \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma) d\sigma$$

implies that

$$t + (1 - F(v_2^*)) \Delta q(v_1^*, v_2^*) < t' + \int_{\frac{v_1^* + v_2^*}{2}}^{v_2^*} (q(v_2^*, \sigma) - q(v_1^*, \sigma)) f(\sigma) d\sigma + (1 - F(v_2^*)) \Delta q(v_1^*, v_2^*).$$

Hence, the buyer is better off in the bonus tournament than in the auction.

B.6.3 Proof of Lemma 2.5

This section provides the proof of Lemma 2.5 from Section 2.5.2.3. Suppose that there are n suppliers and that assumption (A1)" holds. Consider an FPT with two prizes $A_1 > A_2 > 0$,

where the supplier with the highest quality receives A_1 and the supplier with the second-highest quality receives A_2 .⁶ For notational convenience, suppose that $v_1 \leq v_2 \leq \dots \leq v_n$. We first provide an intermediate result.

Lemma B.4. *If v_1, v_2, \dots, v_n is an equilibrium of an FPT with two prizes, then $v_1 = v_2 = v_3$ and $v_{n-2} = v_{n-1} = v_n$.*

Proof. We will prove that $v_1 = v_2 = v_3$. The other claim follows by an analogous argument.

Step 1: $v_1 = v_2$. Suppose not. Then $v_1 < v_2$. Thus, the revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1 + v_2}{2} A_1 + \frac{v_3 - v_2}{2} A_2.$$

Therefore, a deviation to any $v'_1 \in (v_1, v_2)$ increases the probability of winning the first prize, while not affecting the probability of winning the second prize. Hence, it is profitable.

Step 2: $v_1 = v_2 < v_3 = v_4$ cannot be an equilibrium. Denote with $P_{\sigma < v_i}^{i,1}$ the probability that supplier i wins the first prize when $\sigma < v_i$. Analogously define the probabilities of winning when the state is greater than the chosen approach and the probabilities of winning the second prize. By random tie breaking we have $P_{\sigma < v_1}^{1,1} = P_{\sigma < v_2}^{2,1} = P_{\sigma < v_1}^{1,2} = P_{\sigma < v_2}^{2,2}$ and $P_{\sigma > v_1}^{1,1} = P_{\sigma > v_2}^{2,1} = P_{\sigma > v_1}^{1,2} = P_{\sigma > v_2}^{2,2}$. We will show that $P_{\sigma < v_1}^{1,1} = P_{\sigma > v_1}^{1,1}$. Suppose that this was not true. First, suppose $P_{\sigma < v_1}^{1,1} > P_{\sigma > v_1}^{1,1}$. Then, there exist $\varepsilon, \varepsilon', \varepsilon'' > 0$ arbitrarily small such that a deviation $v'_1 = v_1 - \varepsilon$ leads to revenues

$$\Pi_1(v'_1, v_{-1}) = 2(P_{\sigma < v_1}^{1,1} - \varepsilon') A_1 + 2(P_{\sigma > v_1}^{1,1} - \varepsilon'') A_2.$$

For sufficiently small ε this constitutes a profitable deviation. The case $P_{\sigma < v_1}^{1,1} < P_{\sigma > v_1}^{1,1}$ follows by an analogous argument, but the incentives to deviate are even stronger.

Now suppose that $P_{\sigma < v_1}^{1,1} = P_{\sigma > v_1}^{1,1}$ and $v_1 = v_2 < v_3 = v_4$. We will show that this cannot be an equilibrium. In the proposed equilibrium $P_{\sigma < v_1}^{1,1} = v_1/2$ and $P_{\sigma < v_1}^{1,1} + P_{\sigma > v_1}^{1,1} = P_{\sigma < v_1}^{1,2} + P_{\sigma > v_1}^{1,2} = v_1$. Hence, the expected revenue is

$$\Pi_1(v_1, v_{-1}) = v_1 A_1 + v_1 A_2.$$

For any deviation $v'_1 \in (v_2, v_3)$ the probability of winning the first prize is

$$\frac{v'_1 + v_3}{2} - \frac{v'_1 + v_2}{2} = \frac{v_3 - v_2}{2} = v_1$$

where the last equality follows from $P_{\sigma < v_1}^{1,1} = P_{\sigma > v_1}^{1,1}$. Using $v_3 = v_4$, the probability of winning the second prize is

$$\frac{v_2 + v'_1}{2} > v_1$$

thus it follows that $\Pi_1(v'_1, v_{-1}) > \Pi_1(v_1, v_{-1})$.

Step 3: $v_1 = v_2 < v_3 < v_4$ cannot be an equilibrium. The revenue of supplier 1 is

$$\Pi_1(v_1, v_{-1}) = \frac{v_1}{2} A_1 + \frac{v_3 - v_1}{4} A_1 + \frac{v_3 + v_1}{4} A_2 + \frac{v_4 - v_3}{4} A_2. \quad (\text{B.11})$$

⁶Ties are broken randomly, with equal chance of winning for each firm with the respective quality.

Consider a deviation to $v'_1 \in (v_1, v_3)$. The revenue is

$$\Pi_1(v'_1, v_{-1}) = \frac{v_3 - v_1}{2}A_1 + \frac{v'_1 + v_1}{2}A_2 + \frac{v_4 - v_3}{2}A_2.$$

If $\Pi_1(v'_1, v_{-1}) > \Pi_1(v_1, v_{-1})$, then this is a profitable deviation. If $\Pi_1(v'_1, v_{-1}) \leq \Pi_1(v_1, v_{-1})$ is equivalent with

$$\frac{v_1}{2}A_1 - \frac{v_3 - v_1}{4}A_1 + \frac{v_3 - v_1}{4}A_2 - \frac{v'_1}{2}A_2 - \frac{v_4 - v_3}{4}A_2 \geq 0 \quad (\text{B.12})$$

But consider in that case a deviation to $v''_1 = v_1 - \varepsilon$ for small positive ε . The expected revenue is

$$\Pi_1(v''_1, v_{-1}) = \frac{v''_1 + v_1}{2}A_1 + \frac{v_3 - v_1}{2}A_2$$

and $\lim_{\varepsilon \rightarrow 0} \Pi_1(v''_1, v_{-1}) = v_1A_1 + \frac{v_3 - v_1}{2}A_2$. Together with (B.11), this implies

$$\lim_{\varepsilon \rightarrow 0} \Pi_1(v''_1, v_{-1}) - \Pi_1(v_1, v_{-1}) = \frac{v_1}{2}A_1 + \frac{v_3 - v_1}{4}A_2 - \frac{v_3 - v_1}{4}A_1 - \frac{v_1}{2}A_2 - \frac{v_4 - v_3}{4}A_2.$$

Since $v'_1 > v_1$, (B.12) implies $\lim_{\varepsilon \rightarrow 0} \Pi_1(v''_1, v_{-1}) - \Pi_1(v_1, v_{-1}) > 0$. Hence, there always exists $\varepsilon > 0$ small enough such that $\Pi_1(v''_1, v_{-1}) - \Pi_1(v_1, v_{-1}) > 0$. \square

The lemma implies that the maximal number of active approaches in an FPT with two prizes is $n - 4$. By Lemma B.3 an FPT with a single prize implements an equilibrium with $n - 2$ active approaches. By Lemma 2.4(ii), it is possible to implement the socially optimal allocation with $n - 2$ approaches in an FPT with a single prize. Implementing this equilibrium in a single-prize FPT, where the prize size is the sum of the two prizes in an FPT with two prizes, strictly increases the total payoff. On the other hand, the payoff of the suppliers remains the same, so the expected buyer payoff strictly increases.

C Appendix: Chapter 3

C.1 Proof of Proposition 3.1

The proposition is established in two steps. First, we provide conditions for the first-best solution to be such that all N sellers conduct research in every period until a breakthrough is achieved after which search is stopped completely. Second, we show that if Assumption 3.1 holds, then the conditions identified in the first step hold as well.

Let $n(\theta, t)$ be a function which specifies the number of sellers which in the first-best do research given period t and the current highest quality is θ . Gal et al. (1981) and Morgan (1983) have shown that $n(\theta, t)$ is decreasing in θ and increasing in t . Denote with $n_\theta(\theta^k, t) = n(\theta^{k+1}, t) - n(\theta^k, t)$ and $n_t(\theta^k, t) = n(\theta^k, t+1) - n(\theta^k, t)$ the change in the optimal number of sellers due to an increase in θ or t respectively.

Step 1: We begin by showing that $n(\theta^{b-1}, 1) = N$ is a sufficient condition for $n(\theta, t) = N$ for all $\theta < \theta^b$ and all $t \geq 1$. Recall that $n_t(\theta, t) \geq 0$. Thus, we have $n(\theta^{b-1}, t) = N$ for all $t \geq 1$. Further, recall that $n_\theta(\theta, t) \leq 0$. Consequently, $n(\theta^{b-1}, 1) = N$ implies $n(\theta^s, 1) = N$ for all $s \in \{1, \dots, b-1\}$. Analogously, $n(\theta^{b-1}, T) = N$ implies $n(\theta^s, T) = N$ for all $s \in \{1, \dots, b-1\}$. Taking this together we have $n(\theta, t) = N$ for all $\theta < \theta^b$ and all $t \geq 1$.

We next show that $n(\theta^b, T) = 0$ is a sufficient condition for $n(\theta, t) = 0$ for all $\theta \geq \theta^b$ and all $t \geq 1$. It follows from $n_t(\theta, t) \geq 0$ that $n(\theta^b, t) = 0$ for all $t \geq 1$. Further, because $n_\theta(\theta, t) \leq 0$ we must have $n(\theta, t) = 0$ for all $\theta \geq \theta^b$ and all $t \geq 1$.

Step 2: Next, we show that Assumption 3.1 implies that the conditions identified in Step 1 hold. Note that $n(\theta^b, T) = 0$ is equivalent to

$$F(\theta^b)\theta^b + \sum_{j=b+1}^K \left(F(\theta^j) - F(\theta^{j-1}) \right) \theta^j - \theta^b < C.$$

The left-hand side of the inequality is the expected benefit of conducting research with one seller in the last period given that the an innovation of quality θ^b is already available. The right-hand side is the cost of doing research with a single seller. We can rearrange the inequality to obtain

$$\begin{aligned} (F(\theta^b) - 1)\theta^b + (1 - F(\theta^b))\theta^b + \sum_{j=b+1}^K \left(F(\theta^j) - F(\theta^{j-1}) \right) (\theta^j - \theta^b) &< C \\ \sum_{j=b+1}^K \left(F(\theta^j) - F(\theta^{j-1}) \right) (\theta^j - \theta^b) &< C \end{aligned}$$

This inequality is satisfied for any F if

$$(\theta^K - \theta^b) \leq C$$

which holds by Assumption 3.1(i).

We will now show that if Assumption 3.1(ii) holds, then $n(\theta^{b-1}, 1) \geq N$. Note that to demonstrate that $n(\theta^{b-1}, 1) \geq N$ it is enough to show that having N sellers conduct research is better than $N - 1$. First, we cannot have more than N sellers, and second, Morgan (1983, Proposition 2) shows that the expected benefit of conducting research within a period is concave in the number of sellers. Thus, if it is better to have N than $N - 1$ sellers, it is also better than having $N - s$ for $s \geq 1$ sellers. Let $V(\theta, t)$ denote the value of having quality θ in period t given an optimal continuation in subsequent periods. Then, we can write the expected payoff of having N sellers conduct research in period 1 given that we have quality θ^{b-1} as

$$F^N(\theta^{b-1})V(\theta^{b-1}, 2) + \theta^b(1 - F^N(\theta^{b-1})) + M(N) - NC$$

where $M(N) = \sum_{j=b+1}^K (F^N(\theta^j) - F^N(\theta^{j-1})) (\theta^j - \theta^b)$. The expected payoff of having $N - 1$ sellers conduct research is

$$F^{N-1}(\theta^{b-1})V(\theta^{b-1}, 2) + \theta^b(1 - F^{N-1}(\theta^{b-1})) + M(N - 1) - (N - 1)C.$$

Thus, having N firms is better if the difference between the two inequalities is weakly positive, which reads

$$(1 - F(\theta^{b-1}))F^{N-1}(\theta^{b-1}) [\theta^b - V(\theta^{b-1}, 2)] + (M(N) - M(N - 1)) - C \geq 0.$$

Consider now the value θ^b such that the inequality holds with equality, i.e., such that we are indifferent between N and $N - 1$ sellers. This implies that we would rather have N than $N - s$ sellers for $s \geq 2$ by the within-period concavity in the number of sellers. Moreover, since $n_t(\theta, t) \geq 0$ the θ^b which induces indifference in period 1 is such that for any period $t \geq 2$ having N firms is weakly better than having $N - 1$ firms, too. Therefore, the optimal continuation is to always employ N firms. Hence,

$$\begin{aligned} V(\theta^{b-1}, 2) = & F^{N(T-1)}(\theta^{b-1})\theta^{b-1} + (1 - F^{N(T-1)}(\theta^{b-1})) (\theta^b(1 - F^N(\theta^{b-1})) + M(N)) \\ & - NC \sum_{j=1}^{T-1} F^{Nj}(\theta^{b-1})j, \end{aligned}$$

because either we continue to have a quality of θ^{b-1} until the end, or at some point we have a breakthrough.

Therefore,

$$\begin{aligned} \theta^b \geq \bar{\theta} = & \frac{1}{1 - (1 - F^{N(T-1)}(\theta^{b-1}))(1 - F^N(\theta^{b-1}))} \times \left(\frac{C - (M(N) - M(N - 1))}{(1 - F(\theta^{b-1}))F^{N-1}(\theta^{b-1})} \right. \\ & \left. - F^{N(T-1)}(\theta^{b-1})\theta^{b-1} + (1 - F^{N(T-1)}(\theta^{b-1}))M(N) - NC \sum_{j=1}^{T-1} F^{Nj}(\theta^{b-1})j \right) \end{aligned}$$

is a sufficient condition on θ^b .

C.2 Proof of Proposition 3.2

The result follows as a special case (such that $N_t = N_{t+1}$ for all t) of Proposition 3.5.

C.3 Proof of Proposition 3.3

The result follows because the first-best is a global stopping rule with a fixed number of sellers (Proposition 3.1) and a dynamic fixed prize tournament can implement any global stopping rule with a fixed number of sellers (Proposition 3.2).

C.4 Proof of Proposition 3.4

Fix the number of sellers to N . Then any round of research yields a draw from the distribution $F^N = G$. Hence, we can reformulate the problem to one of either one seller with distribution G doing research to no research taking place at all. Then, the proof in Gal et al. (1981, p. 605) with setting $K = 1$ goes through directly proving the result.

C.5 Proof of Proposition 3.5

The tournament $\langle \mathbf{E}, \mathbf{p}, \mathbf{N} \rangle$ induces a game of incomplete information with the set of players being the buyer and the set of sellers \mathcal{N} which is given by all the sellers that are active in the tournament at some point. Abusing notation, let $|\mathcal{N}| = N$. In what follows we prove that there exists a sequential equilibrium in this game in which the sellers will conduct research in every period until they reach an innovation of quality at least θ^g , which they submit to the buyer, who will then stop the tournament. Thus, the equilibrium induces a global stopping rule. We begin by formally describing the game of incomplete information and characterize the equilibrium candidate. We then prove that this equilibrium candidate is indeed a sequential equilibrium using the one-shot deviation principle by Hendon et al. (1996).

The Game

The tournament $\langle \mathbf{E}, \mathbf{p}, \mathbf{N} \rangle$ induces the following extensive form game of incomplete information: $\mathcal{G} = \langle I, H, \alpha, F, (\mathcal{I}_i)_{i \in I}, (u_i)_{i \in I} \rangle$. The set of players is $I = \{B, S_1, \dots, S_N\}$. The set H is the set of histories, where the set of terminal histories is denoted Z and the actions available after the non-terminal history h is denoted $A(h) = \{a : (h, a) \in H\}$. Note that sellers who have been eliminated cannot report an innovation anymore and that sellers who have not yet been added to the tournament cannot do research. The function α assigns to each non-terminal history a member of I , i.e., α is the player function. The set of initial histories is the finite set of the states of the world Θ^{NT} . The true initial history is $\theta \in \Theta$, where each element $\theta_{it} \in \Theta$ (where $i \in 1, \dots, N$ and $t \in 1, \dots, T$) is drawn i.i.d. from the probability distribution F . A seller i who conducts research in period t receives quality equal to θ_{it} . For each player $i \in I$ a partition \mathcal{I}_i of $\{h \in H : \alpha(h) = i\}$ with the property that $A(h) = A(h')$, whenever h and h' are in the same member of the partition. The function $u_i : Z \rightarrow \mathbb{R}$ maps for each player i the payoff at each of

the terminal nodes. For the sellers the payoffs are determined by the research costs they have incurred, the entry fee they pay (or receive) if they enter the contest, and the prize they receive. The buyer's payoff is determined by the quality of the innovation she gets, the entry fees of the participants, and the prize she pays to the winning seller. In what follows we will use the terms doing research and investing (in research) interchangeably.

Timing

Period 0:

- All N invited sellers decide whether to enter or not. If they enter they pay the entry fee E_t according to what period they are supposed to start.

Period $t < T$:

- Stage 1: Each seller simultaneously decides whether to perform research at cost C . Sellers do not observe the actions taken by their competitors.
- Stage 2: Each seller i who conducted research receives quality equal to θ_{it} . All other sellers receive quality 0.
- Stage 3: Having privately observed the value of their innovation, sellers simultaneously decide whether to privately submit their best innovation.
- Stage 4: The buyer observes the set of submissions. If there have been no submissions, the buyer cannot end the tournament. If there have been submissions, the buyer decides whether to declare a winner or not. If a winner is declared the tournament stops, the buyer obtains the winning innovation and the seller who submitted the winning innovation receives the prize p_t . If the tournament continues and the set of sellers has to be reduced to N_{t+1} , the buyer selects N_{t+1} sellers depending on their submission (if necessary randomly) to continue. If the tournament continues and the set of sellers has to be increased to N_{t+1} , $N_{t+1} - N_t$ sellers who paid E_{t+1} entry fee join in the next period.

Period T :

- Stages 1-3: As above.
- Stage 4: The tournament stops and the buyer has to declare a winner whose submissions the buyer then obtains in exchange for the prize p_T . If no seller submitted, the prize is randomly allocated.

Equilibrium Candidate

Denote with $\theta^i|h$ the highest quality available to player i at history h . For sellers, this is the highest quality they have so far discovered. For buyers, this is the highest quality currently submitted. The equilibrium candidate (σ, μ) is defined as follows:

Sellers If $A(h) = \{\text{Invest, Not Invest}\} = \{I, NI\}$ then

$$\sigma^i(h) = \begin{cases} I & \text{if } \theta^i|h < \hat{\theta}(h) \\ NI & \text{else} \end{cases}, \quad (\text{C.1})$$

where $\hat{\theta} : H \rightarrow \Theta$. If $A(h) = \{\text{Submit, Not Submit}\} = \{S, NS\}$ then

$$\sigma^i(h) = \begin{cases} S & \text{if } \theta^i|h \geq \theta^g \\ NS & \text{else} \end{cases}. \quad (\text{C.2})$$

As we already noted, a seller who has not been added to the tournament cannot invest and a seller who has been eliminated cannot submit any innovation.

Equilibrium beliefs of seller i are as follows. Denote a history in the period t' as $h_{t'}$. Let the last period when the buyer i has not observed a deviation by the seller be $t^e|h_{t'}$. This means that in period t^e , seller i did not submit and that in all periods $t^e + 1, \dots, t' - 1$ the seller i submitted a quality over θ^g but the buyer did not end the tournament. The beliefs that the element of state of the world $\tilde{\theta}$ in some period t and for a player j , where the true state of the world is θ_{jt} are given by the following cases.

- Own elements of the state of the world ($i = j$):

$$\mu_{jt}^i(\theta^k|h_{t'}) = \begin{cases} 1 & \text{if } t \leq t', a_{it}|h_{t'} = I \text{ and } \theta^k = \theta_{jt} \\ 0 & \text{if } t \leq t', a_{it}|h_{t'} = I \text{ and } \theta^k \neq \theta_{jt} \\ F(\theta^k) - F(\theta^{k-1}) & \text{else} \end{cases}. \quad (\text{C.3})$$

- Others' elements of the state of the world ($i \neq j$):

$$\mu_{jt}^i(\theta^k|h_{t'}) = \begin{cases} \frac{F(\theta^k) - F(\theta^{k-1})}{F(\theta^g - 1)} & \text{if } t \leq t^e|h_{t'} \text{ and } \theta^k < \theta^g \\ 0 & \text{if } t \leq t^e|h_{t'} \text{ and } \theta^k \geq \theta^g \\ F(\theta^k) - F(\theta^{k-1}) & \text{else} \end{cases}. \quad (\text{C.4})$$

For own elements, the seller learns exact state if he invests, if he does not, or if the chance to invest has not occurred yet, he holds initial beliefs. For the others' elements, once a period starts after a no deviation from the buyer, the seller concludes that everybody has invested up to that point and that nobody has a quality higher than θ^g . This implies that each individual θ_{jt} is drawn from the truncated distribution. If the seller observes that the buyer deviated, he learns nothing about the realization of the state in that period, hence he should hold the initial beliefs. For all the states which have not been revealed yet, the seller holds initial beliefs.

Buyer Consider any information set at which the buyer is moving, i.e., there has been at least one submission. The buyer stops the game if and only if there has been a submission of quality at least θ^g . The buyer's beliefs need to be specified only when there has been at

least one submission, as the buyer doesn't move if there hasn't been any. If there has been a submission, the beliefs are as follows. For any seller who has not submitted an innovation, the buyer believes that research has been conducted in every period, yet the draws were always below θ^g . For a firm which submitted an innovation, the buyer believes that research has been conducted in every period and that the submission is the currently highest quality.

There is no profitable one-shot deviation for the buyer

First, observe that the buyer cannot stop the tournament in any period if no firm submits an innovation. Thus we only need to consider cases where at least one firm has submitted an innovation. Further, whenever the buyer declares a winner, she will always choose the highest submission. Moreover, in period T the tournament ends in any case and consequently the buyer simply declares the highest quality innovation the winner.

Period $T - 1$

Let the expected highest quality after one additional round of research by M firms, given that the current highest quality is $\theta^k \in \Theta$, be given by a function $R : \Theta \times \mathbb{N} \rightarrow [\theta^1, \theta^K]$. That is

$$R(\theta^k, M) = F(\theta^k)^M \theta^k + \sum_{j=k+1}^K (F(\theta^j)^M - F(\theta^{j-1})^M) \theta^j.$$

Suppose the sellers play the candidate equilibrium strategy. Consider the incentives of the buyer in the period $T - 1$, when the highest quality is $\theta^k < \theta^g$. Stopping the tournament results in the payoff $\theta^k - p_{T-1}$ while continuing the tournament yields her $R(\theta^k, N_T) - p_T$. The buyer continues the tournament if and only if

$$R(\theta^k, N_T) - \theta^k > p_T - p_{T-1}.$$

Next, we show that the LHS of the above inequality is strictly decreasing in θ^k .

$$\begin{aligned} & R(\theta^{k+1}, N_T) - \theta^{k+1} - (R(\theta^k, N_T) - \theta^k) \\ &= F(\theta^{k+1})^{N_T} \theta^{k+1} + \sum_{j=k+2}^K (F(\theta^j)^{N_T} - F(\theta^{j-1})^{N_T}) \theta^j - \theta^{k+1} \\ &\quad - F(\theta^k)^{N_T} \theta^k - \sum_{j=k+1}^K (F(\theta^j)^{N_T} - F(\theta^{j-1})^{N_T}) \theta^j + \theta^k \\ &= F(\theta^{k+1})^{N_T} \theta^{k+1} - \theta^{k+1} - F(\theta^k)^{N_T} \theta^k - (F(\theta^{k+1})^{N_T} - F(\theta^k)^{N_T}) \theta^{k+1} + \theta^k \\ &= (\theta^{k+1} - \theta^k)(F(\theta^k)^{N_T} - 1) < 0. \end{aligned}$$

The LHS is strictly decreasing in θ^k and since $R(\theta^g, N_T) - \theta^g = p_T - p_{T-1}$ by construction, the buyer has no incentive to stop the tournament if $\theta^k < \theta^g$. If $\theta^k > \theta^g$ and all firms do research in the period T it still holds $R(\theta^k, N_T) - \theta^k < p_T - p_{T-1}$, hence the buyer does not want to continue the tournament and, therefore, the buyer stops the tournament in the period $T - 1$ if and only if the highest quality is $\theta^k \geq \theta^g$.

If a round of elimination is ahead and there have been more than N_{t+1} submissions, the buyer obviously chooses the best of them to continue. If not, the buyer chooses randomly. If the buyer has to increase the number of sellers, she chooses $N_{t+1} - N_t$ sellers from the set of previously inactive sellers who paid E_{t+1} entry fee, all which only have worthless innovations to begin with.

Period $t \leq T - 2$

Step 1. Denote with $V_t(\mu, \pi|\theta^k)$ the value to the buyer of having the highest quality θ^k in period t given that she follows the equilibrium candidate. Then, it follows that $V_t(\mu, \pi|\theta^g) = \theta^g - p_t$. In this step, we will show that also

$$V_t(\mu, \pi|\theta^g) = F(\theta^g)^{N_{t+1}} V_{t+1}(\mu, \pi|\theta^g) + \sum_{j=g+1}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) V_{t+1}(\mu, \pi|\theta^j)$$

for any t . It suffices to show that

$$\theta^g - p_t = F(\theta^g)^{N_{t+1}} V_{t+1}(\mu, \pi|\theta^g) + \sum_{j=g+1}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) V_{t+1}(\mu, \pi|\theta^j).$$

Obviously, for any $\theta^j \geq \theta^g$ it holds $V_{t+1}(\mu, \pi|\theta^j) = \theta^j - p_{t+1}$. Substituting

$$\begin{aligned} \theta^g - p_t &= F(\theta^g)^{N_{t+1}} (\theta^g - p_{t+1}) + \sum_{j=g+1}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) (\theta^j - p_{t+1}) \\ p_{t+1} - p_t &= F(\theta^g)^{N_{t+1}} \theta^g + \sum_{j=g+1}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) \theta^j - \theta^g \end{aligned}$$

which holds by definition for any t .

Step 2. In this step we show that for any pair θ^k, θ^{k+1} such that $\theta^{k+1} \leq \theta^g$ it holds that

$$V_t(\mu, \pi|\theta^{k+1}) - V_t(\mu, \pi|\theta^k) = F(\theta^k)^{N_{t+1}} (V_{t+1}(\mu, \pi|\theta^{k+1}) - V_{t+1}(\mu, \pi|\theta^k)).$$

We have

$$V_t(\mu, \pi|\theta^k) = F(\theta^k)^{N_{t+1}} V_{t+1}(\mu, \pi|\theta^k) + \sum_{j=k+1}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) V_{t+1}(\mu, \pi|\theta^j)$$

By Step 1, an equivalent expression holds for $V_t(\mu, \pi|\theta^{k+1})$. Expanding $V_t(\mu, \pi|\theta^{k+1}) - V_t(\mu, \pi|\theta^k)$ we get:

$$\begin{aligned} &V_t(\mu, \pi|\theta^{k+1}) - V_t(\mu, \pi|\theta^k) \\ &= F(\theta^{k+1})^{N_{t+1}} V_{t+1}(\mu, \pi|\theta^{k+1}) + \sum_{j=k+2}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) V_{t+1}(\mu, \pi|\theta^j) \\ &\quad - F(\theta^k)^{N_{t+1}} V_{t+1}(\mu, \pi|\theta^k) - \sum_{j=k+1}^K (F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}}) V_{t+1}(\mu, \pi|\theta^j) \end{aligned}$$

$$= F(\theta^k)^{N_{t+1}}(V_{t+1}(\mu, \pi|\theta^{k+1}) - V_{t+1}(\mu, \pi|\theta^k))$$

Step 3. In this step we show that for any $t \leq T-2$ and any $\theta^k < \theta^g$, the buyer does not stop the tournament, i.e., there is no profitable one-shot deviation. Stopping the tournament in period t yields the payoff of $\theta^k - p_t$. Thus, it suffices to show that $V_t(\mu, \pi|\theta^k) > \theta^k - p_t$ for any $\theta^k < \theta^g$. Observe that $V_t(\mu, \pi|\theta^g) - \theta^g + p_t = 0$. We will show that $V_t(\mu, \pi|\theta^k) - \theta^k$ is strictly decreasing in θ^k for any $\theta^k < \theta^g$. The result then follows. This is equivalent to

$$V_t(\mu, \pi|\theta^{k+1}) - \theta^{k+1} - (V_t(\mu, \pi|\theta^k) - \theta^k) < 0,$$

where $\theta^{k+1} \leq \theta^g$. By Step 2 we can write

$$F(\theta^k)^{N_{t+1}}(V_{t+1}(\mu, \pi|\theta^{k+1}) - V_{t+1}(\mu, \pi|\theta^k)) - (\theta^{k+1} - \theta^k) < 0$$

Observe that $V_T(\theta^{k+1}) - V_T(\theta^k) = \theta^{k+1} - \theta^k$. Iterating Step 2 we get

$$F(\theta^k)^{N_{t+1}}(V_{t+1}(\theta^{k+1}) - V_{t+1}(\theta^k)) < (\theta^{k+1} - \theta^k)$$

and thus a one-shot deviation is not profitable.

Step 4. In this step we show that the buyer stops the tournament whenever $\theta^k > \theta^g$, i.e. we show that there is no profitable one-shot deviation in this case either. Consider a quality of $\theta^k \geq \theta^g$ in period t and suppose the buyer does not stop the tournament. She will for sure stop the tournament in period $t+1$, however, as we consider only a one-shot deviation. The buyer stops the tournament if

$$\theta^k - p_t > F(\theta^k)^{N_{t+1}}\theta^k + \sum_{j=k+1}^K \left(F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}} \right) \theta^j - p_{t+1}$$

or, equivalently, if

$$p_{t+1} - p_t > F(\theta^k)^{N_{t+1}}\theta^k + \sum_{j=k+1}^K \left(F(\theta^j)^{N_{t+1}} - F(\theta^{j-1})^{N_{t+1}} \right) \theta^j - \theta^k.$$

From our period T analysis we know that the RHS is strictly decreasing in θ^k , and by construction it is equal to the LHS for $\theta^k = \theta^g$. Hence whenever $\theta^k > \theta^g$, the inequality holds and the buyer stops the tournament.

Step 5. If a round of elimination is ahead and there have been more than N_{t+1} submissions, the buyer obviously chooses the best of them to continue. If not, the buyer chooses randomly. If the buyer has to increase the number of sellers, she chooses $N_{t+1} - N_t$ sellers from the set of previously inactive sellers who paid E_{t+1} entry fee, all which only have worthless innovations to begin with.

There is no profitable one-shot deviation at the research stage for the seller

Let π' be a strategy profile that coincides with the equilibrium candidate π with the exception of the seller i 's action in the investment stage in period t . Thus, it is a one-shot deviation. Recall that

$$p_t = p_1 + \sum_{i=2}^t \Delta(\theta^g, N_i),$$

where

$$\Delta(\theta^g, n) = F(\theta^g)^n \theta^g + \sum_{j=g+1}^K \left(F(\theta^j)^n - F(\theta^{j-1})^n \right) \theta^j - \theta^g,$$

In what follows, we show two things. First, for a sufficiently high p_1 there exists a $\theta \in \Theta$ with $\theta^g \leq \theta < \theta^K$ such that investing is optimal for all $\theta^k \leq \theta$. Second, for each p_1 there exists a $\theta \in \Theta$ with $\theta^g \leq \theta < \theta^K$ such that not investing is optimal for all $\theta^k > \theta$. Notice that in any period t the seller could win an amount $z_t \in Z_t = \{0, p_t, p_t/2, \dots, p_t/N_t\}$. Here, winning zero amounts to the tournament ending in that period. Thus, the probability that the tournament continues is given by

$$\left(1 - \sum_{z \in Z_t} P_t^{\mu, \pi}(z | \theta^k) \right) =: W(\mu, \pi | t, \theta^k).$$

Further, let $P_t^{\mu, \pi}(z | \theta^k)$ denote the probability of winning some $z \in Z_t$. We can then define the seller's expected winnings in period t as

$$Z(\mu, \pi | \theta^k, t) = p_t \mathcal{P}^{\mu, \pi}(p_t | \theta^k),$$

where

$$\mathcal{P}^{\mu, \pi}(p_t | \theta^k) = \sum_{n=0}^{N_t-1} \frac{P_t^{\mu, \pi}(p_t/(n+1) | \theta^k)}{n+1}$$

denotes the probability of winning in period t conditional on reaching period t with quality θ^k . Notice that we have

$$\mathcal{P}^{\mu, \pi}(p_t | \theta^i) = \mathcal{P}^{\mu, \pi}(p_t | \theta^j), \quad \forall \theta^i, \theta^j < \theta^g; t < T.$$

Further, for all $\theta^k < \theta^g$ and $t < s < T$ (i.e., any subsequent period after the deviation except the last one) we have $\mathcal{P}^{\mu, \pi}(p_s | \theta^k) = \mathcal{P}^{\mu, \pi'}(p_s | \theta^k)$. In period t , when the deviation takes place, we have $\mathcal{P}^{\mu, \pi}(p_t | \theta^k) > \mathcal{P}^{\mu, \pi'}(p_t | \theta^k) = 0$ for any $\theta^k < \theta^g$ and in the final period we have $\mathcal{P}^{\mu, \pi}(p_T | \theta^k) \geq \mathcal{P}^{\mu, \pi'}(p_T | \theta^k)$ for any $\theta^k < \theta^g$. Further, notice that the investment costs incurred up to any period $s \geq t$ is greater under π than under π' by exactly C , the investment cost from period t . Note that the probability of winning in period $t < T$ does not depend on the quality at the beginning of the period (since we consider one-shot deviations all qualities will be below the

threshold), because all that matters is whether a quality above the threshold is drawn in period t . Thus, we can drop the quality and write $\mathcal{P}^{\mu,\pi}(p_t|\theta^k) = \mathcal{P}^{\mu,\pi}(p_t)$, but we need to keep in mind that this is only without loss for $t < T$, as the current quality matters for winning in the last period, too.

Moreover, $I_i(\mu, \pi|\theta^k, t) \in \{0, 1\}$ denotes the seller's investment decision. With this in hand we can define the expected utility of the assessment (μ, π) in period t when having a highest quality θ^k by

$$U_i(\mu, \pi|\theta^k, t) = Z(\mu, \pi|\theta^k, t) - I_i(\mu, \pi|\theta^k, t)C + W(\mu, \pi|t, \theta^k)\tilde{U}_i(\mu, \pi|t+1),$$

where

$$\begin{aligned} & \tilde{U}_i(\mu, \pi|t+1) \\ &= I_i(\mu, \pi|\theta^k, t) \left(F(\theta^k)U_i(\mu, \pi|\theta^k, t+1) + \sum_{x=k+1}^{g-1} (F(\theta^x) - F(\theta^{x-1}))U_i(\mu, \pi|\theta^x, t+1) \right) \\ &+ (1 - I_i(\mu, \pi|\theta^k, t))U_i(\mu, \pi|\theta^k, t+1). \end{aligned}$$

Note that the history is not an argument of the expected utility $U_i(\mu, \pi|\theta^k, t)$. The reason is that any investment cost incurred up to period t are sunk and do not matter for the decision in period t and only the highest quality is relevant, but not how (i.e., at what point in the past) the seller got it. Moreover, the continuation utility $\tilde{U}_i(\mu, \pi|t+1)$ is different depending on whether or not the seller invests in period t . In case of investment the seller could start period t with a highest quality ranging from θ^k to θ^{g-1} , but not above, as for any quality above θ^g she would have submitted it in ended the tournament.

Part 1: Investment is optimal up to some θ

We begin by showing that for a sufficiently high p_1 there exists a $\theta \in \Theta$ with $\theta^g \leq \theta < \theta^K$ such that investing is optimal for all $\theta^k \leq \theta$. Thus, we need to show that

$$U_i(\mu, \pi|\theta^k, t) - U_i(\mu, \pi'|\theta^k, t) \geq 0 \tag{C.5}$$

for all $\theta^k \leq \theta$ for sufficiently high p_1 . We will do so by showing that inequality (C.5) is satisfied for any $\theta^k < \theta^K$ for sufficiently high p_1 . We can rewrite the LHS of inequality (C.5) to

$$\begin{aligned} & U_i(\mu, \pi|\theta^k, t) - U_i(\mu, \pi'|\theta^k, t) = \\ & -C + p_t(\mathcal{P}^{\mu,\pi}(p_t) - \mathcal{P}^{\mu,\pi'}(p_t)) \\ & + W(\mu, \pi|t, \theta^k) \left(F(\theta^k)U_i(\mu, \pi|\theta^k, t+1) + \sum_{x=k+1}^{g-1} (F(\theta^x) - F(\theta^{x-1}))U_i(\mu, \pi|\theta^x, t+1) \right) \\ & - W(\mu, \pi'|t, \theta^k)U_i(\mu, \pi|\theta^k, t+1). \end{aligned}$$

Note the following about the term $U_i(\mu, \pi|\theta^k, t+1)$ at the very end. This is the continuation value of the one-shot deviation π' . Yet, the argument in the function is the equilibrium candidate

π . The reason for this is that in period $t + 1$ the only thing that matters is the strategy (and belief) for periods after and including $t + 1$ (and there π and π' coincide) and all that matters from past periods is the highest quality. We need to consider two cases. First the case when the currently highest quality is below θ^g , and second the case when the currently highest quality is at least θ^g .

Case 1 Suppose $\theta^k < \theta^g$. Begin by considering period T . The LHS of inequality (C.5) then simplifies to

$$U_i(\mu, \pi | \theta^k, T) - U_i(\mu, \pi' | \theta^k, T) = \\ -C + p_T(\mathcal{P}^{\mu, \pi}(p_T | \theta^k) - \mathcal{P}^{\mu, \pi'}(p_T | \theta^k))$$

because the tournament ends for sure. Notice that $\mathcal{P}^{\mu, \pi}(p_T | \theta^k) - \mathcal{P}^{\mu, \pi'}(p_T | \theta^k) > 0$. To see this, note that we are comparing the probabilities of winning the tournament (in a tie or outright) for the case of no research in the last period with the case of another round of research in the last period. Clearly, if the seller invests there is a strictly higher chance of winning. Thus, inequality (C.5) is satisfied for sufficiently large p_1 .

Now consider any period $t < T$. We can write the expected utility of the equilibrium strategy as

$$p_t \mathcal{P}^{\mu, \pi}(p_t) - C + \left(\sum_{s=t+1}^T (-C + p_s \mathcal{P}^{\mu, \pi}(p_s)) \prod_{i=t}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu, \pi}(t) \right)$$

and the utility of the one-shot deviation as

$$\sum_{s=t+1}^T \left(-C + p_s \mathcal{P}^{\mu, \pi'}(p_s) \right) F^{-1}(\theta^{g-1}) \prod_{i=t}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu, \pi'}(t),$$

where $Q^{\mu, \pi}(t)$ is the probability that the seller will still be in the tournament given that it continues. This captures the risk of being eliminated. We have $Q^{\mu, \pi'}(t) \leq Q^{\mu, \pi}(t)$ because not investing yields a lower expected innovation and therefore a higher chance of being eliminated.

Recall that we can write prizes as $p_t = p_1 + \sum_{i=2}^T \Delta(\theta^g, N_i)$. Making use of this we can write the expected utility of the equilibrium strategy as

$$p_t \mathcal{P}^{\mu, \pi}(p_t) - C + \left(\sum_{s=t+1}^T (-C + p_s \mathcal{P}^{\mu, \pi}(p_s)) \prod_{i=t}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu, \pi}(t) \right) \\ = p_1 \mathcal{P}^{\mu, \pi}(p_t) + p_1 \left(\sum_{s=t+1}^T \mathcal{P}^{\mu, \pi}(p_s) \prod_{i=t}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu, \pi}(t) \right) + K$$

where K contains all the terms that are not a function of p_1 .

We can proceed accordingly for the expected utility of the one-shot deviation. Then the difference between the equilibrium candidate and the one-shot deviation corresponds exactly to the LHS of inequality (C.5). Dropping all the terms that do not depend on p_1

this difference then reads

$$p_1 \left(\mathcal{P}^{\mu,\pi}(p_t) + \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi}(p_s) F^{N_t}(\theta^{g-1}) Q^{\mu,\pi}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi}(t) \right) \right) \\ - p_1 \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi'}(p_s) F^{N_t-1}(\theta^{g-1}) Q^{\mu,\pi'}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi'}(t) \right)$$

Recall that $\mathcal{P}^{\mu,\pi}(p_s) = \mathcal{P}^{\mu,\pi'}(p_s)$ for all $t < s < T$ and $\mathcal{P}^{\mu,\pi}(p_T) \geq \mathcal{P}^{\mu,\pi'}(p_T)$ and that $Q^{\mu,\pi'}(t) \leq Q^{\mu,\pi}(t)$. Thus, for the inequality to be satisfied we need

$$\begin{aligned} & \mathcal{P}^{\mu,\pi}(p_t) + \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi}(p_s) F^{N_t}(\theta^{g-1}) Q^{\mu,\pi}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi}(t) \right) \\ & - \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi'}(p_s) F^{N_t-1}(\theta^{g-1}) Q^{\mu,\pi'}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi'}(t) \right) \\ & \geq \mathcal{P}^{\mu,\pi}(p_t) + \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi'}(p_s) F^{N_t}(\theta^{g-1}) Q^{\mu,\pi'}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi'}(t) \right) \\ & - \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi'}(p_s) F^{N_t-1}(\theta^{g-1}) Q^{\mu,\pi'}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi'}(t) \right) \\ & = \mathcal{P}^{\mu,\pi}(p_t) + F^{N_t}(\theta^{g-1}) \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi'}(p_s) Q^{\mu,\pi'}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi'}(t) \right) \\ & - F^{N_t-1}(\theta^{g-1}) \left(\sum_{s=t+1}^T \mathcal{P}^{\mu,\pi'}(p_s) Q^{\mu,\pi'}(t) \prod_{i=t+1}^{s-1} F^{N_i}(\theta^{g-1}) Q^{\mu,\pi'}(t) \right) \\ & = \mathcal{P}^{\mu,\pi}(p_t) - F^{N_t-1}(\theta^{g-1})(1 - F(\theta^{g-1}))R \\ & \geq 0 \end{aligned}$$

to be positive. Note that that $R \leq 1$ as R is the total probability of winning the tournament (either tied or outright). Thus, what remains to be shown in order for the inequality (C.5) to be satisfied is that

$$\mathcal{P}^{\mu,\pi}(p_t) + F^{N_t-1}(\theta^{g-1})(F(\theta^{g-1}) - 1) \geq 0.$$

We can write

$$\mathcal{P}^{\mu,\pi}(p_t) = \sum_{k=g}^K (F(\theta^k) - F(\theta^{k-1})) \sum_{n=0}^{N_t-1} \binom{N-1}{n} \frac{F^{N_t-1-n}(\theta^{k-1}) (F(\theta^k) - F(\theta^{k-1}))^n}{n+1}.$$

The lowest quality with which the seller can win in period $t < T$ is θ^g , which she gets with a probability $F(\theta^g) - F(\theta^{g-1})$. In addition, to win outright, all others must draw at most

θ^{g-1} . The probability of winning when tying with one is

$$\binom{N_t-1}{1} \frac{F^{N_t-2}(\theta^{k-1}) (F(\theta^k) - F(\theta^{k-1}))}{2}$$

as $N_t - 2$ sellers must draw at most θ^{g-1} and exactly one must draw θ^g , too, and there are $N_t - 1$ different ways of getting there and then there is a $1/2$ chance of winning the tie.

Proving that inequality (C.5) is satisfied then boils down to proving

$$F^{N_t-1}(\theta^{g-1})(F(\theta^{g-1}) - 1) + \sum_{k=g}^K (F(\theta^k) - F(\theta^{k-1})) \sum_{n=0}^{N_t-1} \binom{N-1}{n} \frac{F^{N-1-n}(\theta^{k-1}) (F(\theta^k) - F(\theta^{k-1}))^n}{n+1} \geq 0.$$

To see that this is indeed the case some steps are needed.

$$\begin{aligned} & F^{N_t-1}(\theta^{g-1})(F(\theta^{g-1}) - 1) \\ & + \sum_{k=g}^K (F(\theta^k) - F(\theta^{k-1})) \sum_{n=0}^{N_t-1} \binom{N_t-1}{n} \frac{F^{N_t-1-n}(\theta^{k-1}) (F(\theta^k) - F(\theta^{k-1}))^n}{n+1} \\ & \geq F^{N_t-1}(\theta^{g-1})(F(\theta^{g-1}) - 1) + \sum_{k=g}^K (F(\theta^k) - F(\theta^{k-1})) F^{N_t-1}(\theta^{k-1}) \\ & \geq F^{N_t-1}(\theta^{g-1})(F(\theta^{g-1}) - 1) + F^{N_t-1}(\theta^{g-1}) \sum_{k=g}^K (F(\theta^k) - F(\theta^{k-1})) \\ & = F^{N_t-1}(\theta^{g-1})(F(\theta^{g-1}) - 1) + F^{N_t-1}(\theta^{g-1})(1 - F(\theta^{g-1})) \\ & = 0. \end{aligned}$$

The first inequality follows by considering only the first element of the binomial sum (that is, only the outright wins).

Case 2 Suppose $\theta^g \leq \theta^k < \theta^K$. In this case, the game will end with certainty in this period and the LHS of inequality (C.5) reads

$$\begin{aligned} & U_i(\mu, \pi | \theta^k, t) - U_i(\mu, \pi' | \theta^k, t) = \\ & -C + (p_1 + \sum_{i=2}^T \Delta(\theta^g, N_i)) (\mathcal{P}^{\mu, \pi}(p_t) - \mathcal{P}^{\mu, \pi'}(p_t)). \end{aligned}$$

Thus, for a sufficiently high p_1 , not investing is not a profitable deviation.

Part 2: Investment is not optimal above some θ

We will now show that for each p_1 there exists a $\theta \in \Theta$ with $\theta^g \leq \theta < \theta^K$ such that not investing is optimal for all $\theta^k > \theta$. It is obvious that for a quality of θ^K it is never optimal to invest, as research is costly. So there will always exist a quality level above which not investing is optimal. What we remains to be shown is that if there is a $\theta < \theta^K$ such that not investing is optimal at θ , then for all $\theta^k \geq \theta$ not investing is optimal, too. We do this by showing that whenever it is optimal not to invest for θ^k , then it is also optimal not to invest for θ^{k+1} . The proof is then

completed by induction.

Consider some $\theta^k \geq \theta \geq \theta^g$. Abusing notation, let $Z(\mu, \pi|\theta^k, t)$ denote the expected winnings *after* the investment stage. Thus, we can write the expected utility of having highest quality θ^k and investing in that period as

$$F(\theta^k)Z(\mu, \pi|\theta^k, t) + \sum_{m=k+1}^K (F(\theta^m) - F(\theta^{m-1}))Z(\mu, \pi|\theta^m, t) - C$$

while the expected utility of having highest quality θ^k and not investing in that period is $Z(\mu, \pi|\theta^k, t)$. By assumption we have

$$Z(\mu, \pi|\theta^k, t) - \left(F(\theta^k)Z(\mu, \pi|\theta^k, t) + \sum_{m=k+1}^K (F(\theta^m) - F(\theta^{m-1}))Z(\mu, \pi|\theta^m, t) - C \right) \geq 0.$$

We now show that this implies

$$Z(\mu, \pi|\theta^{k+1}, t) - \left(F(\theta^{k+1})Z(\mu, \pi|\theta^{k+1}, t) + \sum_{m=k+2}^K (F(\theta^m) - F(\theta^{m-1}))Z(\mu, \pi|\theta^m, t) - C \right) \geq 0.$$

We have

$$\begin{aligned} & Z(\mu, \pi|\theta^{k+1}, t) - \left(F(\theta^{k+1})Z(\mu, \pi|\theta^{k+1}, t) + \sum_{m=k+2}^K (F(\theta^m) - F(\theta^{m-1}))Z(\mu, \pi|\theta^m, t) - C \right) \\ & - Z(\mu, \pi|\theta^k, t) + \left(F(\theta^k)Z(\mu, \pi|\theta^k, t) + \sum_{m=k+1}^K (F(\theta^m) - F(\theta^{m-1}))Z(\mu, \pi|\theta^m, t) - C \right) \\ & = Z(\mu, \pi|\theta^{k+1}, t) - Z(\mu, \pi|\theta^k, t) - F(\theta^{k+1})Z(\mu, \pi|\theta^{k+1}, t) \\ & + F(\theta^k)Z(\mu, \pi|\theta^k, t) + (F(\theta^{k+1}) - F(\theta^k))Z(\mu, \pi|\theta^{k+1}, t) \\ & = (1 - F(\theta^k))(Z(\mu, \pi|\theta^{k+1}, t) - Z(\mu, \pi|\theta^k, t)) \\ & \geq 0 \end{aligned}$$

and therefore not investing is optimal for θ^{k+1} given that not investing was optimal at θ^k .

There is no profitable one-shot deviation at the submission stage for the seller

Let π' denote the one-shot deviation of seller i in period t at the submission stage. Observe that submitting an innovation that has quality below θ^g is never profitable, unless a round of elimination is ahead. Then, any seller should submit their highest innovation as this increases the chance of being allowed to continue. If no round of elimination is ahead, submitting below θ^g is not profitable. Thus we only need to consider the decision of a seller who has an innovation of quality $\theta^k \geq \theta^g$. Let $S_i(\mu, \pi|\theta^k, t) \in \{0, 1\}$ denote seller i 's submission decision in period t with highest quality θ^k . Given that we consider the submission stage with $\theta^k \geq \theta^g$ we can write utility as follows

$$U_i(\mu, \pi|\theta^k, t) = S_i(\mu, \pi|\theta^k, t)Z(\mu, \pi|\theta^k, t) + (1 - S_i(\mu, \pi|\theta^k, t))Q^{\mu, \pi}(t)\tilde{U}_i(\mu, \pi|t+1),$$

where

$$Z(\mu, \pi|\theta^k, t) = p_t \sum_{n=0}^{N-1} \frac{P_t^{\mu, \pi}(p_t/(n+1)|\theta^k)}{n+1},$$

and

$$\begin{aligned} \tilde{U}_i(\mu, \pi|t+1) = & I_i(\mu, \pi|\theta^k, t+1) \left((F(\theta^k)U_i(\mu, \pi|\theta^k, t+1) + \right. \\ & \left. \sum_{x=k+1}^K (F(\theta^x) - F(\theta^{x-1}))U_i(\mu, \pi|\theta^x, t+1) - C \right) + (1 - I_i(\mu, \pi|\theta^k, t+1))U_i(\mu, \pi|\theta^k, t+1). \end{aligned}$$

The term $P_t^{\mu, \pi}(z|\theta^k)$ captures the probability of winning prize t in period t when the seller's highest quality is θ^k after the investment stage in t . Thus, $Z(\mu, \pi|\theta^k, t)$ captures the expected winning after the investment stage. Moreover, the cost of investment in period t is already sunk at the submission stage and does not show up. The term $\tilde{U}_i(\mu, \pi|t+1)$ corresponds to the continuation value the seller receives when she does not submit. Recall that submitting will end the game for sure, as the buyer is playing according to the equilibriums strategy and the seller has a quality $\theta^k \geq \theta^g$. Even if the seller does not submit, the game may still end if another seller submitted a sufficiently high quality or because the seller is eliminated. Thus we weight the continuation value by that probability that the contest indeed continues to the next period and denote this $Q^{\mu, \pi}(t)$. The continuation value itself differs depending on whether the seller will invest in period t or not.

Submitting is profitable if

$$U_i(\mu, \pi|\theta^k, t) - U_i(\mu, \pi'|\theta^k, t) \geq 0. \quad (\text{C.6})$$

Step 1. Suppose θ^k is sufficiently high so that the seller would not invest in the following period. We can then rewrite the left-hand side of equation (C.6) to

$$\begin{aligned} & U_i(\mu, \pi|\theta^k, t) - U_i(\mu, \pi'|\theta^k, t) \\ &= Z(\mu, \pi|\theta^k, t) - Q^{\mu, \pi'}(t)Z(\mu, \pi|\theta^k, t+1) \\ &= p_1 \sum_{n=0}^{N-1} \frac{[P_t^{\mu, \pi}(p_t/(n+1)|\theta^k) - Q^{\mu, \pi'}(t)P_{t+1}^{\mu, \pi'}(p_{t+1}/(n+1)|\theta^k)]}{n+1} + R_1 - R_2, \end{aligned}$$

where the terms R_1 and R_2 do not depend on p_1 . Further, the term in the brackets in the sum is strictly positive. Hence, increasing p_1 increases the difference $U_i(\mu, \pi|\theta^k, t) - U_i(\mu, \pi'|\theta^k, t)$ and thus, for sufficiently large p_1 , the inequality is satisfied.

Step 2. Suppose θ^k is sufficiently low so that the seller would still invest in the following period. If the seller submits, she will get any prize $z_t \in Z_t = \{0, p_t, p_t/2, \dots, p_t/N_t\}$ and the game will end for sure, as she is submitting a quality $\theta^k \geq \theta^g$. Suppose the state of the world is such that she would receive a price $z_t \in Z_t \setminus \{p_t\}$, i.e., she will not win outright. Not submitting would mean that the contest ends and she receives no prize, as there is (at least) one other seller

with a quality $\theta \geq \theta^k \geq \theta^g$. Since we have $z_t \geq 0$ for all $z_t \in Z_t \setminus \{p_t\}$, submitting yields a weakly higher payoff in those states of the world than not submitting. Suppose next that the state of the world is such that the highest valuation among the other sellers is θ and we have $\theta^g \leq \theta < \theta^k$. Thus, the seller would win outright if she submits and if she does not submit she receives nothing and the tournament ends. Thus, submitting yields a strictly higher payoff than not submitting. Finally, suppose the state of the world is such that the highest quality among the other seller is θ and we have $\theta < \theta^g$. Thus, submitting would yield a prize of p_t while not submitting would yield an expected winning of

$$Q^{\mu, \pi'}(t)Z(\mu, \pi|\theta^k, t+1)$$

if she does not invest in the subsequent period or

$$Q^{\mu, \pi'}(t) \left(F(\theta^k)Z(\mu, \pi|\theta^k, t+1) + \sum_{x=k+1}^K (F(\theta^x) - F(\theta^{x-1}))Z(\mu, \pi|\theta^x, t+1) \right)$$

if she submits. Given that she chooses optimally between submitting and not submitting, submitting would yield Z^* , which denotes the maximum of the two above expressions. Essentially, in this state of the world the seller is trading-off winning the higher prize p_{t+1} in the next period with a probability below 1 against winning p_t for sure. Thus, for a sufficiently high prize p_1 she will always choose to submit.

C.6 Proof of Proposition 3.6

The proof proceeds in two steps. In the Step 1 we show that extending T is always beneficial for the buyer in case of a dynamic prize tournament. In Step 2 we show by example that extending T may be harmful for the buyer in case of a fixed prize dynamic tournament.

Step 1: The expected costs in a dynamic prize tournament, implementing a global stopping rule θ^g in a T -period tournament are given by

$$EK^g(\theta^g, N, T) = \left(\sum_{t=1}^{T-1} tF^{(t-1)N}(\theta^{g-1})(1 - F^N(\theta^{g-1})) + TF^{(T-1)N}(\theta^{g-1}) \right) NC.$$

Thus, the marginal cost of extending the tournament to $T+1$ periods is

$$\begin{aligned} EK^g(\theta^g, N, T+1) - EK^g(\theta^g, N, T) &= \\ &= \left(\sum_{t=1}^T tF^{(t-1)N}(\theta^{g-1})(1 - F^N(\theta^{g-1})) + (T+1)F^{TN}(\theta^{g-1}) \right) NC \\ &\quad - \left(\sum_{t=1}^{T-1} tF^{(t-1)N}(\theta^{g-1})(1 - F^N(\theta^{g-1})) + TF^{(T-1)N}(\theta^{g-1}) \right) NC \\ &= \left(-TF^{(T-1)N}(\theta^{g-1})(F^N(\theta^{g-1})) + (T+1)F^{TN}(\theta^{g-1}) \right) NC \\ &= F^{TN}(\theta^{g-1})NC \end{aligned}$$

and expected quality by $EQ^g(\theta^g, N, T) = \sum_{k=1}^K \theta^k h^g(\theta^k | \theta^g, N, T)$. where

$$h^g(\theta^k | \theta^g, N, T) = \begin{cases} F^{NT}(\theta^k) - F^{NT}(\theta^{k-1}) & k < g, \\ \sum_{t=1}^T F^{N(t-1)}(\theta^{g-1}) (F^N(\theta^k) - F^N(\theta^{k-1})) & k \geq g. \end{cases}$$

Then, the marginal benefit of extending the tournament to $T + 1$ periods is

$$\begin{aligned} EQ^g(\theta^g, N, T + 1) - EQ^g(\theta^g, N, T) &= \\ &= \sum_{k=1}^K \theta^k h^g(\theta^k | \theta^g, N, T + 1) - \sum_{k=1}^K \theta^k h^g(\theta^k | \theta^g, N, T) \\ &= \sum_{k=1}^K \theta^k (h^g(\theta^k | \theta^g, N, T + 1) - h^g(\theta^k | \theta^g, N, T)) \\ &= \sum_{k=1}^{g-1} \theta^k (F^{NT}(\theta^k)(F^N(\theta^k) - 1) - F^{NT}(\theta^{k-1})(F^N(\theta^{k-1}) - 1)) \\ &\quad + \sum_{k=g}^K \theta^k F^{NT}(\theta^{g-1}) (F^N(\theta^k) - F^N(\theta^{k-1})) \end{aligned} \quad (C.7)$$

The seller benefits from extending the tournament if

$$F^{TN}(\theta^{g-1})NC \leq (EQ^g(\theta^g, N, T + 1) - EQ^g(\theta^g, N, T)).$$

From the optimality of θ^g we know that

$$NC \leq \sum_{j=g+1}^K \theta^j (F^N(\theta^j) - F^N(\theta^{j-1})) - \theta^g (1 - F^N(\theta^g))$$

Thus, to show that the seller benefits from extending the tournament, it is sufficient to show that

$$\begin{aligned} F^{TN}(\theta^{g-1}) \left(\sum_{j=g+1}^K \theta^j (F^N(\theta^j) - F^N(\theta^{j-1})) - \theta^g (1 - F^N(\theta^g)) \right) \\ \leq (EQ^g(\theta^g, N, T + 1) - EQ^g(\theta^g, N, T)). \end{aligned} \quad (C.8)$$

Combining (C.7) and (C.8) and simplifying, we get that the sufficient condition is

$$F^{TN}(\theta^{g-1})\theta^g(1 - F^N(\theta^{g-1})) \geq \sum_{k=1}^{g-1} \theta^k (F^{NT}(\theta^k)(1 - F^N(\theta^k)) - F^{NT}(\theta^{k-1})(1 - F^N(\theta^{k-1})))$$

Recall that $F(\theta^0) = 0$ and note that we can write this sum as

$$\begin{aligned} &\sum_{k=1}^{g-1} \theta^k (F^{NT}(\theta^k)(1 - F^N(\theta^k)) - F^{NT}(\theta^{k-1})(1 - F^N(\theta^{k-1}))) \\ &= \theta^1 F^{NT}(\theta^1)(1 - F^N(\theta^1)) - \theta^2 F^{NT}(\theta^1)(1 - F^N(\theta^1)) \\ &\quad + \theta^2 F^{NT}(\theta^2)(1 - F^N(\theta^2)) - \theta^3 F^{NT}(\theta^2)(1 - F^N(\theta^2)) \end{aligned}$$

$$\begin{aligned}
& \vdots \\
& + \theta^{g-2} F^{NT}(\theta^{g-2})(1 - F^N(\theta^{g-2})) - \theta^{g-1} F^{NT}(\theta^{g-2})(1 - F^N(\theta^{g-2})) \\
& + \theta^{g-1} F^{NT}(\theta^{g-1})(1 - F^N(\theta^{g-1})) \\
& = - \sum_{k=1}^{g-2} (\theta^{k+1} - \theta^k) F^{NT}(\theta^k)(1 - F^N(\theta^k)) + \theta^{g-1} F^{NT}(\theta^{g-1})(1 - F^N(\theta^{g-1}))
\end{aligned}$$

This allows us to rewrite the sufficient condition to

$$(\theta^g - \theta^{g-1}) F^{TN}(\theta^{g-1})(1 - F^N(\theta^{g-1})) \geq - \sum_{k=1}^{g-2} (\theta^{k+1} - \theta^k) F^{NT}(\theta^k)(1 - F^N(\theta^k))$$

which always holds because $\theta^{k+1} > \theta^k$.

Step 2: To construct an example of a harmful extension of T in case of a fixed prize tournament we consider a setting with $N = 2$, $\Theta = \{0, 1\}$ and extend $T = 2$ to $T + 1 = 3$. Let the probability of drawing $\theta^1 = 1$ be given by π . We will choose parameters such that the optimal individual threshold is $\theta^i = 1$. The expected costs in the case of $T = 2$ are given by

$$EK(2, 2) = 2(\pi C + 2(1 - \pi)C)$$

and the expected quality

$$EQ(2, 2) = 1 - (1 - \pi)^4.$$

In the case of $T = 3$ we have

$$EK(2, 3) = 2(\pi C + 2(1 - \pi)\pi C + 3(1 - \pi)^2 C)$$

and the expected quality

$$EQ(2, 3) = 1 - (1 - \pi)^6.$$

Hence, the change in expected surplus for the buyer is given by

$$EQ(2, 2) - EK(2, 2) - EQ(2, 3) + EK(2, 3) = (1 - \pi)^2 (2C - (2 - \pi)\pi(1 - \pi)^2)$$

which is negative for $C = 1/10$ and $\pi = 2/5$. Since

$$EQ(2, 2) - EK(2, 2) = \frac{16}{25} - \frac{8}{25} > 0$$

the individual threshold is $\theta^i = 1$ is indeed optimal.

D Appendix: Chapter 4

D.1 Proof of Theorem 4.1

D.1.1 Proof of Lemma 4.1

If-statement. We first show that (IC-R) is implied by (i) - (iii). Note that (IC-R) can be rewritten as

$$S(e, \theta) - S(e', \theta') \geq (\theta - \theta') S_u(e', \theta') \quad \forall (e, \theta), (e', \theta') \in E \times \Theta.$$

Using (i) and (iii), this is equivalent to the requirement that, $\forall e' \in E$ and $\forall \theta, \theta' \in \Theta$,

$$\int_{\theta'}^{\theta} (S_u(e', s) - S_u(e', \theta')) ds \geq 0,$$

and this inequality indeed holds since $S_u(e', \theta)$ is non-decreasing in θ by (ii).

Only-if-statement. We now proceed to prove that (IC-R) implies (i) - (iii). Note that for the special case $\theta' = \theta$, (IC-R) is reduced to the requirement that $S(e, \theta) \geq S(e', \theta) \quad \forall e, e' \in E$. Interchanging e and e' , we immediately obtain (i). Next, consider the special case where $e' = e$. For this case, (IC-R) requires that, $\forall \theta, \theta' \in \Theta$,

$$S(e, \theta) \geq -S_t(e, \theta') + \theta S_u(e, \theta') \tag{IC_{\theta, \theta'}}$$

and

$$S(e, \theta') \geq -S_t(e, \theta) + \theta' S_u(e, \theta). \tag{IC_{\theta', \theta}}$$

Summing up $(IC_{\theta, \theta'})$ and $(IC_{\theta', \theta})$ we obtain

$$(\theta - \theta') (S_u(e, \theta) - S_u(e, \theta')) \geq 0 \quad \forall \theta, \theta' \in \Theta.$$

Thus, $S_u(e, \theta)$ must be non-decreasing in θ , which is condition (ii). The envelope formula in (iii) follows directly from Theorem 2 of Milgrom and Segal (2002), where absolute continuity of $S(e, \theta)$ holds because the set of transfer profiles is bounded. ■

D.1.2 Proof of Lemma 4.2

By Lemma 4.1, credibility implies that, $\forall e, e' \in E$ and $\forall \theta \in \Theta$,

$$\delta(e, e', \theta) = S(e, \theta) - S(e', \theta) = \int_{\underline{\theta}}^{\theta} (S_u(e, s) - S_u(e', s)) ds = 0.$$

This implies that, for any fixed $e, e' \in E$, $S_u(e, \theta) = S_u(e', \theta)$ for almost every $\theta \in \Theta$. It then also immediately follows that $S_t(e, \theta) = S_t(e', \theta)$ for almost every $\theta \in \Theta$. Now choose an arbitrary $e' \in E$ and define the functions x and \hat{x} by

$$x(\theta) = S_t(e', \theta), \quad \hat{x}(\theta) = S_u(e', \theta) \quad \forall \theta \in \Theta.$$

It follows that, for any $e \in E$, $S_t(e, \theta) = x(\theta)$ and $S_u(e, \theta) = \hat{x}(\theta)$ for almost all $\theta \in \Theta$. ■

D.1.3 Proof of Lemma 4.3

Suppose $\Phi = (\mu^{e, \theta})_{(e, \theta) \in E \times \Theta}$ implements σ . In particular, Φ is credible, so by Lemma 4.2 there exists a pair of function x and \hat{x} such that, $\forall e \in E$,

$$\mathbb{E}_{\mu^{e, \theta}} \left[\sum_{i=1}^n t_i \right] = x(\theta), \quad \mathbb{E}_{\mu^{e, \theta}} \left[\sum_{i=1}^n u(t_i) \right] = \hat{x}(\theta),$$

for almost every $\theta \in \Theta$. Since Φ implements σ , we also have $\forall i \in I$ and $\forall \sigma'_i \in \Delta \mathbb{R}_+$,

$$\mathbb{E}_\sigma \left[\mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e, \theta}} [u(t_i)] \right] - c(e_i) \right] \geq \mathbb{E}_{(\sigma'_i, \sigma_{-i})} \left[\mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e, \theta}} [u(t_i)] \right] - c(e_i) \right].$$

The expected payoff of the principal with (σ, Φ) is given by

$$\Pi_P(\sigma, \Phi) = \mathbb{E}_\sigma \left[\sum_{i=1}^n e_i - \mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e, \theta}} \left[\sum_{i=1}^n t_i \right] \right] \right] = \mathbb{E}_\sigma \left[\sum_{i=1}^n e_i \right] - \mathbb{E}_\tau [x(\theta)].$$

For every $e \in E$, define a probability measure $\mu^e \in \Delta T$ such that

$$\mu^e(A) = \mathbb{E}_\tau [\mu^{e, \theta}(A)]$$

for all measurable subsets $A \subseteq T$.¹ Now construct an alternative contract $\hat{\Phi}$ by setting $\hat{\mu}^{e, \theta} = \mu^e$ for all $(e, \theta) \in E \times \Theta$. This contract satisfies the property of θ -independence stated in the lemma. Since, $\forall (e, \theta) \in E \times \Theta$,

$$\begin{aligned} \hat{S}_t(e, \theta) &= \mathbb{E}_{\hat{\mu}^{e, \theta}} \left[\sum_{i=1}^n t_i \right] = \mathbb{E}_{\mu^e} \left[\sum_{i=1}^n t_i \right] = \mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e, \theta}} \left[\sum_{i=1}^n t_i \right] \right] = \mathbb{E}_\tau [x(\theta)], \\ \hat{S}_u(e, \theta) &= \mathbb{E}_{\hat{\mu}^{e, \theta}} \left[\sum_{i=1}^n u(t_i) \right] = \mathbb{E}_{\mu^e} \left[\sum_{i=1}^n u(t_i) \right] = \mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e, \theta}} \left[\sum_{i=1}^n u(t_i) \right] \right] = \mathbb{E}_\tau [\hat{x}(\theta)], \end{aligned}$$

by Lemma 1 it is straightforward to check that $\hat{\Phi}$ is credible. Furthermore, note that

$$\begin{aligned} \Pi_i(\sigma', \hat{\Phi}) &= \mathbb{E}_{\sigma'} \left[\mathbb{E}_\tau \left[\mathbb{E}_{\hat{\mu}^{e, \theta}} [u(t_i)] \right] - c(e_i) \right] \\ &= \mathbb{E}_{\sigma'} \left[\mathbb{E}_\tau \left[\mathbb{E}_{\mu^e} [u(t_i)] \right] - c(e_i) \right] \\ &= \mathbb{E}_{\sigma'} \left[\mathbb{E}_\tau \left[\mathbb{E}_{\mu^{e, \theta}} [u(t_i)] \right] - c(e_i) \right] = \Pi_i(\sigma', \Phi) \end{aligned}$$

¹The assumption discussed in footnote 15 ensures that the expectation (as well as the ones in the proof of the next lemma) is well-defined. It is also easy to show that μ^e is indeed a probability measure.

for all σ' and $i \in I$, which implies that $\hat{\Phi}$ implements σ because Φ implements σ .

Finally, from the above arguments we also obtain that the principal's expected payoff is $\mathbb{E}_\sigma [\sum_{i=1}^n e_i] - \mathbb{E}_\tau [x(\theta)]$ with both (σ, Φ) and $(\sigma, \hat{\Phi})$. ■

D.1.4 Proof of Lemma 4.4

Suppose $\Phi = (\mu^e)_{e \in E}$ implements σ . We first construct a probability measure $\eta \in \Delta T$ by

$$\eta(A) = \mathbb{E}_\sigma [\mu^e(A)]$$

for all measurable subsets $A \subseteq T$. Furthermore, for each $i \in I$ we construct a probability measure $\eta^{(i)} \in \Delta T$ by setting

$$\eta^{(i)}(A) = \mathbb{E}_\sigma [\mu^{(0, e_{-i})}(A)]$$

for all measurable subsets $A \subseteq T$. We now construct an alternative contract $\hat{\Phi} = (\hat{\mu}^e)_{e \in E}$ as follows. For $e = \bar{e}$, we let $\hat{\mu}^e = \eta$. For any $e = (e_i, \bar{e}_{-i})$ with $e_i \neq \bar{e}_i$, we let $\hat{\mu}^e = \eta^{(i)}$. For all remaining e , we let $\hat{\mu}^e = \mu^e$.

We first show that $\hat{\Phi}$ is credible. Since Φ is credible and its transfers are independent of θ , by Lemma 4.2 there exist $x, \hat{x} \in \mathbb{R}_+$ such that $\mathbb{E}_{\mu^e} [\sum_{i=1}^n t_i] = x$ and $\mathbb{E}_{\mu^e} [\sum_{i=1}^n u(t_i)] = \hat{x}$ for all $e \in E$. First consider $\hat{\mu}^e$ for $e = \bar{e}$. We obtain

$$\mathbb{E}_{\hat{\mu}^{\bar{e}}} \left[\sum_{i=1}^n t_i \right] = \mathbb{E}_\eta \left[\sum_{i=1}^n t_i \right] = \mathbb{E}_\sigma \left[\mathbb{E}_{\mu^e} \left[\sum_{i=1}^n t_i \right] \right] = \mathbb{E}_\sigma [x] = x$$

and, by the analogous argument, $\mathbb{E}_{\hat{\mu}^{\bar{e}}} [\sum_{i=1}^n u(t_i)] = \hat{x}$. Now consider $\hat{\mu}^e$ for $e = (e_i, \bar{e}_{-i})$ with $e_i \neq \bar{e}_i$. We obtain

$$\mathbb{E}_{\hat{\mu}^{(e_i, \bar{e}_{-i})}} \left[\sum_{i=1}^n t_i \right] = \mathbb{E}_{\eta^{(i)}} \left[\sum_{i=1}^n t_i \right] = \mathbb{E}_\sigma \left[\mathbb{E}_{\mu^{(0, e_{-i})}} \left[\sum_{i=1}^n t_i \right] \right] = \mathbb{E}_\sigma [x] = x$$

and, by the analogous argument, $\mathbb{E}_{\hat{\mu}^{(e_i, \bar{e}_{-i})}} [\sum_{i=1}^n u(t_i)] = \hat{x}$. Since $\hat{\Phi}$ and Φ are identical for all other e , we can conclude that $\mathbb{E}_{\hat{\mu}^e} [\sum_{i=1}^n t_i] = x$ and $\mathbb{E}_{\hat{\mu}^e} [\sum_{i=1}^n u(t_i)] = \hat{x}$ for all $e \in E$. It is then straightforward to check that $\hat{\Phi}$ is credible by using Lemma 4.1.

We next show that, in $\hat{\Phi}$, for each agent $i \in I$ it is a best response to play \bar{e}_i when the remaining agents are playing \bar{e}_{-i} , which implies that $\hat{\Phi}$ implements \bar{e} . This claim holds because, $\forall i \in I$ and $\forall e'_i \neq \bar{e}_i$,

$$\begin{aligned} \Pi_i(\bar{e}, \hat{\Phi}) &= \mathbb{E}_\eta [u(t_i)] - c(\bar{e}_i) \\ &= \mathbb{E}_\sigma [\mathbb{E}_{\mu^e} [u(t_i)]] - c(\mathbb{E}_\sigma [e_i]) \\ &\geq \mathbb{E}_\sigma [\mathbb{E}_{\mu^e} [u(t_i)]] - \mathbb{E}_\sigma [c(e_i)] \\ &\geq \mathbb{E}_\sigma [\mathbb{E}_{\mu^{(0, e_{-i})}} [u(t_i)]] \\ &\geq \mathbb{E}_\sigma [\mathbb{E}_{\mu^{(0, e_{-i})}} [u(t_i)]] - c(e'_i) \\ &= \mathbb{E}_{\eta^{(i)}} [u(t_i)] - c(e'_i) = \Pi_i((e'_i, \bar{e}_{-i}), \hat{\Phi}), \end{aligned} \tag{D.1}$$

where the first inequality follows the convexity of c and the second inequality follows from the fact that Φ implements σ .

Finally, from the above arguments we also obtain that the principal's expected payoff is $\sum_{i=1}^n \bar{e}_i - x$ with both (σ, Φ) and $(\bar{e}, \hat{\Phi})$. ■

D.1.5 Proof of Lemma 4.5

Suppose $\Phi = (\mu^e)_{e \in E}$ implements \bar{e} . We now construct an alternative contract $\hat{\Phi} = (\hat{\mu}^e)_{e \in E}$ as follows. For $e = \hat{e}$, we define $\hat{\mu}^e$ by generating a profile of prizes $t = (t_1, \dots, t_n)$ according to $\mu^{\bar{e}}$ and then allocating these prizes randomly and uniformly among the agents. For any $e = (e_i, \hat{e}_{-i})$ with $e_i \neq \hat{e}_i$, we let $\hat{\mu}^e$ be given as follows. A number j is drawn uniformly from I and then a profile of prizes $t = (t_1, \dots, t_n)$ is generated according to $\mu^{(0, \bar{e}_{-j})}$. The deviating agent i gets the prize t_j and the remaining $n - 1$ prizes are allocated randomly and uniformly among the non-deviating agents. Note that, by construction, this punishment rule for unilateral deviations does not depend on the identity of the agent being punished. For all remaining e , we let $\hat{\mu}^e = \mu^e$.

We first show that $\hat{\Phi}$ is credible. By Lemma 4.2, credibility and θ -independence of Φ imply that there exists $x, \hat{x} \in \mathbb{R}_+$ such that $\mathbb{E}_{\mu^e} [\sum_{i=1}^n t_i] = x$ and $\mathbb{E}_{\mu^e} [\sum_{i=1}^n u(t_i)] = \hat{x}$ for all $e \in E$. Now first consider $\hat{\mu}^e$ for $e = \hat{e}$. We obtain

$$\begin{aligned} \mathbb{E}_{\hat{\mu}^{\hat{e}}} \left[\sum_{i=1}^n t_i \right] &= \mathbb{E}_{\mu^{\bar{e}}} \left[\sum_{i=1}^n t_i \right] = x, \\ \mathbb{E}_{\hat{\mu}^{\hat{e}}} \left[\sum_{i=1}^n u(t_i) \right] &= \mathbb{E}_{\mu^{\bar{e}}} \left[\sum_{i=1}^n u(t_i) \right] = \hat{x}. \end{aligned}$$

Now consider $\hat{\mu}^e$ for any $e = (e_i, \hat{e}_{-i})$ with $e_i \neq \hat{e}_i$. We obtain

$$\begin{aligned} \mathbb{E}_{\hat{\mu}^{(e_i, \hat{e}_{-i})}} \left[\sum_{i=1}^n t_i \right] &= \sum_{j=1}^n \frac{1}{n} \mathbb{E}_{\mu^{(0, \bar{e}_{-j})}} \left[\sum_{i=1}^n t_i \right] = \frac{1}{n} \sum_{j=1}^n x = x, \\ \mathbb{E}_{\hat{\mu}^{(e_i, \hat{e}_{-i})}} \left[\sum_{i=1}^n u(t_i) \right] &= \sum_{j=1}^n \frac{1}{n} \mathbb{E}_{\mu^{(0, \bar{e}_{-j})}} \left[\sum_{i=1}^n u(t_i) \right] = \frac{1}{n} \sum_{j=1}^n \hat{x} = \hat{x}. \end{aligned}$$

Since $\hat{\Phi}$ and Φ are identical for all other e , we can conclude that $\mathbb{E}_{\hat{\mu}^e} [\sum_{i=1}^n t_i] = x$ and $\mathbb{E}_{\hat{\mu}^e} [\sum_{i=1}^n u(t_i)] = \hat{x}$ for all $e \in E$. It is then straightforward to check that $\hat{\Phi}$ is credible by using Lemma 4.1.

We next show that, in $\hat{\Phi}$, for each agent $i \in I$ it is a best response to play \hat{e}_i when the remaining agents are playing \hat{e}_{-i} , which implies that $\hat{\Phi}$ implements \hat{e} . To prove this claim, note that

$$\mathbb{E}_{\mu^{\bar{e}}} [u(t_i)] - c(\bar{e}_i) \geq \mathbb{E}_{\mu^{(0, \bar{e}_{-i})}} [u(t_i)]$$

holds for all $i \in I$ because Φ implements \bar{e} . Summing over all $i \in I$ and dividing by n yields

$$\mathbb{E}_{\mu^{\bar{e}}} \left[\frac{1}{n} \sum_{k=1}^n u(t_k) \right] - \frac{1}{n} \sum_{k=1}^n c(\bar{e}_k) \geq \frac{1}{n} \sum_{k=1}^n \mathbb{E}_{\mu^{(0, \bar{e}_{-k})}} [u(t_k)].$$

We now obtain, $\forall i \in I$ and $\forall e_i \neq \hat{e}_i$,

$$\begin{aligned}
\Pi_i(\hat{e}, \hat{\Phi}) &= \mathbb{E}_{\hat{\mu}^{\hat{e}}} [u(t_i)] - c(\hat{e}_i) \\
&= \mathbb{E}_{\mu^{\bar{e}}} \left[\frac{1}{n} \sum_{k=1}^n u(t_k) \right] - c(\hat{e}_i) \\
&\geq \mathbb{E}_{\mu^{\bar{e}}} \left[\frac{1}{n} \sum_{k=1}^n u(t_k) \right] - \frac{1}{n} \sum_{k=1}^n c(\bar{e}_k) \\
&\geq \frac{1}{n} \sum_{k=1}^n \mathbb{E}_{\mu^{(0, \bar{e}_{-k})}} [u(t_k)] - c(e_i) \\
&= \mathbb{E}_{\mu^{(e_i, \hat{e}_{-i})}} [u(t_i)] - c(e_i) = \Pi_i((e_i, \hat{e}_{-i}), \hat{\Phi}),
\end{aligned}$$

where the first inequality follows from convexity of c . Hence the claim follows.

Finally, from the above arguments we also obtain that the principal's expected payoff is $\sum_{i=1}^n \bar{e}_i - x$ with both (\bar{e}, Φ) and $(\hat{e}, \hat{\Phi})$. ■

D.1.6 Proof of Lemma 4.6

Suppose $\Phi = (\mu^e)_{e \in E}$ implements the symmetric profile \hat{e} . From the proof of Lemma 4.5 we know that it is without loss of generality to assume that Φ has the following form. If $e = \hat{e}$, a profile of prizes $t = (t_1, \dots, t_n)$ is generated according to some probability measure π and these prizes are randomly and uniformly allocated to the agents. If $e = (e_i, \hat{e}_{-i})$ with $e_i \neq \hat{e}_i$ for some $i \in I$, a profile of prizes $t^d = (t_1^d, \dots, t_n^d)$ is generated according to some (i -independent) probability measure ρ and agent i gets t_n^d , while the remaining $n - 1$ prizes are randomly and uniformly allocated among the other agents. For all other effort profiles e , the transfer rule can be chosen as for \hat{e} . Thus, we have

$$\mathbb{E}_{\mu^e} [u(t_i)] = \mathbb{E}_{\pi} \left[\frac{1}{n} \sum_{k=1}^n u(t_k) \right], \quad \mathbb{E}_{\mu^{(e_i, \hat{e}_{-i})}} [u(t_i)] = \mathbb{E}_{\rho} [u(t_n^d)].$$

Furthermore, by Lemma 4.2, credibility and θ -independence of Φ imply that there exist $x, \hat{x} \in \mathbb{R}_+$ such that $\mathbb{E}_{\mu^e} [\sum_{i=1}^n t_i] = x$ and $\mathbb{E}_{\mu^e} [\sum_{i=1}^n u(t_i)] = \hat{x}$ for all $e \in E$.

Now construct a contest C_y with prize profile y as follows. Define \underline{t}^d as the certainty equivalent of a deviating agent's random transfers in contract Φ , i.e., $u(\underline{t}^d) = \mathbb{E}_{\rho} [u(t_n^d)]$. Note that $\underline{t}^d \leq \mathbb{E}_{\rho} [t_n^d]$ by concavity of u . Then define the prize profile

$$y = \left(\frac{x - \underline{t}^d}{n-1}, \dots, \frac{x - \underline{t}^d}{n-1}, \underline{t}^d \right).$$

The allocation rule of C_y is as follows. If $e = \hat{e}$, the prizes are randomly and uniformly allocated among all agents. If $e = (e_i, \hat{e}_{-i})$ with $e_i \neq \hat{e}_i$ for some $i \in I$, the deviating agent i obtains \underline{t}^d and all other agents obtain $(x - \underline{t}^d)/(n-1)$. For all other effort profiles e , the prizes are again randomly and uniformly allocated among all agents.

Since C_y is a contest, it is credible. Furthermore, $\forall i \in I$ and $\forall e_i \neq \hat{e}_i$,

$$\begin{aligned}
\Pi_i(\hat{e}, C_y) &= \left(\frac{n-1}{n}\right) u\left(\frac{x - \underline{t}^d}{n-1}\right) + \frac{1}{n} u(\underline{t}^d) - c(\hat{e}_i) \\
&\geq \left(\frac{n-1}{n}\right) u\left(\frac{\mathbb{E}_\rho[\sum_{k=1}^n t_k^d] - \mathbb{E}_\rho[t_n^d]}{n-1}\right) + \frac{1}{n} \mathbb{E}_\rho[u(t_n^d)] - c(\hat{e}_i) \\
&= \left(\frac{n-1}{n}\right) u\left(\mathbb{E}_\rho\left[\sum_{k=1}^{n-1} \frac{1}{n-1} t_k^d\right]\right) + \frac{1}{n} \mathbb{E}_\rho[u(t_n^d)] - c(\hat{e}_i) \\
&\geq \left(\frac{n-1}{n}\right) \mathbb{E}_\rho\left[u\left(\sum_{k=1}^{n-1} \frac{1}{n-1} t_k^d\right)\right] + \frac{1}{n} \mathbb{E}_\rho[u(t_n^d)] - c(\hat{e}_i) \\
&\geq \left(\frac{n-1}{n}\right) \mathbb{E}_\rho\left[\sum_{k=1}^{n-1} \frac{1}{n-1} u\left(t_k^d\right)\right] + \frac{1}{n} \mathbb{E}_\rho[u(t_n^d)] - c(\hat{e}_i) \\
&= \mathbb{E}_\rho\left[\frac{1}{n} \sum_{k=1}^{n-1} u\left(t_k^d\right)\right] + \mathbb{E}_\rho\left[\frac{1}{n} u\left(t_n^d\right)\right] - c(\hat{e}_i) \\
&= \mathbb{E}_\rho\left[\frac{1}{n} \sum_{k=1}^n u\left(t_k^d\right)\right] - c(\hat{e}_i) \\
&= \mathbb{E}_\pi\left[\frac{1}{n} \sum_{k=1}^n u(t_k)\right] - c(\hat{e}_i) \\
&\geq \mathbb{E}_\rho\left[u(t_n^d)\right] - c(e_i) \\
&= u(\underline{t}^d) - c(e_i) = \Pi_i((e_i, \hat{e}_{-i}), C_y),
\end{aligned}$$

where the first inequality follows from $\underline{t}^d \leq \mathbb{E}_\rho[t_n^d]$, the second and third inequalities follow from concavity of u , and the last inequality follows from the fact that Φ implements \hat{e} . We can thus conclude that C_y also implements \hat{e} .

Finally, from the above arguments we also obtain that the principal's expected payoff is $\sum_{i=1}^n \hat{e}_i - x$ with both (\hat{e}, Φ) and (\hat{e}, C_y) . ■

D.2 Proof of Theorem 4.2

Only-if-statement. Suppose (σ^*, C_y^*) solves (P). We first claim that σ^* must be a pure-strategy effort profile. By contradiction, suppose there exists $j \in I$ such that σ_j^* is not a Dirac measure. We can now proceed exactly as in the proof of Lemma 4.4 to construct a contract $\hat{\Phi}$ (in fact, a contest) that implements a pure-strategy profile \bar{e} . The only difference to the proof of Lemma 4.4 is that we let $\bar{e}_j = \mathbb{E}_{\sigma_j^*}[e_j] + \epsilon$ for some $\epsilon > 0$ (but still $\bar{e}_i = \mathbb{E}_{\sigma_i^*}[e_i]$ for all $i \neq j$). Credibility of $\hat{\Phi}$ and the fact that \bar{e}_i is a best response to \bar{e}_{-i} for all $i \neq j$ follow exactly as in the proof of Lemma 4.4. The fact that \bar{e}_j is a best response to \bar{e}_{-j} for sufficiently small $\epsilon > 0$ follows because the first inequality in (D.1) is strict for j when $\epsilon = 0$, because c is strictly convex and σ_j^* is not a Dirac measure. Since the principal's payoff with $(\bar{e}, \hat{\Phi})$ is increased by ϵ , (σ^*, C_y^*) cannot have been a solution to (P).

Now suppose (\bar{e}, C_y^*) solves (P), where \bar{e} may still be asymmetric. Denote $x = \sum_{k=1}^n y_k$. We next show that whenever $y_n > 0$, there exists another contest $C_{\tilde{y}}$ with $\sum_{k=1}^n \tilde{y}_k = x$ that

implements an effort profile \tilde{e} with $\sum_{i=1}^n \tilde{e}_i > \sum_{i=1}^n \bar{e}_i$, and hence (\bar{e}, C_y^*) cannot have been a solution to (P). Denote by $p_i^k(e)$ the probability that agent i receives prize y_k in C_y^* when the effort profile is e . Note that

$$\Pi_i(\bar{e}, C_y^*) = \sum_{k=1}^n p_i^k(\bar{e})u(y_k) - c(\bar{e}_i) \geq \sum_{k=1}^n p_i^k(0, \bar{e}_{-i})u(y_k) \geq u(y_n),$$

because C_y^* implements \bar{e} . Now consider an agent $j \in I$ for which $p_j^n(\bar{e}) < 1$. Construct a contest $C_{\tilde{y}}$ with a profile of prizes \tilde{y} given by $\tilde{y}_1 = y_1 + \delta$, $\tilde{y}_n = y_n - \delta$, and $\tilde{y}_k = y_k$ for all $k \neq 1, n$, where $\delta \in (0, y_n]$. Note that $\sum_{k=1}^n \tilde{y}_k = x$. Let effort profile \tilde{e} be such that $\tilde{e}_j = \bar{e}_j + \epsilon$ and $\tilde{e}_i = \bar{e}_i$ for all $i \neq j$, where $\epsilon > 0$. Note that $\sum_{i=1}^n \tilde{e}_i > \sum_{i=1}^n \bar{e}_i$. The rule of contest $C_{\tilde{y}}$ is the following. If the effort profile is \tilde{e} , then the prizes \tilde{y} are allocated such that each agent i receives prize \tilde{y}_k with probability $\tilde{p}_i^k(\tilde{e}) = p_i^k(\bar{e})$. If some agent i unilaterally deviates from \tilde{e} , then agent i receives the prize \tilde{y}_n , while the prizes $\tilde{y}_1, \dots, \tilde{y}_{n-1}$ are allocated randomly and uniformly among the remaining agents. Otherwise, the allocation of prizes can be chosen arbitrarily. For sufficiently small $\epsilon > 0$ we then have, $\forall i \in I$ and $\forall e_i \in \mathbb{R}_+$,

$$\begin{aligned} \Pi_i(\tilde{e}, C_{\tilde{y}}) &= \sum_{k=1}^n \tilde{p}_i^k(\tilde{e})u(\tilde{y}_k) - c(\tilde{e}_i) \\ &= \Pi_i(\bar{e}, C_y^*) + p_i^1(\bar{e})(u(y_1 + \delta) - u(y_1)) \\ &\quad + p_i^n(\bar{e})(u(y_n - \delta) - u(y_n)) + c(\bar{e}_i) - c(\tilde{e}_i) \\ &\geq u(y_n) + p_i^1(\bar{e})(u(y_1 + \delta) - u(y_1)) \\ &\quad + p_i^n(\bar{e})(u(y_n - \delta) - u(y_n)) + c(\bar{e}_i) - c(\tilde{e}_i) \\ &\geq u(y_n - \delta) = u(\tilde{y}_n) \geq \Pi_i((e_i, \tilde{e}_{-i}), C_{\tilde{y}}), \end{aligned}$$

where the second inequality holds because

$$u(y_n) + p_i^n(\bar{e})(u(y_n - \delta) - u(y_n)) \geq u(y_n - \delta)$$

for all $i \in I$, with strict inequality for j . Hence $C_{\tilde{y}}$ implements \tilde{e} .

When studying the set of all contest solutions to (P), we thus need to consider only pure-strategy effort profiles \bar{e} and contests C_y with $y_n = 0$. Fix a sum of prizes $x \in [0, \bar{T}]$. Let e^x be the (unique) effort level that solves

$$\frac{n-1}{n}u\left(\frac{x}{n-1}\right) - c(e^x) = 0.$$

Note that, by the assumptions on u and c , the solution e^x is differentiable, strictly increasing and strictly concave in x . We now claim that ne^x is an upper bound on the sum of efforts implementable with a contest C_y that has $\sum_{k=1}^n y_k = x$ and $y_n = 0$, and it can be reached only by implementing the symmetric effort profile (e^x, \dots, e^x) . Suppose first that C_y implements an effort profile \bar{e} with $\sum_{i=1}^n \bar{e}_i \geq ne^x$ but $\bar{e} \neq (e^x, \dots, e^x)$. Note that

$$\Pi_i(\bar{e}, C_y) = \sum_{k=1}^n p_i^k(\bar{e})u(y_k) - c(\bar{e}_i) \geq u(y_n) - c(0) = 0,$$

because C_y implements \bar{e} . Summing these inequalities over all agents we obtain

$$\sum_{i=1}^n \sum_{k=1}^n p_i^k(\bar{e}) u(y_k) - \sum_{i=1}^n c(\bar{e}_i) = \sum_{k=1}^{n-1} u(y_k) - \sum_{i=1}^n c(\bar{e}_i) \geq 0.$$

However, due to weak concavity of u and strict convexity of c we also have

$$\sum_{k=1}^{n-1} u(y_k) - \sum_{i=1}^n c(\bar{e}_i) < (n-1)u\left(\frac{x}{n-1}\right) - nc(e^x) = 0,$$

a contradiction. Observe next that (e^x, \dots, e^x) can indeed be implemented. For instance, let $y = (x/(n-1), \dots, x/(n-1), 0)$ and choose the rules of C_y as follows. If the effort profile is (e^x, \dots, e^x) , then the prizes are allocated randomly and uniformly across the agents. If some agent i unilaterally deviates from (e^x, \dots, e^x) , then agent i receives the prize 0, while each other agent receives $x/(n-1)$. Otherwise, the allocation of prizes can be chosen arbitrarily. It follows immediately from the definition of e^x that this contest indeed implements (e^x, \dots, e^x) .

Given any sum of prizes x , the highest payoff that the principal can achieve is thus given by $\Pi_P(x) = ne^x - x$, and the problem is reduced to a choice of $x \in [0, \bar{T}]$. Since Π_P is continuous in x , it follows that a solution exists. Furthermore, since Π_P is differentiable and strictly concave, the first-order condition $\partial \Pi_P / \partial x = 0$ that is stated in part (i) of the theorem uniquely characterizes a value $\bar{x} > 0$ (given the assumptions on u and c), and the optimal value of x is given by $x^* = \min\{\bar{x}, \bar{T}\}$. The resulting implemented optimal effort level is then given by $e^* = e^{x^*}$.

We complete the proof of the only-if-statement by showing that any optimal contest has the profile of prizes $y = (x^*/(n-1), \dots, x^*/(n-1), 0)$ whenever u is strictly concave. By contradiction, let C_y be a contest that implements (e^*, \dots, e^*) with $\sum_{k=1}^n y_k = x^*$ and $y_n = 0$ but $y_1 \neq y_{n-1}$. Proceeding as before, summing the inequalities $\Pi_i((e^*, \dots, e^*), C_y) \geq 0$ over all agents yields $\sum_{k=1}^{n-1} u(y_k) - nc(e^*) \geq 0$. Strict concavity of u , however, implies that $\sum_{k=1}^{n-1} u(y_k) - nc(e^*) < (n-1)u(x^*/(n-1)) - nc(e^*) = 0$, a contradiction.

If-statement. We showed above that the upper bound on the principal's payoff is given by $ne^* - x^*$. Thus, any contest which implements (e^*, \dots, e^*) with the prize sum x^* attains the upper bound. ■

D.3 Proof of Corollary 4.1

Each optimal contest induces individual efforts of e^* and pays a sum of x^* , as characterized in Theorem 4.2. Now consider the principal's first-best problem. If the agents' efforts were directly observable and verifiable, then the principal could ask for individual efforts of e and would have to compensate the agents with a transfer sum x such that $u(x/n) - c(e) = 0$. Put differently, for a given transfer sum x the maximal achievable individual effort is

$$e^x = c^{-1}\left(u\left(\frac{x}{n}\right)\right),$$

and the first-best problem is to maximize $ne^x - x$ by choice of $x \in [0, \bar{T}]$. With the same arguments as in the proof of Theorem 4.2, this yields $x^{FB} = \min\{\tilde{x}, \bar{T}\}$, where \tilde{x} is given by

$$u'\left(\frac{\tilde{x}}{n}\right) = c'\left(c^{-1}\left(u\left(\frac{\tilde{x}}{n}\right)\right)\right).$$

The resulting optimal effort level is

$$e^{FB} = c^{-1}\left(u\left(\frac{x^{FB}}{n}\right)\right).$$

Now suppose that the agents are risk-neutral, i.e., the function u is linear. The conditions characterizing (e^*, x^*) in Theorem 4.2 then coincide with those characterizing (e^{FB}, x^{FB}) above, which implies $(e^*, x^*) = (e^{FB}, x^{FB})$. Then suppose that the agents are risk-averse, i.e., the function u is strictly concave. If $x^* \neq x^{FB}$ there is nothing to prove. Hence assume $x^* = x^{FB}$. Inspection of the conditions that define e^* and e^{FB} then immediately reveals that $e^* < e^{FB}$. ■

D.4 Proof of Theorem 4.3

The proof proceeds in two steps. Step 1 shows that (e^*, \dots, e^*) is an equilibrium of the contest C_y^* described in the theorem. Step 2 shows that no other equilibria exist. The structure of the arguments in Step 2 is reminiscent of equilibrium characterization proofs in all-pay auctions without censoring (see Baye et al., 1996).

Step 1. Consider deviations e'_i of agent i from (e^*, \dots, e^*) . If $e'_i > e^*$, we obtain

$$\begin{aligned} \Pi_i((e^*, \dots, e^*), C_y^*) &= \frac{n-1}{n} u\left(\frac{x^*}{n-1}\right) - c(e^*) \\ &> \frac{n-1}{n} u\left(\frac{x^*}{n-1}\right) - c(e'_i) \\ &= \Pi_i((e^*, \dots, e'_i, \dots, e^*), C_y^*). \end{aligned}$$

If $e'_i < e^*$, we obtain

$$\begin{aligned} \Pi_i((e^*, \dots, e^*), C_y^*) &= \frac{n-1}{n} u\left(\frac{x^*}{n-1}\right) - c(e^*) \\ &= 0 \geq -c(e'_i) = \Pi_i((e^*, \dots, e'_i, \dots, e^*), C_y^*). \end{aligned}$$

Thus, the contest C_y^* implements the effort profile (e^*, \dots, e^*) .

Step 2. By contradiction, suppose C_y^* also implements some other profile $\sigma \neq (e^*, \dots, e^*)$. Denote the support of σ_i by L_i , so $e_i \in L_i$ if and only if every open neighbourhood N of e_i satisfies $\sigma_i(N) > 0$. We first show that it must be that $L_i \subseteq [0, e^*]$ for all $i \in I$. Suppose not, so there exists an agent i and an effort level $e_i > e^*$ such that $\sigma_i((e_i - \epsilon, e_i + \epsilon)) > 0 \quad \forall \epsilon > 0$. Fix $\bar{\epsilon} > 0$ such that $e_i - \bar{\epsilon} > e^*$. Note that the expected payoff of agent i playing $e'_i \geq e^*$ with

probability one, while the other agents play σ_{-i} , is

$$\Pi_i(e'_i, \sigma_{-i}) = \left[1 - \prod_{j \neq i} \sigma_j([e^*, \infty)) + \prod_{j \neq i} \sigma_j([e^*, \infty)) \frac{n-1}{n} \right] u\left(\frac{x^*}{n-1}\right) - c(e'_i).$$

where we omit the dependence on C_y^* to simplify notation. Since c is strictly increasing, we have $\Pi_i(e^*, \sigma_{-i}) > \Pi_i(e'_i, \sigma_{-i})$ for all $e'_i > e^*$. Hence $\Pi_i(e^*, \sigma_{-i}) > \Pi_i(\bar{e}_i, \sigma_{-i})$ for all $\bar{e}_i \in (e_i - \bar{\epsilon}, e_i + \bar{\epsilon})$. Since $\sigma_i((e_i - \bar{\epsilon}, e_i + \bar{\epsilon})) > 0$, agent i could strictly increase his expected payoff by shifting the mass from this interval to e^* . Thus, σ is not an equilibrium. From now on, we only consider the cases where $L_i \subseteq [0, e^*] \forall i \in I$. Let $\underline{e}_i = \min L_i$. Since the proposed profile σ is different from (e^*, \dots, e^*) , it must be that $\underline{e} = \min_{i \in I} \underline{e}_i < e^*$.

First, suppose that $\underline{e} > 0$. Furthermore suppose that $\sigma_j(\{\underline{e}\}) > 0$ for exactly one agent $j \in I$, or that $\sigma_i(\{\underline{e}\}) = 0$ for all $i \in I$. In the latter case let j be such that $\underline{e}_j = \underline{e}$. Then there exists some $\bar{\epsilon} > 0$ such that

$$\Pi_j(\underline{e} + \epsilon, \sigma_{-j}) \leq \left[1 - \prod_{i \neq j} \sigma_i((\underline{e} + \epsilon, \infty)) \right] u\left(\frac{x^*}{n-1}\right) - c(\underline{e} + \epsilon) < 0$$

for all $\epsilon < \bar{\epsilon}$. Intuitively, the probability that agent j wins a positive prize approaches zero as ϵ approaches zero (by right continuity of $\sigma_i((\underline{e} + \epsilon, \infty))$ in ϵ and $\sigma_i((\underline{e}, \infty)) = 1$), while the cost of effort at \underline{e} is strictly positive. Hence agent j could strictly increase his expected payoff by shifting the mass $\sigma_j([\underline{e}, \underline{e} + \bar{\epsilon})) > 0$ from $[\underline{e}, \underline{e} + \bar{\epsilon})$ to 0. Next suppose that $\sigma_i(\{\underline{e}\}) > 0$ for at least two agents $i = j, k$. Then there exists a small $\epsilon > 0$ such that

$$\begin{aligned} \Pi_j(\underline{e}, \sigma_{-j}) &\leq \left[1 - \left(1 - \frac{1}{2} \sigma_k(\{\underline{e}\}) \right) \prod_{i \neq j, k} \sigma_i((\underline{e}, \infty)) \right] u\left(\frac{x^*}{n-1}\right) - c(\underline{e}) \\ &< \left[1 - \prod_{i \neq j} \sigma_i((\underline{e}, \infty)) \right] u\left(\frac{x^*}{n-1}\right) - c(\underline{e} + \epsilon) \\ &\leq \Pi_j(\underline{e} + \epsilon, \sigma_{-j}). \end{aligned}$$

The intuition is that a small upward deviation from \underline{e} increases the probability of winning discretely, while marginally increasing the effort costs. Hence agent j could strictly increase his expected payoff by shifting the mass $\sigma_j(\{\underline{e}\}) > 0$ from \underline{e} to $\underline{e} + \epsilon$. We conclude that there does not exist an equilibrium $\sigma \neq (e^*, \dots, e^*)$ with $\underline{e} > 0$.

Second, suppose that $\underline{e} = 0$. Consider first the case where $\sigma_i(\{0\}) = 0$ for all $i \in I$, that is, no agent places an atom on 0. If there is an agent j such that $\underline{e}_k > 0$ for all $k \neq j$, then there exists some $\bar{\epsilon} > 0$ such that $\Pi_j(\epsilon, \sigma_{-j}) = -c(\epsilon)$ for all $\epsilon < \bar{\epsilon}$. Agent j could then strictly increase his expected payoff by shifting the mass $\sigma_j((0, \bar{\epsilon})) > 0$ from $(0, \bar{\epsilon})$ to 0. Thus, there have to be at least two agents j and k with $\underline{e}_j = \underline{e}_k = 0$. But in this case, observe that

$$\Pi_j(\epsilon, \sigma_{-j}) \leq \left[\left(1 - \prod_{i \neq j} \sigma_i((\epsilon, \infty)) \right) u\left(\frac{x^*}{n-1}\right) - c(\epsilon) \right]$$

and

$$\lim_{\epsilon \rightarrow 0} \left[\left(1 - \prod_{i \neq j} \sigma_i((\epsilon, \infty)) \right) u\left(\frac{x^*}{n-1}\right) - c(\epsilon) \right] = 0.$$

Thus for every $\bar{\Pi} > 0$ there exists $\bar{\epsilon} > 0$ such that $\Pi_j(\epsilon, \sigma_{-j}) < \bar{\Pi}$ for all $\epsilon < \bar{\epsilon}$. Intuitively, both the probability of winning and the costs approach zero as $\epsilon \rightarrow 0$. However, it must be that $\Pi_j(e^*, \sigma_{-j}) > 0$ since $\Pi_j(e^*, \dots, e^*) = 0$ and the probability that j wins a positive prize is strictly greater if the other agents play σ_{-j} , because at least agent k exerts efforts lower than e^* with strictly positive probability. Hence agent j could strictly increase his expected payoff by shifting the mass $\sigma_j((0, \bar{\epsilon})) > 0$ from $(0, \bar{\epsilon})$ to e^* , for some sufficiently small $\bar{\epsilon} > 0$. The only remaining case is $\sigma_j(\{0\}) > 0$ for at least one agent $j \in I$. Observe that there can only be one such agent, since otherwise a small upward deviation from 0 would lead to a discrete increase in the probability of winning a positive prize, analogous to the argument above. Then it must be that $\Pi_j(\sigma_j, \sigma_{-j}) = 0$ since $\Pi_j(0, \sigma_{-j}) = 0$. This can only be the maximum payoff of agent j if all other agents exert deterministic efforts equal to e^* , since otherwise $\Pi_j(e^*, \sigma_{-j}) > 0$. In this case, agent j is indifferent between playing 0 or e^* , and all other effort levels yield strictly lower payoffs. This implies $\sigma_j(\{0\}) + \sigma_j(\{e^*\}) = 1$. Now consider an agent $k \neq j$. Observe that a deviation by agent k to some ϵ with $0 < \epsilon < e^*$ leads to payoffs

$$\Pi_k(\epsilon, \sigma_{-k}) = \sigma_j(\{0\})u\left(\frac{x^*}{n-1}\right) - c(\epsilon).$$

Thus a sufficiently small $\epsilon > 0$ will be a profitable deviation whenever

$$\sigma_j(\{0\})u\left(\frac{x^*}{n-1}\right) > \sigma_j(\{0\})u\left(\frac{x^*}{n-1}\right) + (1 - \sigma_j(\{0\}))\frac{n-1}{n}u\left(\frac{x^*}{n-1}\right) - c(e^*).$$

This can be reformulated to

$$0 > -\sigma_j(\{0\})\frac{n-1}{n}u\left(\frac{x^*}{n-1}\right),$$

which always holds because $\sigma_j(\{0\}) > 0$. ■

D.5 Proof of Theorem 4.4

Consider a nested contest with prize profile $y = (x^*/(n-1), \dots, x^*/(n-1), 0)$ and the general success function (4.1). We will show that, for an appropriate choice of f , the effort profile (e^*, \dots, e^*) is an equilibrium. The proof proceeds in three steps. In Step 1, we derive the agents' payoff function in the nested contest. Step 2 introduces the specific value $r^*(n)$ stated in the theorem. In Step 3, we then complete the proof that the resulting contest indeed implements the desired effort profile.

Step 1. Let $p(e_i)$ denote the probability that agent i wins none of the $n-1$ positive prizes, given that all other agents exert effort e^* . Furthermore, let u^* be the utility derived from a

positive prize. Then, the expected payoff of agent i , when all other agents exert e^* , is given by

$$\begin{aligned}\Pi_i(e_i) &= [1 - p(e_i)] u^* - c(e_i) \\ &= \left[1 - \frac{(n-1)! f(e^*)^{n-1}}{\prod_{k=1}^{n-1} [f(e_i) + (n-k)f(e^*)]} \right] u^* - c(e_i) \\ &= \left[1 - \prod_{k=1}^{n-1} \frac{(n-k)f(e^*)}{[f(e_i) + (n-k)f(e^*)]} \right] u^* - c(e_i) \\ &= \left[1 - \prod_{k=1}^{n-1} \frac{(n-k)f(e^*)}{[f(e_i) + (n-k)f(e^*)]} \right] \left(\frac{n}{n-1} \right) c(e^*) - c(e_i).\end{aligned}$$

Now suppose $f(e_i) = c(e_i)^r$ for some $r \geq 0$. It is easy to see that $\Pi_i(0) = \Pi_i(e^*) = 0$ for any r . We will show in the next two steps that $\Pi_i(e_i) \leq 0$ for all e_i when $r = r^*(n) = (n-1)/(H_n - 1)$, where $H_n = \sum_{k=1}^n 1/k$ is the n -th harmonic number. This implies that (e^*, \dots, e^*) is an equilibrium.

Step 2. Consider any $e_i > 0$ (we already know the value of Π_i for $e_i = 0$). To determine the sign of $\Pi_i(e_i)$, we can equivalently examine the sign of

$$\Pi_i(e_i) \left[\frac{n-1}{nc(e^*)} \right] = \left[1 - \prod_{k=1}^{n-1} \frac{(n-k)c(e^*)^r}{[c(e_i)^r + (n-k)c(e^*)^r]} \right] - \left(\frac{n-1}{n} \right) \frac{c(e_i)}{c(e^*)}.$$

Make the change of variables $y^* = c(e^*)^r$ and $y = c(e_i)^r$ to obtain

$$F(y|r) = \left[1 - \prod_{k=1}^{n-1} \frac{(n-k)y^*}{[y + (n-k)y^*]} \right] - \frac{n-1}{n} \left(\frac{y}{y^*} \right)^{\frac{1}{r}}.$$

After the additional variable substitution $x = y^*/y$ we obtain

$$F(x|r) = \left[1 - \prod_{k=1}^{n-1} \frac{(n-k)x}{[1 + (n-k)x]} \right] - \frac{n-1}{n} \left(\frac{1}{x} \right)^{\frac{1}{r}}.$$

Showing that $F(x|r) \leq 0$ for all $x > 0$, $x \neq 1$, is then sufficient to ensure that the contest with parameter r implements the optimum.

Fix any x and let us look for $r(x)$ such that $F(x|r(x)) = 0$. Since F is strictly increasing in r whenever $x \in (0, 1)$, we obtain that $F(x|r) \leq 0$ for any fixed $x \in (0, 1)$ whenever $r \leq r(x)$, so $r(x)$ gives an upper bound on the possible values of r . Similarly, since F is strictly decreasing in r whenever $x \in (1, \infty)$, we obtain that $F(x|r) \leq 0$ for any fixed $x \in (1, \infty)$ whenever $r \geq r(x)$, so $r(x)$ gives a lower bound on the possible values of r . Thus it is sufficient to find a value r^* such that $r(x) \geq r^*$ for all $x \in (0, 1)$ and $r(x) \leq r^*$ for all $x \in (1, \infty)$.

Rewriting the equation $F(x|r(x)) = 0$, we have

$$\left[1 - \prod_{k=1}^{n-1} \frac{(n-k)x}{[1 + (n-k)x]} \right] = \frac{n-1}{n} \left(\frac{1}{x} \right)^{\frac{1}{r(x)}}$$

$$\begin{aligned}
\log \left[1 - \prod_{k=1}^{n-1} \frac{(n-k)x}{[1 + (n-k)x]} \right] &= \log \left(\frac{n-1}{n} \right) - \frac{1}{r(x)} \log(x) \\
\frac{1}{r(x)} \log(x) &= \log \left(\frac{n-1}{n} \right) - \log \left[1 - \frac{(n-1)!x^{n-1}}{\prod_{k=1}^{n-1} [1 + (n-k)x]} \right] \\
\frac{1}{r(x)} \log(x) &= \log \left[\frac{n-1}{n} \frac{\prod_{k=1}^{n-1} [1 + (n-k)x]}{\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1}} \right] \\
r(x) &= \frac{\log(x)}{\log \left[\frac{n-1}{n} \frac{\prod_{k=1}^{n-1} [1 + (n-k)x]}{\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1}} \right]}.
\end{aligned}$$

Denote

$$g(x) = \frac{n-1}{n} \frac{\prod_{k=1}^{n-1} [1 + (n-k)x]}{\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1}}$$

so that

$$r(x) = \frac{\log(x)}{\log(g(x))}.$$

Note that $g(x) > 0$ for any $x > 0$. We will first show that $\lim_{x \nearrow 1} r(x) = \lim_{x \searrow 1} r(x) = r^*(n) = (n-1)/(H_n - 1)$. Note that for $x = 1$ both the denominator and the numerator of $r(x)$ equal zero. Hence we use l'Hôpital's rule. Observe that

$$\begin{aligned}
(\log(g(x)))' &= \frac{g'(x)}{g(x)} \\
&= \frac{\left(\frac{\partial}{\partial x} \prod_{k=1}^{n-1} [1 + (n-k)x] \right) \left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right)}{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right) \prod_{k=1}^{n-1} [1 + (n-k)x]} \\
&\quad - \frac{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] \right) \frac{\partial}{\partial x} \left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right)}{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right) \prod_{k=1}^{n-1} [1 + (n-k)x]} \\
&= \frac{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] \right) \frac{\partial}{\partial x} ((n-1)!x^{n-1})}{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right) \prod_{k=1}^{n-1} [1 + (n-k)x]} \\
&\quad - \frac{((n-1)!x^{n-1}) \left(\frac{\partial}{\partial x} \prod_{k=1}^{n-1} [1 + (n-k)x] \right)}{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right) \prod_{k=1}^{n-1} [1 + (n-k)x]} \\
&= \frac{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] \right) (n-1) ((n-1)!x^{n-2})}{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right) \prod_{k=1}^{n-1} [1 + (n-k)x]} \\
&\quad - \frac{((n-1)!x^{n-1}) \left(\sum_{k=1}^{n-1} (n-k) \prod_{j \neq k} [1 + (n-j)x] \right)}{\left(\prod_{k=1}^{n-1} [1 + (n-k)x] - (n-1)!x^{n-1} \right) \prod_{k=1}^{n-1} [1 + (n-k)x]}.
\end{aligned}$$

We evaluate this at $x = 1$, that is,

$$\begin{aligned}
(\log(g(x)))'|_{x=1} &= \frac{\left(\prod_{k=1}^{n-1} [1 + (n - k)]\right) (n - 1)(n - 1)!}{\left(\prod_{k=1}^{n-1} [1 + (n - k)] - (n - 1)!\right) \prod_{k=1}^{n-1} [1 + (n - k)]} \\
&\quad - \frac{(n - 1)! \left(\sum_{k=1}^{n-1} (n - k) \prod_{j \neq k} [1 + (n - j)]\right)}{\left(\prod_{k=1}^{n-1} [1 + (n - k)] - (n - 1)!\right) \prod_{k=1}^{n-1} [1 + (n - k)]} \\
&= \frac{n!(n - 1)(n - 1)!}{(n! - (n - 1)!) n!} \\
&\quad - \frac{(n - 1)! \left(\sum_{k=1}^{n-1} (n - k) \prod_{j \neq k} [1 + (n - j)]\right)}{(n! - (n - 1)!) n!} \\
&= 1 - \frac{\left(\sum_{k=1}^{n-1} (n - k) \prod_{j \neq k} [1 + (n - j)]\right)}{(n - 1) n!} \\
&= 1 - \frac{n! \left(\sum_{k=1}^{n-1} \frac{n - k}{n - k + 1}\right)}{(n - 1) n!} \\
&= \frac{n - 1 - \left(\sum_{k=1}^{n-1} \frac{n - k}{n - k + 1}\right)}{n - 1} \\
&= \frac{1 + \sum_{k=1}^{n-1} \frac{n - k + 1}{n - k + 1} - \sum_{k=1}^{n-1} \frac{n - k}{n - k + 1} - 1}{n - 1} \\
&= \frac{1 + \sum_{k=1}^{n-1} \frac{1}{n - k + 1} - 1}{n - 1} \\
&= \frac{H_n - 1}{n - 1}.
\end{aligned}$$

Thus we have

$$\lim_{x \nearrow 1} r(x) = \lim_{x \searrow 1} r(x) = \frac{1/x}{(\log(g(x)))'|_{x=1}} = \frac{n - 1}{H_n - 1}.$$

To complete the proof of the theorem, it is now sufficient to show that $r(x)$ is weakly monotonically decreasing on $(0, 1)$ and on $(1, \infty)$. We will do this in the next step.

Step 3. To show monotonicity of $r(x)$, we will apply a suitable version of the l'Hôpital montone rule. Proposition 1.1 in Pinelis (2002) (together with Corollary 1.2 and Remark 1.3) implies that $r(x) = \log(x)/\log(g(x))$ is weakly decreasing on $(0, 1)$ and $(1, \infty)$ if

$$\frac{(\log(x))'}{(\log(g(x)))'} = \frac{g(x)}{xg'(x)}$$

is weakly decreasing.² We will thus show that

$$\left(\frac{g(x)}{xg'(x)}\right)' = \frac{[g'(x)x - g(x)]g'(x) - xg(x)g''(x)}{(xg'(x))^2} \leq 0.$$

²Proposition 1.1 is applicable because $\log(x)$ and $\log(g(x))$ are differentiable on the respective intervals and $\lim_{x \rightarrow 1} \log(x) = \lim_{x \rightarrow 1} \log(g(x)) = 0$ holds. The remaining prerequisite $(\log(g(x)))' = g'(x)/g(x) > 0$ also holds, because $g(x) > 0$ and $g'(x) > 0$ according to Lemma D.1 below.

For this, it is sufficient to show the following three conditions:

- (a) $g'(x) > 0$,
- (b) $g''(x) \geq 0$,
- (c) $g'(x)x - g(x) \leq 0$.

We will verify these conditions in the following three lemmas. To do this, consider the function g . We can write

$$\begin{aligned} \prod_{k=1}^{n-1} [1 + (n-k)x] &= (n-1)!x^{n-1} + a_{n-2}x^{n-2} + a_{n-3}x^{n-3} + \dots + a_1x + 1 \\ &= (n-1)!x^{n-1} + \gamma(x), \end{aligned}$$

where a_1, \dots, a_{n-2} are strictly positive coefficients (that depend on n), so that γ is a polynomial of degree $n-2$ which is strictly positive for all $x > 0$.³ We can then rewrite

$$g(x) = \frac{n-1}{n} \frac{(n-1)!x^{n-1} + \gamma(x)}{\gamma(x)}.$$

Lemma D.1. *Condition $g'(x) > 0$ is satisfied.*

Proof. Observe that

$$\begin{aligned} g'(x) &= \frac{n-1}{n} \frac{(n-1)(n-1)!x^{n-2}\gamma(x) - (n-1)!x^{n-1}\gamma'(x)}{\gamma(x)^2} \\ &= \frac{n-1}{n} \frac{(n-1)!x^{n-2}[(n-1)\gamma(x) - x\gamma'(x)]}{\gamma(x)^2}, \end{aligned}$$

and, since

$$\begin{aligned} (n-1)\gamma(x) &= (n-1)a_{n-2}x^{n-2} + (n-1)a_{n-3}x^{n-3} + \dots + (n-1)a_1x + n-1 \text{ and} \\ x\gamma'(x) &= (n-2)a_{n-2}x^{n-2} + (n-3)a_{n-3}x^{n-3} + \dots + a_1x, \end{aligned}$$

it follows that $(n-1)\gamma(x) - x\gamma'(x) > 0$, which implies that $g'(x) > 0$. □

Lemma D.2. *Condition $g''(x) \geq 0$ is satisfied.*

Proof. Observe that

$$g''(x) = \frac{(n-1)(n-1)!}{n} \left[\frac{(n-1)x^{n-2}\gamma(x) - x^{n-1}\gamma'(x)}{\gamma(x)^2} \right]',$$

so that $g''(x) \geq 0$ is equivalent to

³To avoid confusion, the formula should be read as $\gamma(x) = 1$ for $n = 2$ and as $\gamma(x) = a_1x$ for $n = 3$.

$$\begin{aligned}
0 &\leq \left[\frac{(n-1)x^{n-2}\gamma(x) - x^{n-1}\gamma'(x)}{\gamma(x)^2} \right]' \\
&= \frac{[(n-2)(n-1)x^{n-3}\gamma(x) + (n-1)x^{n-2}\gamma'(x) - (n-1)x^{n-2}\gamma'(x) - x^{n-1}\gamma''(x)]\gamma(x)^2}{\gamma(x)^4} \\
&\quad - \frac{[(n-1)x^{n-2}\gamma(x) - x^{n-1}\gamma'(x)]2\gamma(x)\gamma'(x)}{\gamma(x)^4} \\
&= \frac{[(n-2)(n-1)x^{n-3}\gamma(x) - x^{n-1}\gamma''(x)]\gamma(x)^2}{\gamma(x)^4} \\
&\quad - \frac{[(n-1)x^{n-2}\gamma(x) - x^{n-1}\gamma'(x)]2\gamma(x)\gamma'(x)}{\gamma(x)^4} \\
&= \frac{\gamma(x)x^{n-3}}{\gamma(x)^4} \left[(n-2)(n-1)\gamma(x)^2 - x^2\gamma''(x)\gamma(x) - 2(n-1)x\gamma(x)\gamma'(x) + 2x^2\gamma'(x)^2 \right].
\end{aligned}$$

The expression in the square bracket is a polynomial of degree $(2n-4)$. We will show that all coefficients of this polynomial are positive, which implies that the polynomial, and hence also $g''(x)$, is non-negative.

Using the auxiliary definitions $a_0 = 1$ and $a_\kappa = 0$ for $\kappa < 0$, the coefficient multiplying x^{2n-j} in this polynomial, for any $4 \leq j \leq 2n$, is given by

$$\begin{aligned}
&\sum_{k=2}^{j-2} (n-2)(n-1)a_{n-k}a_{n-j+k} - \sum_{k=2}^{j-2} (n-k)(n-k-1)a_{n-k}a_{n-j+k} \\
&\quad - \sum_{k=2}^{j-2} 2(n-1)(n-k)a_{n-k}a_{n-j+k} + \sum_{k=2}^{j-2} 2(n-k)(n-j+k)a_{n-k}a_{n-j+k} \\
&= \sum_{k=2}^{j-2} (n^2 - 3n + 2)a_{n-k}a_{n-j+k} - \sum_{k=2}^{j-2} (n^2 - 2nk - n + k^2 + k)a_{n-k}a_{n-j+k} \\
&\quad - \sum_{k=2}^{j-2} 2(n^2 - nk - n + k)a_{n-k}a_{n-j+k} + \sum_{k=2}^{j-2} 2(n^2 - nj + jk - k^2)a_{n-k}a_{n-j+k} \\
&= \sum_{k=2}^{j-2} (2 + 4nk - 3k^2 - 3k - 2nj + 2jk)a_{n-k}a_{n-j+k}.
\end{aligned}$$

Let $\varphi(n, j, k) = 2 + 4nk - 3k^2 - 3k - 2nj + 2jk$. We will show that $\sum_{k=2}^{j-2} \varphi(n, j, k)a_{n-k}a_{n-j+k} \geq 0$. For $n = 2$ and $n = 3$, this condition can easily be verified directly. Hence we suppose that $n > 3$ from now on.

Observe that for any k there is $k' = j - k$ such that $a_{n-k}a_{n-j+k} = a_{n-k'}a_{n-j+k'}$. Hence we first consider the case where j is odd, so that we can write

$$\sum_{k=2}^{j-2} \varphi(n, j, k)a_{n-k}a_{n-j+k} = \sum_{k=2}^{\frac{j-1}{2}} [\varphi(n, j, k) + \varphi(n, j, j-k)]a_{n-k}a_{n-j+k}.$$

Since $\varphi(n, j, k) + \varphi(n, j, j-k)$ is an integer, we can think of this expression as a long sum where each of the terms $a_{n-k}a_{n-j+k}$ appears exactly $|\varphi(n, j, k) + \varphi(n, j, j-k)|$ times, added or subtracted depending on the sign of $\varphi(n, j, k) + \varphi(n, j, j-k)$. Now note that $\sum_{k=2}^{(j-1)/2} [\varphi(n, j, k) + \varphi(n, j, j-k)]$

$\varphi(n, j, j - k) = 0$ holds. This follows because we can write

$$\begin{aligned}
& \sum_{k=2}^{\frac{j-1}{2}} [\varphi(n, j, k) + \varphi(n, j, j - k)] \\
&= \sum_{k=2}^{j-2} \varphi(n, j, k) \\
&= \sum_{k=2}^{j-2} (2 - 2nj) + (4n - 3 + 2j) \sum_{k=2}^{j-2} k - 3 \sum_{k=2}^{j-2} k^2 \\
&= (j - 3)(2 - 2nj) + (4n - 3 + 2j) \frac{j(j-3)}{2} - 3 \frac{(j-3)(2j^2 - 3j + 4)}{6} \\
&= (j - 3) \left(2 - 2nj + 2nj - \frac{3j}{2} + j^2 - j^2 + \frac{3j}{2} - 2 \right) \\
&= 0.
\end{aligned}$$

Thus, for each instance where a term $a_{n-k'}a_{n-j+k'}$ is subtracted in the long sum, we can find a term $a_{n-k''}a_{n-j+k''}$ which is added. We claim that the respective terms which are added are weakly larger than the terms which are subtracted. This claim follows once we show that both $\varphi(n, j, k) + \varphi(n, j, j - k)$ and $a_{n-k}a_{n-j+k}$ are weakly increasing in k within the range of the sum. In that case, the terms which are subtracted are those for small k and the terms which are added are those for large k , and the latter are weakly larger. The same argument in fact applies when j is even, so that we can write

$$\begin{aligned}
& \sum_{k=2}^{j-2} \varphi(n, j, k) a_{n-k} a_{n-j+k} \\
&= \sum_{k=2}^{\frac{j-2}{2}} [\varphi(n, j, k) + \varphi(n, j, j - k)] a_{n-k} a_{n-j+k} + \varphi(n, j, j/2) a_{n-j/2}^2.
\end{aligned}$$

Importantly, for the last term we have

$$\begin{aligned}
\varphi(n, j, j/2) &= 2 - 2nj - 3 \left(\frac{j}{2} \right)^2 + \frac{j}{2} (4n - 3 + 2j) \\
&= 2 - j^2 \frac{3}{4} - j \frac{3}{2} + j^2 \\
&= 2 + j \left(\frac{j}{4} - \frac{3}{2} \right) \\
&> 0,
\end{aligned}$$

so that the last and largest term $a_{n-j/2}^2 = a_{n-j/2} a_{n-j/2}$ is indeed also added.

We first show that $\varphi(n, j, k) + \varphi(n, j, j - k)$ is weakly increasing in k in the relevant range. We have

$$\begin{aligned}
& \varphi(n, j, k) + \varphi(n, j, j - k) \\
&= (2 - 2nj - 3k^2 + k(4n - 3 + 2j)) + (2 - 2nj - 3(j - k)^2 + (j - k)(4n - 3 + 2j)) \\
&= 4 - 4nj - 3(2k^2 + j^2 - 2jk) + j(4n - 3 + 2j).
\end{aligned}$$

Treating k as a real variable, we obtain

$$\begin{aligned}\frac{\partial}{\partial k} [\varphi(n, j, k) + \varphi(n, j, j - k)] &= -3(4k - 2j) \\ &= -6(2k - j) > 0\end{aligned}$$

for all $k < j/2$, so the claim follows.

We now show that $a_{n-k}a_{n-j+k}$ is weakly increasing in k in the relevant range. Formally, we show that $a_{n-k}a_{n-j+k} \leq a_{n-k-1}a_{n-j+k+1}$ for any $k < j/2$. Observe that we can write

$$\begin{aligned}a_1 &= \sum_{k_1=1}^{n-1} (n - k_1), \\ a_2 &= \sum_{k_2=1}^{n-2} \sum_{k_1=k_2+1}^{n-1} (n - k_2)(n - k_1), \\ &\vdots \\ a_j &= \sum_{k_j=1}^{n-j} \sum_{k_{j-1}=k_j+1}^{n-j+1} \cdots \sum_{k_1=k_2+1}^{n-1} (n - k_j)(n - k_{j-1}) \cdots (n - k_1).\end{aligned}$$

Intuitively, each summand in the definition of a_j is the product of j different elements chosen from the set $\{(n-1), (n-2), \dots, 1\}$, and the nested summation goes over all the different possibilities in which these j elements can be chosen. Using simplified notation for the nested summation, we can thus write (where α , β , λ , and η take the role of the indices of summation, like k in the expression above):

$$\begin{aligned}a_{n-k} &= \sum (n - \alpha_{n-k})(n - \alpha_{n-k-1}) \cdots (n - \alpha_1), \\ a_{n-j+k} &= \sum (n - \beta_{n-j+k})(n - \beta_{n-j+k-1}) \cdots (n - \beta_1), \\ a_{n-k-1} &= \sum (n - \lambda_{n-k-1})(n - \lambda_{n-k-2}) \cdots (n - \lambda_1), \\ a_{n-j+k+1} &= \sum (n - \eta_{n-j+k+1})(n - \eta_{n-j+k}) \cdots (n - \eta_1).\end{aligned}$$

Rewriting the inequality $a_{n-k}a_{n-j+k} \leq a_{n-k-1}a_{n-j+k+1}$ using this notation, we obtain

$$\begin{aligned}&\sum (n - \alpha_{n-k})(n - \alpha_{n-k-1}) \cdots (n - \alpha_1)(n - \beta_{n-j+k})(n - \beta_{n-j+k-1}) \cdots (n - \beta_1) \\ &\leq \sum (n - \lambda_{n-k-1})(n - \lambda_{n-k-2}) \cdots (n - \lambda_1)(n - \eta_{n-j+k+1})(n - \eta_{n-j+k}) \cdots (n - \eta_1).\end{aligned}$$

Observe that each summand of the LHS sum is the product of $(n-k) + (n-j+k) = 2n-j$ elements, all of them chosen from the set $\{(n-1), (n-2), \dots, 1\}$. The first $n-k$ elements are all different from each other, and the last $n-j+k$ elements are all different from each other. Thus, since $n-k > n-j+k$ when $k < j/2$, in each summand at most $n-j+k$ elements can appear twice. Furthermore, the LHS sum goes over all the different combinations that satisfy this property. Similarly, each summand of the RHS sum is the product of $(n-k-1) + (n-j+k+1) = 2n-j$ elements, all of them chosen from the same set $\{(n-1), (n-2), \dots, 1\}$. The first $n-k-1$ elements are all different from each other, and the last $n-j+k+1$ elements are all different from

each other. Thus, (weakly) more than $n - j + k$ elements can appear twice in these summands.⁴ Since the RHS sum goes over all the different combinations that satisfy this property, for each summand on the LHS there exists an equal summand on the RHS. This shows that the inequality indeed holds. \square

Lemma D.3. *Condition $g'(x)x - g(x) \leq 0$ is satisfied.*

Proof. We have

$$g'(x)x - g(x) = \frac{n-1}{n} \left[\frac{(n-1)!x^{n-1}[(n-1)\gamma(x) - x\gamma'(x)]}{\gamma(x)^2} - \frac{(n-1)!x^{n-1} + \gamma(x)}{\gamma(x)} \right],$$

and therefore $g'(x)x - g(x) \leq 0$ if and only if

$$\begin{aligned} 0 &\geq (n-1)!x^{n-1}[(n-1)\gamma(x) - x\gamma'(x)] - (n-1)!x^{n-1}\gamma(x) - \gamma(x)^2 \\ &= (n-1)!x^{n-1}(n-2)\gamma(x) - (n-1)!x^n\gamma'(x) - \gamma(x)^2 \\ &= (n-1)![(n-2)a_{n-2}x^{2n-3} + (n-2)a_{n-3}x^{2n-4} + \cdots + (n-2)a_1x^n + (n-2)x^{n-1} \\ &\quad - (n-2)a_{n-2}x^{2n-3} - (n-3)a_{n-3}x^{2n-4} - \cdots - a_1x^n] - \gamma(x)^2 \\ &= (n-1)![a_{n-3}x^{2n-4} + 2a_{n-4}x^{2n-5} + \cdots + (n-3)a_1x^n + (n-2)x^{n-1}] - \gamma(x)^2 \\ &= (n-1)![a_{n-3}x^{2n-4} + 2a_{n-4}x^{2n-5} + \cdots + (n-3)a_1x^n + (n-2)x^{n-1}] \\ &\quad - \sum_{j=4}^{n+1} \sum_{k=2}^{j-2} a_{n-k}a_{n-j+k}x^{2n-j} - \rho, \end{aligned}$$

where $\rho \geq 0$ is some positive remainder of $\gamma(x)^2$. To show $g'(x)x - g(x) \leq 0$, it is therefore sufficient to ignore ρ and show that the overall coefficient on x^{2n-j} in the last expression is not positive. That is, it is sufficient to show that, for all $j \in \{4, \dots, n+1\}$,

$$(n-1)!(j-3)a_{n-j+1} - \sum_{k=2}^{j-2} a_{n-k}a_{n-j+k} \leq 0.$$

Observe that the sum has exactly $(j-3)$ elements. Then, it is sufficient to show that, for all $k \in \{2, \dots, j-2\}$,

$$(n-1)!a_{n-j+1} \leq a_{n-k}a_{n-j+k}. \quad (\text{D.2})$$

To demonstrate condition (D.2), we will first write the values of the coefficients a_j in a different way. Instead of summing over all possibilities in which j different elements from the set $\{(n-1), (n-2), \dots, 1\}$ can be chosen, we can sum over the $n-j-1$ elements not chosen, and divide the factorial $(n-1)!$ by the product of these elements. This yields

$$\begin{aligned} a_{n-2} &= \sum_{k_1=1}^{n-1} \frac{(n-1)!}{n-k_1}, \\ a_{n-3} &= \sum_{k_2=1}^{n-2} \sum_{k_1=k_2+1}^{n-1} \frac{(n-1)!}{(n-k_2)(n-k_1)}, \end{aligned}$$

⁴The inequality $n-k-1 \geq n-j+k+1$ can be rearranged to $k \leq j/2-1$, which follows from $k < j/2$, except if j is odd and $k = (j-1)/2$. Thus, typically, up to $n-j+k+1$ elements can appear twice. If j is odd and $k = (j-1)/2$, up to $n-k-1$ elements can appear twice, which is identical to $n-j+k$ in that case.

$$\begin{aligned}
& \vdots \\
a_{n-j} &= \sum_{k_{j-1}=1}^{n-j+1} \sum_{k_{j-2}=k_{j-1}+1}^{n-j+2} \cdots \sum_{k_1=k_2+1}^{n-1} \frac{(n-1)!}{(n-k_{j-1})(n-k_{j-2}) \cdots (n-k_1)}, \\
& \vdots \\
a_1 &= \sum_{k_{n-2}=1}^2 \sum_{k_{n-3}=k_{n-2}+1}^3 \cdots \sum_{k_1=k_2+1}^{n-1} \frac{(n-1)!}{(n-k_{n-2})(n-k_{n-3}) \cdots (n-k_1)}.
\end{aligned}$$

Rewriting condition (D.2), we then have

$$\begin{aligned}
& \sum_{\lambda_{j-2}=1}^{n-j+2} \sum_{\lambda_{j-3}=\lambda_{j-2}+1}^{n-j+3} \cdots \sum_{\lambda_1=\lambda_2+1}^{n-1} \frac{((n-1)!)^2}{(n-\lambda_{j-2})(n-\lambda_{j-3}) \cdots (n-\lambda_1)} \\
& \leq \left[\sum_{\alpha_{k-1}=1}^{n-k+1} \sum_{\alpha_{k-2}=\alpha_{k-1}+1}^{n-k+2} \cdots \sum_{\alpha_1=\alpha_2+1}^{n-1} \frac{(n-1)!}{(n-\alpha_{k-1})(n-\alpha_{k-2}) \cdots (n-\alpha_1)} \right] \\
& \times \left[\sum_{\beta_{j-k-1}=1}^{n-j+k+1} \sum_{\beta_{j-k-2}=\beta_{j-k-1}+1}^{n-j+k+2} \cdots \sum_{\beta_1=\beta_2+1}^{n-1} \frac{(n-1)!}{(n-\beta_{j-k-1})(n-\beta_{j-k-2}) \cdots (n-\beta_1)} \right].
\end{aligned}$$

Observe that for each summand on the LHS, the denominator is a product of $j-2$ different elements from the set $\{(n-1), (n-2), \dots, 1\}$. In fact, the LHS sum goes over all the different possibilities in which these $j-2$ elements can be chosen. On the RHS, after multiplication, the denominator of each summand is a product of $(k-1) + (j-k-1) = j-2$ elements from the same set, where replication of some elements may be possible (but is not necessary). Since the RHS sum goes over all these different possibilities, for each summand on the LHS there exists an equal summand on the RHS. This shows that the inequality holds. \square \blacksquare

D.6 Proof of Proposition 4.1

Observe first that $c(e^*)/c'(e^*) < e^*$ holds due to strict convexity of c and $c(0) = 0$. We can therefore write the probability that agent 1 wins the prize in the described contest, holding the effort $e_2 = e^*$ fixed, as a piecewise function

$$p(e_1) = \begin{cases} 1 & \text{if } e_1 > e^* + \frac{c(e^*)}{c'(e^*)}, \\ \frac{1}{2} + \frac{1}{2} \frac{c'(e^*)}{c(e^*)} (e_1 - e^*) & \text{if } e^* - \frac{c(e^*)}{c'(e^*)} \leq e_1 \leq e^* + \frac{c(e^*)}{c'(e^*)}, \\ 0 & \text{if } e_1 < e^* - \frac{c(e^*)}{c'(e^*)}. \end{cases}$$

Then, the expected payoff of agent 1 is given by

$$\Pi_1(e_1) = p(e_1)u^* - c(e_1) = p(e_1)2c(e^*) - c(e_1).$$

It follows that $\Pi_1(e^*) = 0$. We now consider the three types of deviations from e^* .

Case 1: $e_1 < e^* - c(e^*)/c'(e^*)$. It follows immediately that $\Pi_1(e_1) \leq 0$ in this range, which implies that these deviations are not profitable.

Case 2: $e^* - c(e^*)/c'(e^*) \leq e_1 \leq e^* + c(e^*)/c'(e^*)$. Observe that $\Pi'_1(e_1) = c'(e^*) - c'(e_1)$ in this range. Hence the first-order condition yields the unique solution $e_1 = e^*$. Since $\Pi''_1(e_1) = -c''(e_1) < 0$, this is indeed the maximum over this range.

Case 3: $e_1 > e^* + c(e^*)/c'(e^*)$. We have $\Pi_1(e_1) < \Pi_1(e^* + c(e^*)/c'(e^*))$ for this range. Hence, by the arguments for the previous case, these deviations are not profitable either.

We conclude that $e_1 = e^*$ is a best response to $e_2 = e^*$. The argument for agent 2 is symmetric, which implies that the contest implements (e^*, e^*) . ■

D.7 Proof of Proposition 4.2

Suppose that the condition $\sigma_1^2 + \sigma_2^2 - 2\sigma_{12} \leq 2/(\pi\beta^2)$ is satisfied. Consider a contest as described in the proposition. We proceed in two steps. Step 1 derives an expression for agent i 's expected payoff as a function of the effort profile e . Step 2 shows that $e_i = e^*$ is a best response when agent $j \neq i$ chooses $e_j = e^*$.

Step 1. Given an effort profile e , the probability that agent 1 wins the prize is

$$p(e) = \Pr \left[\frac{\tilde{\eta}\tilde{e}_1}{\tilde{e}_2} \geq 1 \right] = \Pr \left[\frac{\tilde{\eta}\tilde{\eta}_1 e_1}{\tilde{\eta}_2 e_2} \geq 1 \right] = \Pr \left[\frac{\tilde{\eta}_2}{\tilde{\eta}\tilde{\eta}_1} \leq \frac{e_1}{e_2} \right].$$

Since the variables $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}$ are log-normally distributed, it follows that the compound variable $\tilde{\eta}_2/(\tilde{\eta}\tilde{\eta}_1)$ is also log-normal, with location parameter $\nu = \nu_2 - \nu_1 - \nu_\eta = 0$ and scale parameter $\sigma^2 = \sigma_1^2 + \sigma_2^2 - \sigma_{12} + \sigma_\eta^2 = 2/(\pi\beta^2)$. The cdf of the log-normal distribution is given by $F(x) = \Phi((\log x - \nu)/\sigma)$, where Φ is the cdf of the standard normal distribution. Thus we can write

$$p(e) = \Phi \left(\log(e_1/e_2)\beta\sqrt{\frac{\pi}{2}} \right).$$

For the probability that agent 2 wins the prize we obtain

$$\begin{aligned} 1 - p(e) &= 1 - \Phi \left(\log(e_1/e_2)\beta\sqrt{\frac{\pi}{2}} \right) \\ &= \Phi \left(-\log(e_1/e_2)\beta\sqrt{\frac{\pi}{2}} \right) \\ &= \Phi \left(\log(e_2/e_1)\beta\sqrt{\frac{\pi}{2}} \right). \end{aligned}$$

Hence the expected payoff of agent $i = 1, 2$ is

$$\begin{aligned} \Pi_i(e) &= \Phi \left(\log(e_i/e_j)\beta\sqrt{\frac{\pi}{2}} \right) u^* - c(e_i) \\ &= \Phi \left(\log(e_i/e_j)\beta\sqrt{\frac{\pi}{2}} \right) 2\gamma e^{*\beta} - \gamma e_i^\beta. \end{aligned}$$

Step 2. Suppose $e_j = e^*$ and consider the choice of agent $i \neq j$. We immediately obtain $\Pi_i(e^*, e^*) = 0$. We will now show that $\Pi_i(e_i, e^*) \leq 0$ always holds, i.e.,

$$\Phi \left(\log(e_i/e^*)\beta\sqrt{\frac{\pi}{2}} \right) \leq \frac{1}{2} \left(\frac{e_i}{e^*} \right)^\beta$$

for all $e_i \in \mathbb{R}_+$. After the change of variables $x = \log(e_i/e^*)\beta\sqrt{\pi/2}$ this becomes the requirement that

$$\Phi(x) \leq \frac{1}{2}e^{x\sqrt{2/\pi}} \quad (\text{D.3})$$

for all $x \in \mathbb{R}$. Inequality (D.3) is satisfied for $x = 0$, where LHS and RHS both take a value of $1/2$. Furthermore, the LHS function and the RHS function are tangent at $x = 0$, because their derivatives are both equal to $1/\sqrt{2\pi}$ at this point. It then follows immediately that inequality (D.3) is also satisfied for all $x > 0$, because the LHS is strictly concave in x in this range, while the RHS is strictly convex. We now consider the remaining case where $x < 0$. We use the fact that $\Phi(x) = \text{erfc}(-x/\sqrt{2})/2$, where

$$\text{erfc}(y) = \frac{2}{\sqrt{\pi}} \int_y^\infty e^{-t^2} dt$$

is the complementary error function (see e.g. Chang et al., 2011). After the change of variables $y = -x/\sqrt{2}$ we thus need to verify

$$\text{erfc}(y) \leq e^{-2y/\sqrt{\pi}} \quad (\text{D.4})$$

for all $y > 0$. Inequality (D.4) is satisfied for $y = 0$, where LHS and RHS both take a value of 1. Now observe that the derivative of the LHS with respect to y is given by $-2e^{-y^2}/\sqrt{\pi}$, while the derivative of the RHS is $-2e^{-2y/\sqrt{\pi}}/\sqrt{\pi}$. The condition that the former is weakly smaller than the latter can be rearranged to $y \leq 2/\sqrt{\pi}$, which implies that (D.4) is satisfied for $0 < y \leq 2/\sqrt{\pi}$. For larger values of y , we can use a Chernoff bound for the complementary error function. Theorem 1 in Chang et al. (2011) implies that

$$\text{erfc}(y) \leq e^{-y^2}$$

for all $y \geq 0$. The inequality $e^{-y^2} \leq e^{-2y/\sqrt{\pi}}$ can be rearranged to $y \geq 2/\sqrt{\pi}$. This implies that (D.4) is satisfied also for $y > 2/\sqrt{\pi}$, which completes the proof. ■

Part IV

Bibliography

Bibliography

- ACEMOGLU, D. AND F. ZILIBOTTI (1997): “Was Prometheus Unbound by Chance? Risk, Diversification, and Growth,” *Journal of Political Economy*, 105, 709–751.
- AGHION, P., N. BLOOM, R. BLUNDELL, R. GRIFFITH, AND P. HOWITT (2005): “Competition and Innovation: An Inverted-U Relationship,” *The Quarterly Journal of Economics*, 120, 701–728.
- AGHION, P., C. HARRIS, P. HOWITT, AND J. VICKERS (2001): “Competition, Imitation and Growth With Step-by-step Innovation,” *The Review of Economic Studies*, 68, 467–492.
- AGHION, P. AND J. TIROLE (1997): “Formal and Real Authority in Organizations,” *Journal of Political Economy*, 105, 1–29.
- AKCIGIT, U. AND W. R. KERR (2010): “Growth through Heterogeneous Innovations,” *PIER Working Paper No. 10-035*.
- AKCIGIT, U. AND Q. LIU (2016): “The Role of Information in Innovation and Competition,” *Journal of the European Economic Association*, 14, 828–870.
- ALONSO, R. AND N. MATOUSCHEK (2008): “Optimal Delegation,” *Review of Economic Studies*, 75, 259–293.
- AMADOR, M. AND K. BAGWELL (2013): “The Theory of Optimal Delegation with an Application to Tariff Caps,” *Econometrica*, 81, 1541–1599.
- ARMSTRONG, M. AND J. VICKERS (2010): “A Model of Delegated Project Choice,” *Econometrica*, 78, 213–244.
- ARROW, K. (1962): “Economic Welfare and the Allocation of Resources for Invention,” in *The rate and direction of inventive activity: Economic and social factors*, Princeton University Press, 609–626.
- ARVE, M. AND D. MARTIMORT (2016): “Dynamic Procurement under Uncertainty: Optimal Design and Implications for Incomplete Contracts,” *American Economic Review*, 106, 3238–

3274.

- BAJARI, P. AND S. TADELIS (2001): “Incentives Versus Transaction Costs: A Theory of Procurement Contracts,” *RAND Journal of Economics*, 32, 387–407.
- BARTLING, B., E. FEHR, AND H. HERZ (2014): “The Intrinsic Value of Decision Rights,” *Econometrica*, 82, 2005–2039.
- BAYE, M. R., D. KOVENOCK, AND C. G. DE VRIES (1996): “The All-Pay Auction with Complete Information,” *Economic Theory*, 8, 291–305.
- BENKERT, J.-M. AND I. LETINA (2016): “Designing Dynamic Research Contests,” *University of Zurich, Department of Economics, Working Paper No. 235*.
- BERGER, J., C. HARBRING, AND D. SLIWKA (2013): “Performance Appraisals and the Impact of Forced Distribution - An Experimental Investigation,” *Management Science*, 59, 54–68.
- BERNARDIN, H. J., D. K. COOKE, AND P. VILLANOVA (2000): “Conscientiousness and Agreeableness as Predictors of Rating Leniency,” *Journal of Applied Psychology*, 85, 232.
- BESTER, H. AND D. KRÄHMER (2008): “Delegation and Incentives,” *RAND Journal of Economics*, 39, 664–682.
- BESTER, H. AND E. PETRAKIS (1993): “The Incentives for Cost Reduction in a Differentiated Industry,” *International Journal of Industrial Organization*, 11, 519–534.
- BHATTACHARYA, S. AND D. MOOKHERJEE (1986): “Portfolio Choice in Research and Development,” *RAND Journal of Economics*, 17, 594–605.
- BIMPIKIS, K., S. EHSANI, AND M. MOSTAGIR (2014): “Designing Dynamic Contests,” Mimeo.
- BLUME, A. (2003): “Bertrand Without Fudge,” *Economics Letters*, 78, 167–168.
- BOL, J. C. (2011): “The Determinants and Performance Effects of Managers’ Performance Evaluation Biases,” *Accounting Review*, 86, 1549–1575.
- BONATTI, A. AND H. RANTAKARI (2016): “The Politics of Compromise,” *The American Economic Review*, 106, 229–259.
- BOUDREAU, K. J., N. LACETERA, AND K. R. LAKHANI (2011): “Incentives And Problem Uncertainty in Innovation Contests: An Empirical Analysis,” *Management Science*, 57, 843–863.

- BOUDREAU, K. J., K. R. LAKHANI, AND M. MENIETTI (2016): “Performance Responses to Competition Across Skill Levels in Rank-Order Tournaments: Field Evidence and Implications for Tournament Design,” *The RAND Journal of Economics*, 47, 140–165.
- BREUER, K., P. NIEKEN, AND D. SLIWKA (2013): “Social Ties and Subjective Performance Evaluations: An Empirical Investigation,” *Review of Managerial Science*, 7, 141–157.
- CABRAL, L. (1994): “Bias in Market R&D Portfolios,” *International Journal of Industrial Organization*, 12, 533–547.
- CALLANDER, S. AND B. HARSTAD (2015): “Experimentation in Federal Systems,” *The Quarterly Journal of Economics*, 130, 951–1002.
- CAPPELEN, A. W., A. D. HOLE, E. Ø. SØRENSEN, AND B. TUNGODDEN (2007): “The Pluralism of Fairness Ideals: An Experimental Approach,” *The American Economic Review*, 97, 818–827.
- CARMICHAEL, H. L. (1983): “The Agent-Agents Problem: Payment by Relative Output,” *Journal of Labor Economics*, 50–65.
- CHANG, S.-H., P. C. COSMAN, AND L. B. MILSTEIN (2011): “Chernoff-Type Bounds for the Gaussian Error Function,” *IEEE Transactions on Communications*, 59, 2939–2944.
- CHATTERJEE, K. AND R. EVANS (2004): “Rivals’ Search for Buried Treasure: Competition and Duplication in R&D,” *RAND Journal of Economics*, 35, 160–183.
- CHE, Y.-K. AND I. GALE (2003): “Optimal Design of Research Contests,” *The American Economic Review*, 93, 646–671.
- CLARK, D. J. AND C. RIIS (1996): “A Multi-Winner Nested Rent-Seeking Contest,” *Public Choice*, 87, 177–184.
- CRAWFORD, V. P. AND J. SOBEL (1982): “Strategic Information Transmission,” *Econometrica*, 1431–1451.
- DASGUPTA, P. AND E. MASKIN (1987): “The Simple Economics of Research Portfolios,” *The Economic Journal*, 97, 581–595.
- DE VARO, J. (2006): “Internal Promotion Competitions in Firms,” *RAND Journal of Economics*, 521–542.
- DECHENAUX, E., D. KOVENOCK, AND R. M. SHEREMETA (2015): “A Survey of Experimental

- Research on Contests, All-Pay Auctions and Tournaments,” *Experimental Economics*, 18, 609–669.
- DESSEIN, W. (2002): “Authority and Communication in Organizations,” *Review of Economic Studies*, 69, 811–838.
- DING, W. AND E. G. WOLFSTETTER (2011): “Prizes and Lemons: Procurement of Innovation under Imperfect Commitment,” *RAND Journal of Economics*, 42, 664–680.
- EASTERLY, W. (2003): “Can Foreign Aid Buy Growth?” *Journal of Economic Perspectives*, 17, 23–48.
- (2009): “Can the West Save Africa?” *Journal of Economic Literature*, 47, 373–447.
- ERAT, S. AND V. KRISHNAN (2012): “Managing Delegated Search over Design Spaces,” *Management Science*, 58, 606–623.
- FARRELL, J., R. J. GILBERT, AND M. L. KATZ (2003): “Market Structure, Organizational Structure, and R&D Diversity,” in *Economics for an Imperfect World: Essays in Honor of Joseph E. Stiglitz*, ed. by R. Arnott, B. Greenwald, R. Kanbur, and B. Nalebuff, Cambridge: MIT Press.
- FEHR, E., H. HERZ, AND T. WILKENING (2013): “The Lure of Authority: Motivation and Incentive Effects of Power,” *American Economic Review*, 103, 1325–1359.
- FERSHTMAN, C. AND A. RUBINSTEIN (1997): “A Simple Model of Equilibrium in Search Procedures,” *Journal of Economic Theory*, 441, 432–441.
- FRANKEL, A. (2014): “Aligned Delegation,” *American Economic Review*, 104, 66–83.
- FU, Q. AND J. LU (2012): “The Optimal Multi-Stage Contest,” *Economic Theory*, 51, 351–382.
- FULLERTON, R. L., B. G. LINSTER, M. MCKEE, AND S. SLATE (2002): “Using Auctions to Reward Tournament Winners: Theory and Experimental Investigations,” *RAND Journal of Economics*, 33, 62–84.
- FULLERTON, R. L. AND R. P. MCAFEE (1999): “Auctioning Entry into Tournaments,” *Journal of Political Economy*, 107, 573–605.
- GAL, S., M. LANDSBERGER, AND B. LEVYKSON (1981): “A Compound Strategy for Search in the Labor Market,” *International Economic Review*, 22, 597–608.

- GANUZA, J.-J. (2007): "Competition and Cost Overruns in Procurement," *The Journal of Industrial Economics*, 55, 633–660.
- GANUZA, J.-J. AND E. HAUKE (2006): "Allocating Ideas: Horizontal Competition in Tournaments," *Journal of Economics & Management Strategy*, 15, 763–787.
- GANUZA, J.-J. AND L. PECHLIVANOS (2000): "Heterogeneity-Promoting Optimal Procurement," *Economics Letters*, 67, 105–112.
- GERSHKOV, A. AND M. PERRY (2009): "Tournaments with Midterm Reviews," *Games and Economic Behavior*, 66, 162–190.
- GIEBE, T. AND O. GÜRTLER (2012): "Optimal Contracts for Lenient Supervisors," *Journal of Economic Behavior & Organization*, 81, 403–420.
- GILBERT, R. (2006): "Looking for Mr. Schumpeter: Where Are We in the Competition-Innovation Debate?" in *Innovation Policy and the Economy*, ed. by J. Adam, J. Lerner, and S. Stern, The MIT Press, vol. 6.
- GOLTSMAN, M., J. HÖRNER, G. PAVLOV, AND F. SQUINTANI (2009): "Mediation, Arbitration and Negotiation," *Journal of Economic Theory*, 144, 1397–1420.
- GREEN, B. AND C. R. TAYLOR (forthcoming): "Breakthroughs, Deadlines and Self-Reported Progress: Contracting for Multistage Projects," *American Economic Review*.
- GREEN, J. R. AND N. L. STOKEY (1983): "A Comparison of Tournaments and Contracts," *Journal of Political Economy*, 349–364.
- GUO, Y. (2016): "Dynamic Delegation of Experimentation," *American Economic Review*, 106, 1969–2008.
- HALAC, M., N. KARTIK, AND Q. LIU (forthcoming): "Contests for Experimentations," *Journal of Political Economy*.
- HENDON, E., H. J. JACOBSEN, AND B. SLOTH (1996): "The One-Shot-Deviation Principle for Sequential Rationality," *Games and Economic Behavior*, 12, 274–282.
- HOLMSTRÖM, B. (1977): "On Incentives and Control in Organizations," Ph.D. thesis, Stanford University.
- (1984): "On the Theory of Delegation," in *Bayesian Models in Economic Theory*, ed. by M. Boyer and R. Kihlstrom, Amsterdam: Elsevier, 115–141.

- IVANOV, M. (2010): “Informational Control and Organizational Design,” *Journal of Economic Theory*, 145, 721–751.
- JIA, H., S. SKAPERDAS, AND S. VAIDYA (2013): “Contest Functions: Theoretical Foundations and Issues in Estimation,” *International Journal of Industrial Organization*, 31, 211–222.
- KAPLAN, T. R. (2012): “Communication of Preferences in Contests for Contracts,” *Economic Theory*, 51, 487–503.
- KARTIK, N. (2011): “A Note on Undominated Bertrand Equilibria,” *Economics Letters*, 111, 125–126.
- KELLNER, C. (2015): “Tournaments as A Response to Ambiguity Aversion in Incentive Contracts,” *Journal of Economic Theory*, 159, 627–655.
- KLEIN, A. H. AND A. SCHMUTZLER (forthcoming): “Optimal Effort Incentives in Dynamic Tournaments,” *Games and Economic Behavior*.
- KLETTE, T. AND D. DE MEZA (1986): “Is the Market Biased against Risky R&D?” *RAND Journal of Economics*, 17, 133–139.
- KOLOTLIN, A., H. LI, AND W. LI (2013): “Optimal Limited Authority for Principal,” *Journal of Economic Theory*, 148, 2344–2382.
- KONRAD, K. A. (2001): “Privacy and Time-Consistent Optimal Labor Income Taxation,” *Journal of Public Economics*, 79, 503–519.
- (2009): *Strategy and Dynamics in Contests*, Oxford University Press.
- (2014): “Search Duplication in Research and Design Spaces — Exploring the Role of Local Competition,” *International journal of industrial organization*, 37, 222–228.
- KOVÁČ, E. AND T. MYLOVANOV (2009): “Stochastic Mechanisms in Settings without Monetary Transfers: The Regular Case,” *Journal of Economic Theory*, 144, 1373–1395.
- KRÄHMER, D. AND E. KOVÁČ (2016): “Optimal Sequential Delegation,” *Journal of Economic Theory*, 163, 849–888.
- KRISHNA, V. AND J. MORGAN (2008): “Contracting for Information under Imperfect Commitment,” *RAND Journal of Economics*, 39, 905–925.
- KRUSE, T. AND P. STRACK (2015): “Optimal Stopping with Private Information,” *Journal of*

- Economic Theory*, 159, 702–727.
- KWON, I. (2010): “R&D Portfolio and Market Structure,” *The Economic Journal*, 120, 313–323.
- LANG, M., C. SEEL, AND P. STRACK (2014): “Deadlines in Stochastic Contests,” *Journal of Mathematical Economics*, 52, 134–142.
- LAZEAR, E. P. AND S. ROSEN (1981): “Rank-Order Tournaments as Optimum Labor Contracts,” *Journal of Political Economy*, 89, 841–864.
- LETINA, I. (2016): “The Road not Taken: Competition and the R&D Portfolio,” *RAND Journal of Economics*, 47, 433–460.
- LETINA, I. AND A. SCHMUTZLER (2015): “Designing Institutions for Diversity,” *University of Zurich, Department of Economics, Working Paper No. 200*.
- (2016): “Inducing Variety: A Theory of Innovation Contests,” *Mimeo*.
- MACLEOD, B. W. (2003): “Optimal Contracting with Subjective Evaluation,” *The American Economic Review*, 93, 216–240.
- MALCOMSON, J. M. (1984): “Work Incentives, Hierarchy, and Internal Labor Markets,” *Journal of Political Economy*, 92, 486–507.
- (1986): “Rank-Order Contracts for a Principal with Many Agents,” *Review of Economic Studies*, 53, 807–817.
- MANKIW, N. G. AND M. D. WHINSTON (1986): “Free Entry and Social Inefficiency,” *RAND Journal of Economics*, 17, 48–58.
- MELUMAD, N. D. AND T. SHIBANO (1991): “Communication in Settings with No Transfers,” *RAND Journal of Economics*, 173–198.
- MILGROM, P. AND I. SEGAL (2002): “Envelope Theorems for Arbitrary Choice Sets,” *Econometrica*, 70, 583–601.
- MOLDOVANU, B. AND A. SELA (2001): “The Optimal Allocation of Prizes in Contests,” *American Economic Review*, 542–558.
- (2006): “Contest Architecture,” *Journal of Economic Theory*, 126, 70–96.
- MORGAN, P. B. (1983): “Search and Optimal Sample Size,” *The Review of Economic Studies*, 50, 659–675.

- MORGAN, P. B. AND R. MANNING (1985): "Optimal Search," *Econometrica*, 53, 923–944.
- NALEBUFF, B. J. AND J. E. STIGLITZ (1983): "Prizes and Incentives: Towards a General Theory of Compensation and Competition," *Bell Journal of Economics*, 21–43.
- NETZER, N. AND F. SCHEUER (2010): "Competitive Markets without Commitment," *Journal of Political Economy*, 118, 1079–1109.
- OLSZEWSKI, W. AND R. SIEGEL (2016): "Effort-Maximizing Contests," *mimeo*.
- PEI, H. D. (2015a): "A Blame Shifting Theory of Delegation," *mimeo*.
- (2015b): "Communication with Endogenous Information Acquisition," *Journal of Economic Theory*, 160, 132–149.
- PÉREZ-CASTRILLO, D. AND D. WETTSTEIN (2016): "Discrimination in a Model of Contests with Incomplete Information about Ability," *International Economic Review*, 57, 881–914.
- PINELIS, I. (2002): "L'Hospital Type Rules for Monotonicity, with Applications," *Journal of Inequalities in Pure and Applied Mathematics*, 3.
- PRENDERGAST, C. (1999): "The Provision of Incentives in Firms," *Journal of Economic Literature*, 37, 7–63.
- PRENDERGAST, C. AND R. H. TOPEL (1996): "Favoritism in Organizations," *Journal of Political Economy*, 104, 958–978.
- QUIRMBACH, H. C. (1993): "R&D: Competition, Risk, and Performance," *RAND Journal of Economics*, 24, 157–197.
- RAHMAN, D. (2012): "But Who Will Monitor the Monitor?" *American Economic Review*, 102, 2767–2797.
- RAJAN, R. G. AND A. SUBRAMANIAN (2008): "Aid and Growth: What Does the Cross-Country Evidence Really Show?" *Review of Economics and Statistics*, 90, 643–665.
- REYNOLDS, S. S. AND R. M. ISAAC (1992): "Stochastic Innovation and Product Market Organization." *Economic Theory*, 2, 525–545.
- RIECK, T. (2010): "Information Disclosure in Innovation Contests," Bonn Econ Discussion Papers 16/2010, Bonn Graduate School of Economics.
- ROBINSON, C. K. (1999): "Leap-Frog and Other Forms of Innovation: Protecting the Future

- for High-Tech and Emerging Industries through Merger Enforcement.” *Address before the American Bar Association, Chicago, IL, June 10, 1999.*
- RUBINFELD, D. L. AND J. HOVEN (2001): “Innovation and Antitrust Enforcement,” in *Dynamic Competition and Public Policy: Technology, Innovation and Antitrust Issues*, ed. by J. Ellig, New York: Cambridge University Press.
- SAH, R. K. AND J. E. STIGLITZ (1987): “The Invariance of Market Innovation to the Number of Firms,” *RAND Journal of Economics*, 18, 98–108.
- SCHMIDT, K. M. (1997): “Managerial Incentives and Product Market Competition,” *The Review of Economic Studies*, 64, 191–213.
- SCHMUTZLER, A. (2013): “Competition and Investment — A Unified Approach?” *International Journal of Industrial Economics*, 31, 477–487.
- SCHÖTTNER, A. (2008): “Fixed-Prize Tournaments Versus First-Price Auctions in Innovation Contests,” *Economic Theory*, 35, 57–71.
- SEGAL, I. AND M. D. WHINSTON (2007): “Antitrust in Innovative Industries,” *The American Economic Review*, 97, 1703–1730.
- SENA, V. (2004): “The Return of the Prince of Denmark: A Survey on Recent Developments in the Economics of Innovation,” *The Economic Journal*, 114, F312–F332.
- SIEGEL, R. (2009): “All-Pay Contests,” *Econometrica*, 77, 71–92.
- SVENSSON, J. (2003): “Why Conditional Aid Does Not Work and What Can Be Done About It?” *Journal of Development Economics*, 70, 381–402.
- SZALAY, D. (2005): “The Economics of Clear Advice and Extreme Options,” *Review of Economic Studies*, 72, 1173–1198.
- TANNER, N. (2014): “Optimal Delegation Under Uncertain Bias,” *mimeo*.
- TAYLOR, C. R. (1995): “Digging for Golden Carrots: An Analysis of Research Tournaments,” *The American Economic Review*, 872–890.
- TERWIESCH, C. AND Y. XU (2008): “Innovation Contests, Open Innovation, and Multiagent Problem Solving,” *Management Science*, 54, 1529–1543.
- U.S. DEPARTMENT OF JUSTICE AND THE FEDERAL TRADE COMMISSION (2010): “Hori-

- zontal Merger Guidelines,” <http://www.justice.gov/atr/public/guidelines/hmg-2010.html>, accessed on May 19, 2015.
- VAN CAYSEELE, P. (1998): “Market Structure and Innovation: A Survey of the Last Twenty Years,” *De Economist*, 391–417.
- VIVES, X. (2008): “Innovation and Competitive Pressure,” *The Journal of Industrial Economics*, 56, 419–469.

Part V

Curriculum Vitae

Curriculum Vitae

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